A short course on Altimetry

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National Oceanography Centre
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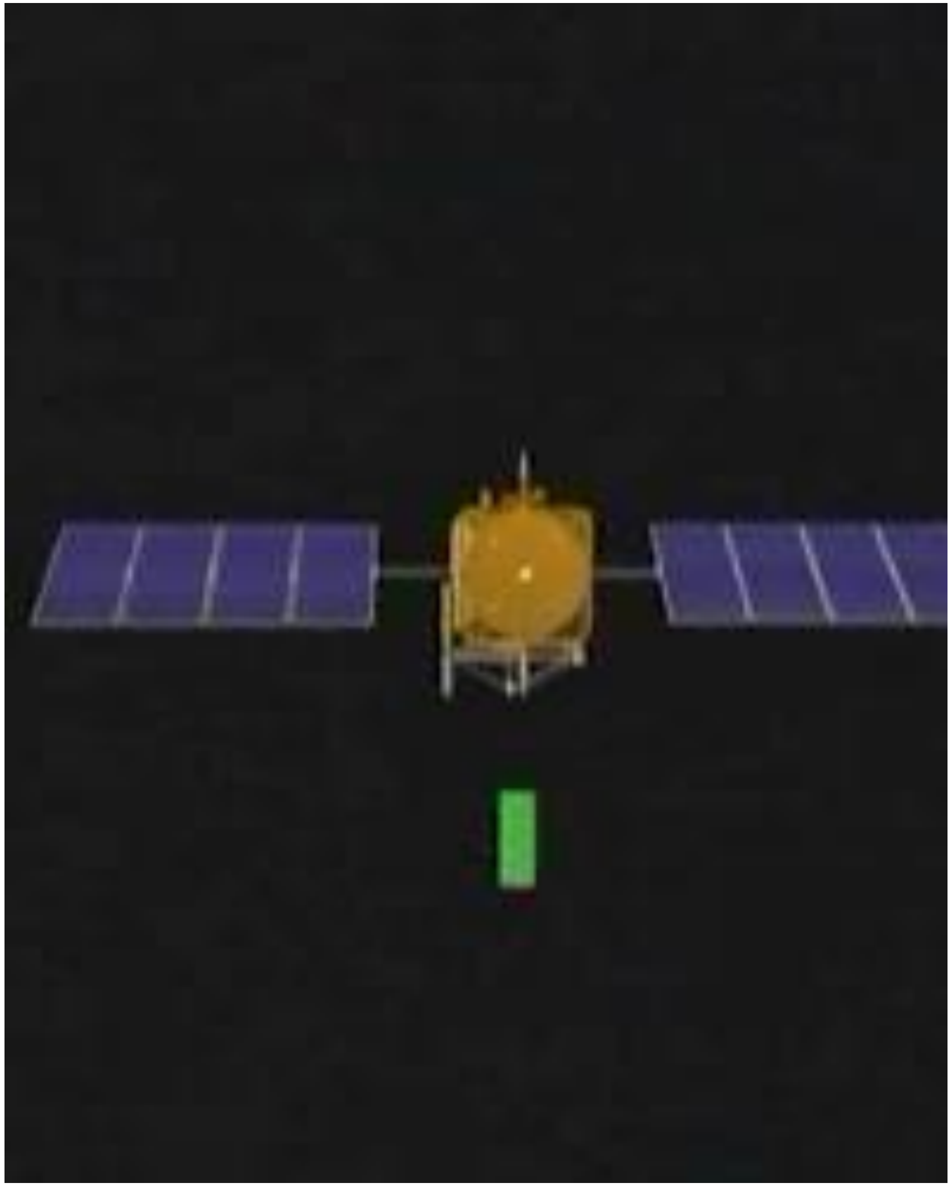
Outline

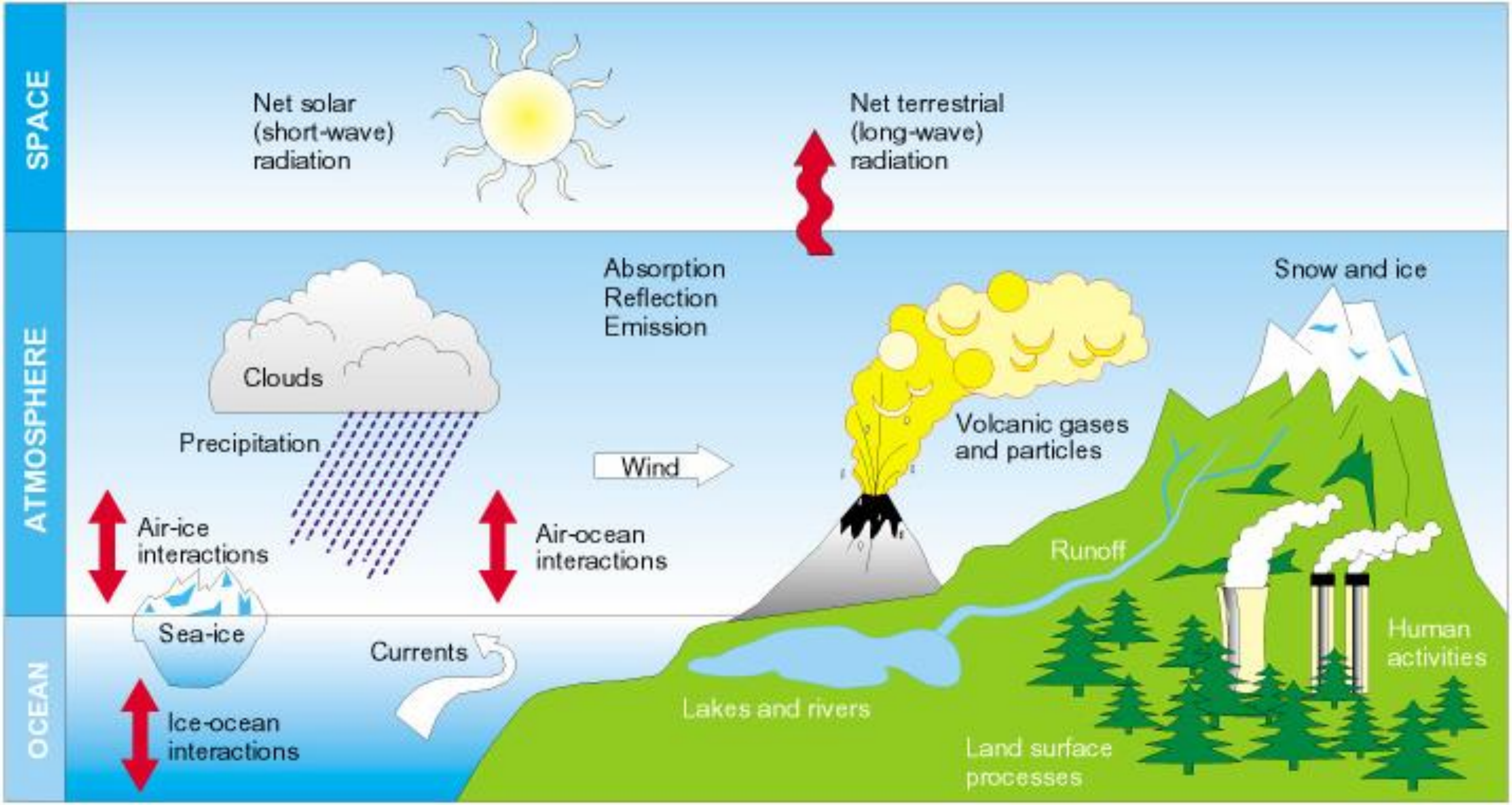
- Rationale
  - why we need altimetry
- A1 – Principles of altimetry
  - how it works in principle
  - New techniques
- A2 – Altimeter Data Processing
  - From satellite height to surface height: corrections
  - (or how it is made accurate)
- A3 – Altimetry and Oceanography
- A4 – Geophysical parameters and applications
  - what quantities we measure
  - how we use them!

Rationale for Radar Altimetry over the oceans

- Climate change
  - oceans are a very important component of the climate system
- Altimeters monitor **currents / ocean circulation**…
- …that can be used to estimate **heat** storage and transport
- … and to assess the interaction between **ocean and atmosphere**
- We also get interesting by-products: **wind/waves, rain**

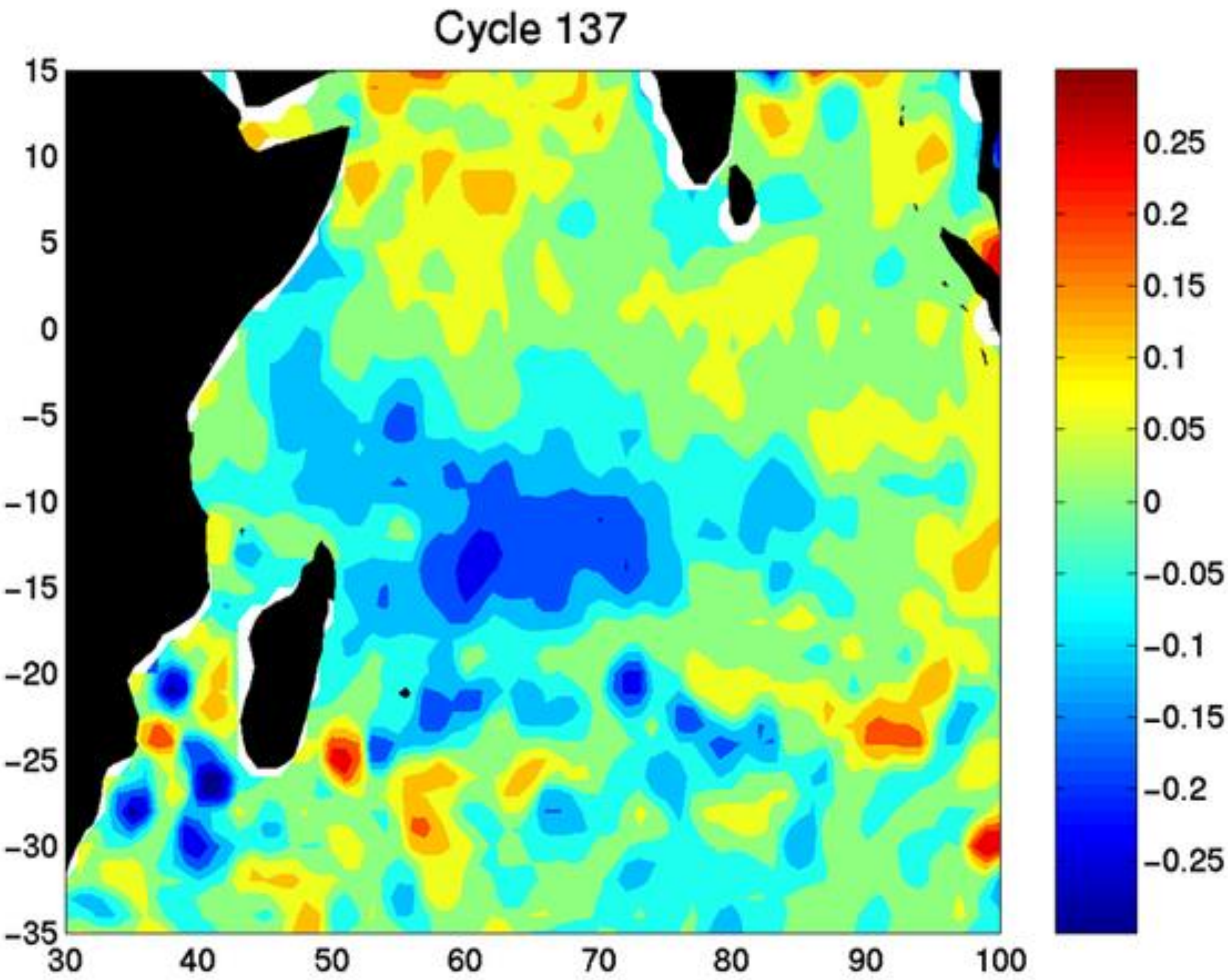


The Climate System



courtesy N. Noreiks, L. Bengtsson, MPI AV/Global/0101

The sea is not flat….



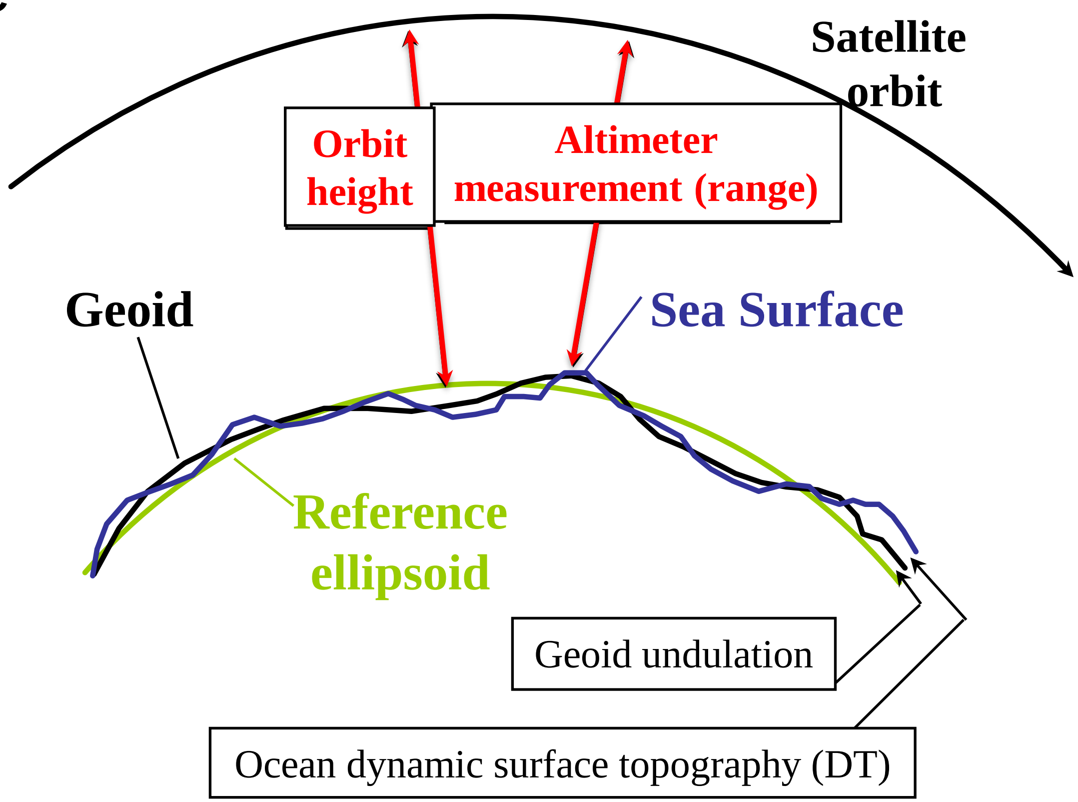
Surface dynamical features of height = **tens of cm** over lengths = **hundreds of kms**

P. Cipollini, H. Snaith – A short course on Altimetry

Altimetry 1 – principles & instruments

Basic Principles

- The altimeter is a radar at vertical incidence
- The signal returning to the satellite is from quasi-specular reflection
- Measure distance between satellite and sea (**range**)
- Determine position of satellite (precise **orbit**)
- Hence determine **height** of sea surface
- Oceanographers require height relative to **geoid**

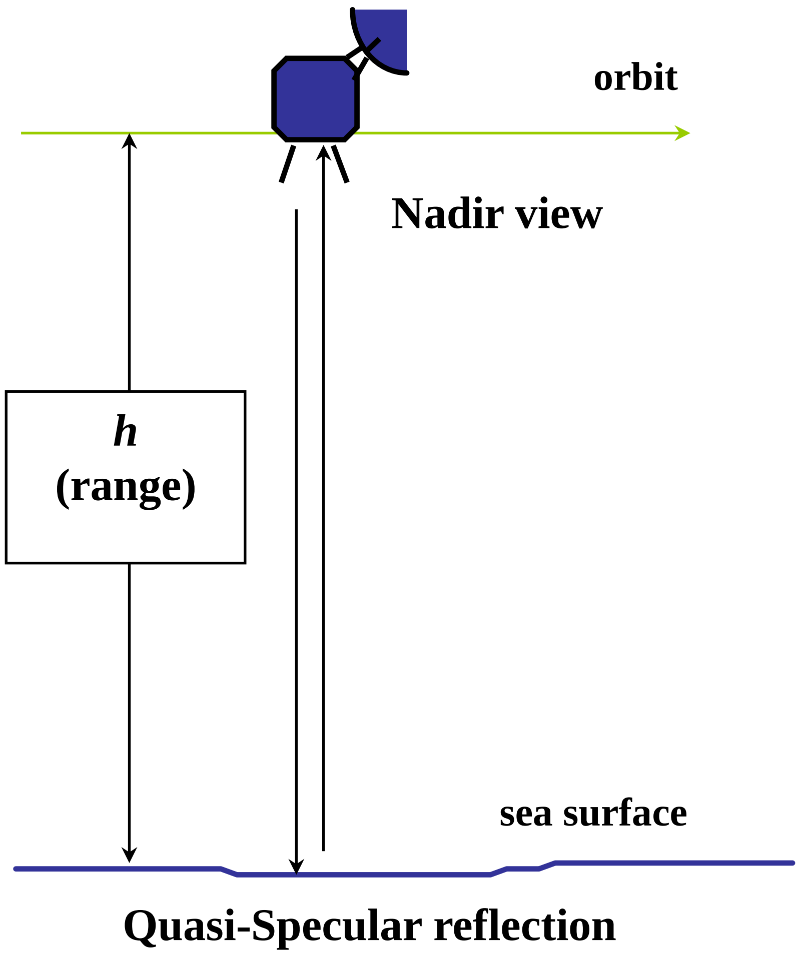


DT= Orbit - Range - Geoid

Measuring ocean topography with radar

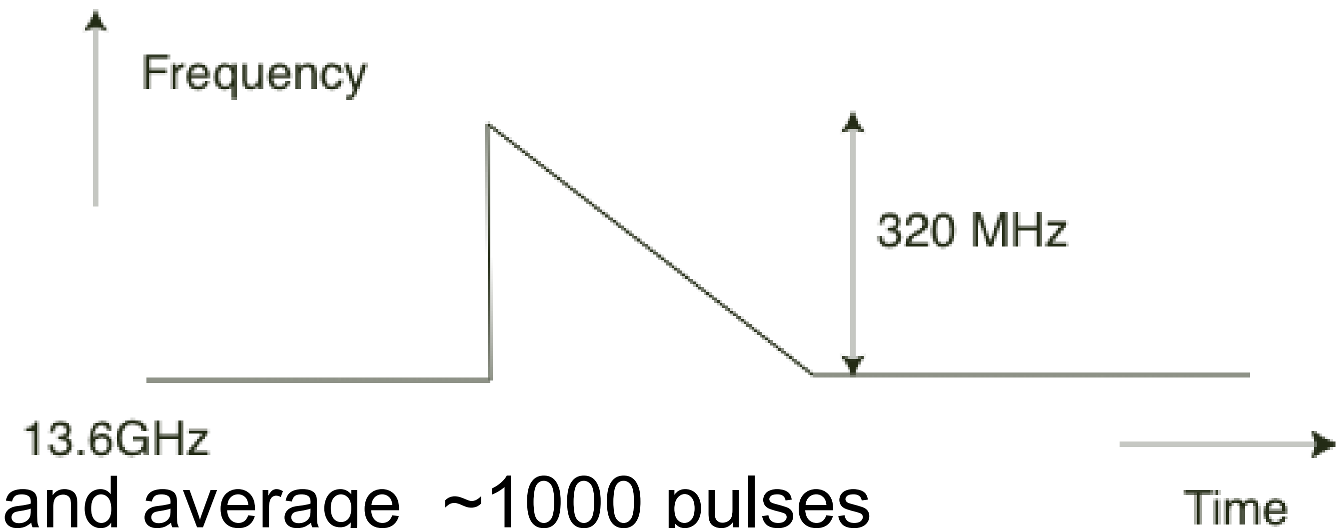
- Measure travel time, 2T, from emit to return
- $h = T \times c$ ($c \approx 3x10^8$ m/s)
- **Resolution to ~1cm** would need a pulse of $3x10^{-10}$s (0.3 nanoseconds)

- 0.3ns… That would be a pulse bandwidth of >3 GHz… Impossible!



Chirp, chirp….

- So we have to use tricks: **chirp** pulse compression



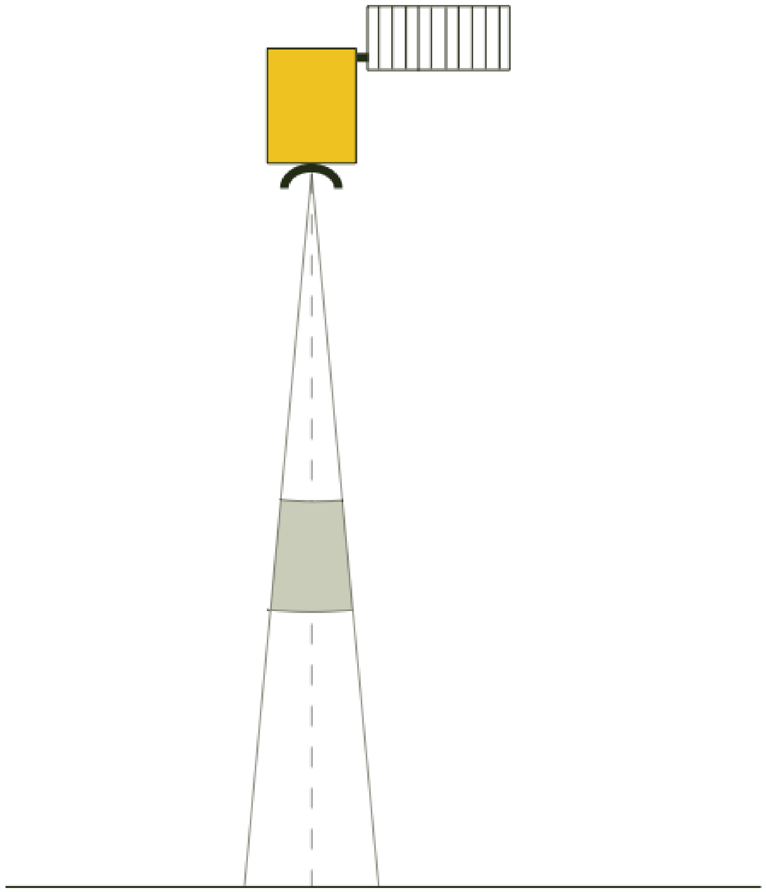
- …and average ~1000 pulses
- It is also necessary to apply a number of corrections for atmospheric and surface effects

Beam- and Pulse- Limited Altimeters

- In principle here are two types of altimeter:
  - beam-limited
  - pulse-limited

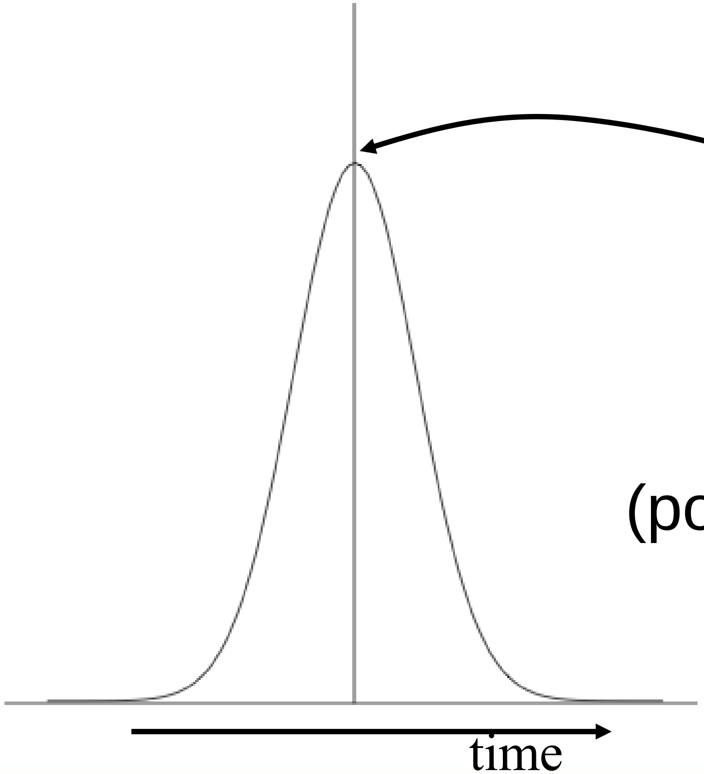
Beam-Limited Altimeter

- Return pulse is dictated by the width of the beam



Beam-Limited altimeter

- A plot of return power versus time for a beam-limited altimeter looks like the *heights* of the specular points, i.e. the probability density function (pdf) of the specular scatterers



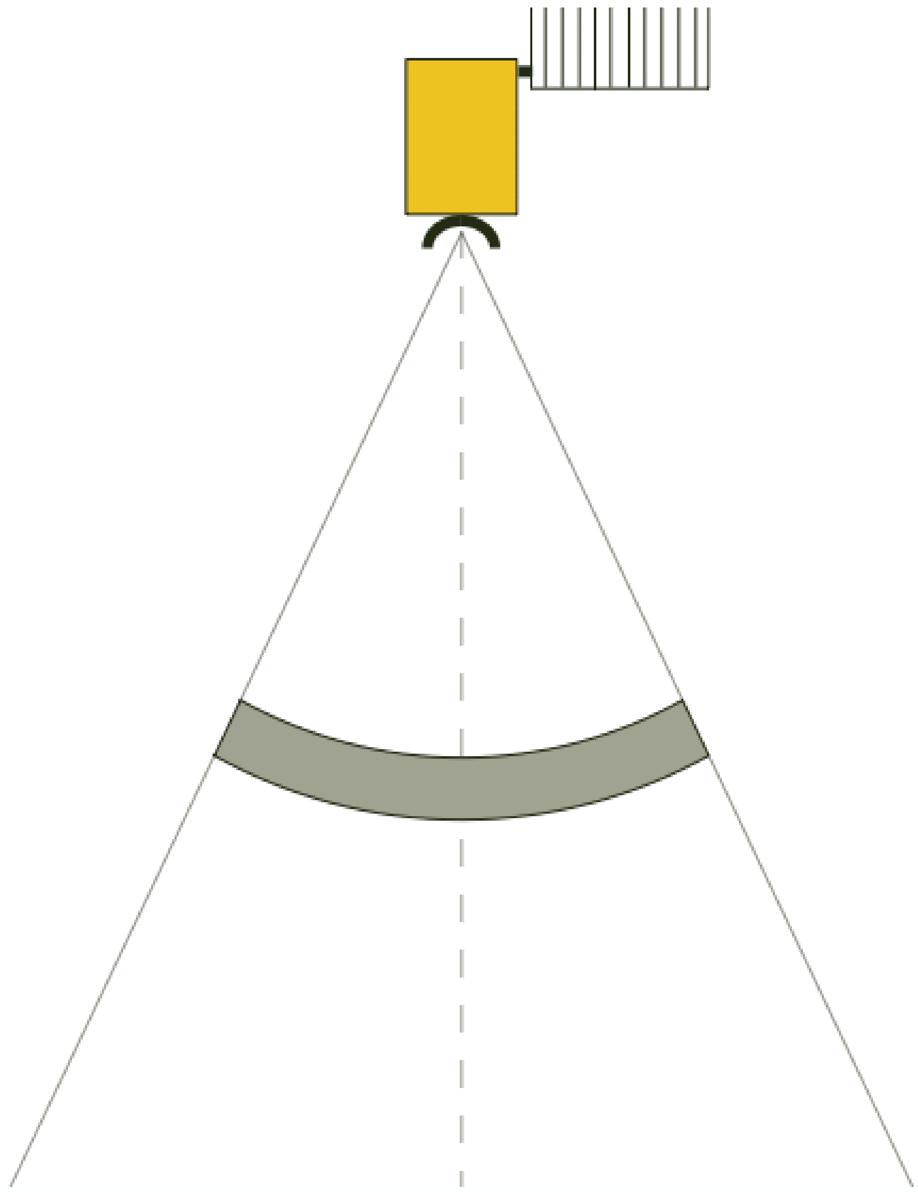
The **tracking point**
(point taken for the range measurement)
is the maximum of the curve

Beam-Limited: technological problems

- Narrow beams require very large antennae and are impractical in space
  - For a **5 km** footprint a beam width of about **0.3°** is required.
  - For a 13.6 GHz altimeter this would imply a **5 m** antenna.
- Even more important: highly sensitivity to mispointing, which affects both amplitude and measured range
- New missions like ESA's CryoSat (launched 8 Apr 2010) and Sentinel-3 use synthetic aperture techniques (delay-Doppler Altimeter) that "can be seen as" a beam-limited instrument in the along-track direction.

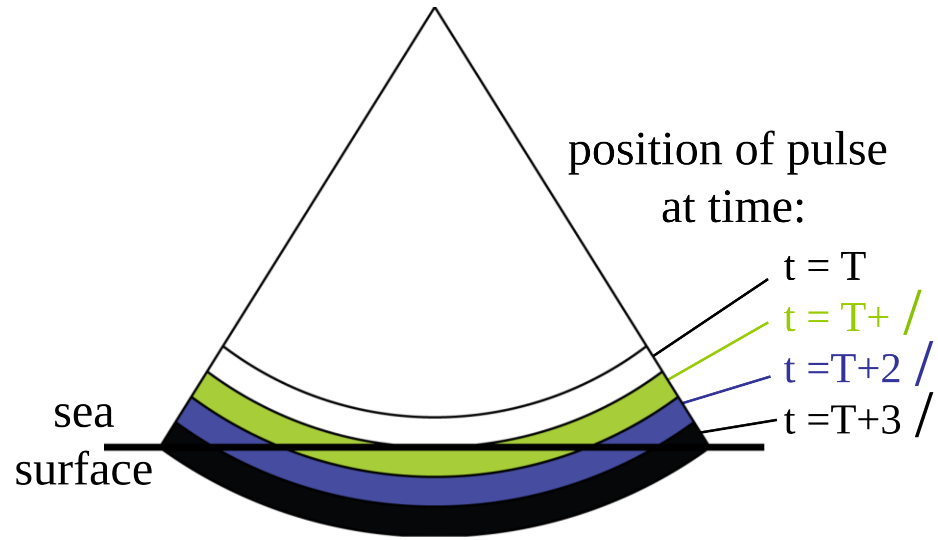
Pulse-Limited Altimeter

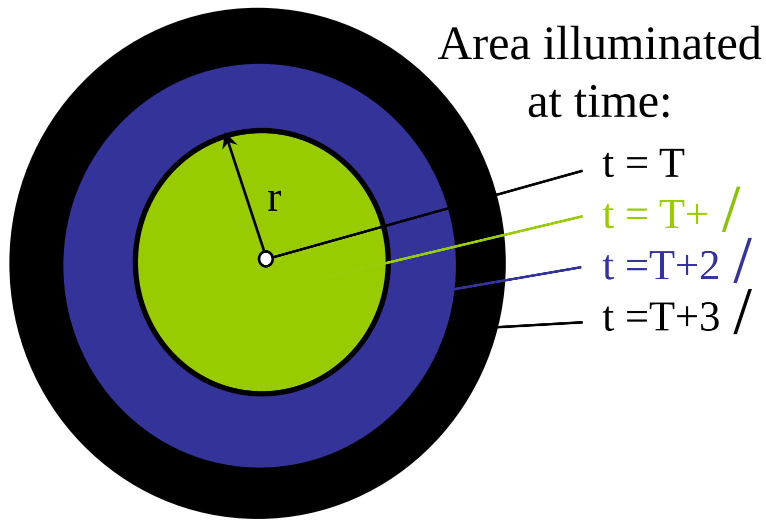
- In a pulse-limited altimeter the shape of the return is dictated by the length (width) of the pulse

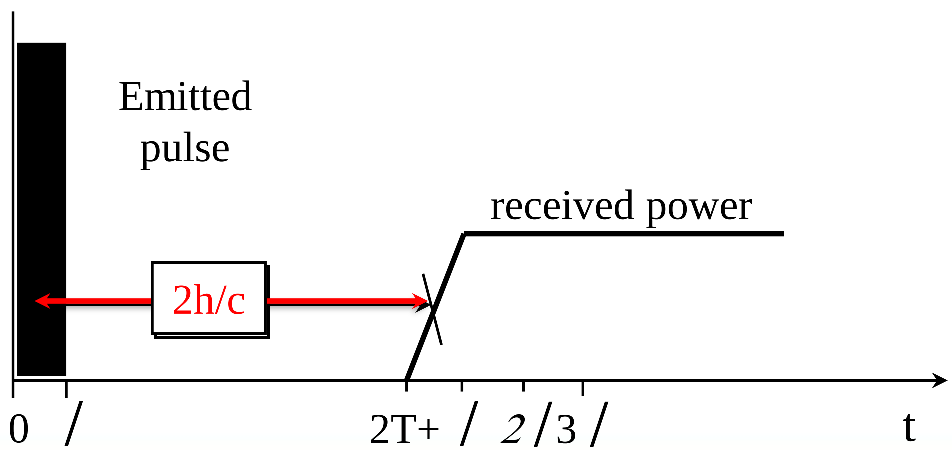


The “pulse-limited” footprint

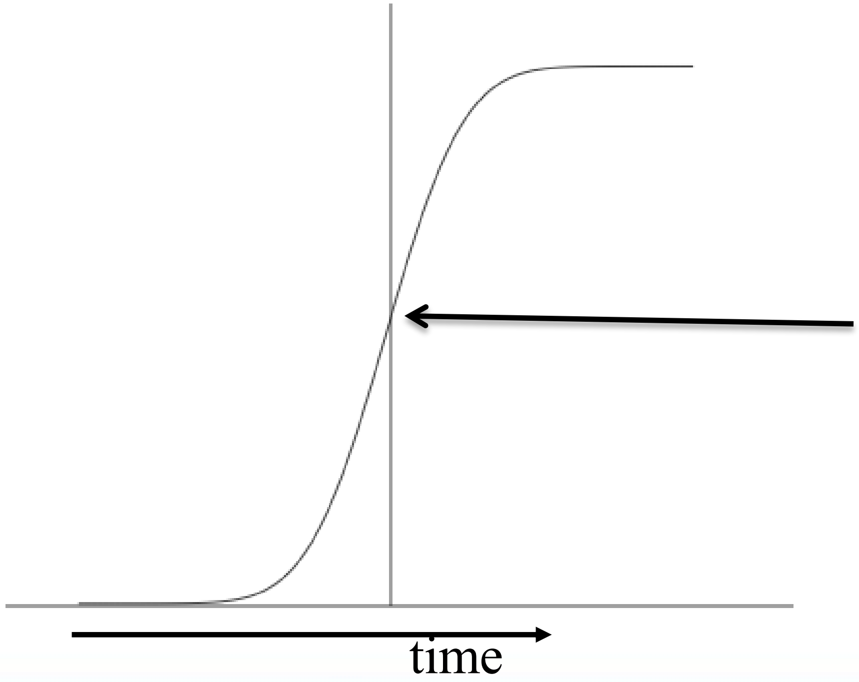
- Full illumination when rear of pulse reaches the sea – then area illuminated stays constant
- Area illuminated has radius $r = \sqrt{(2hc\ /)}$
- Measure interval between mid-pulse emission and time to reach half full height







- A plot of return power versus time for a pulse-limited altimeter looks like the *integral* of the heights of the specular points, i.e. the cumulative distribution function (cdf) of the specular scatterers



The tracking point is the half power point of the curve

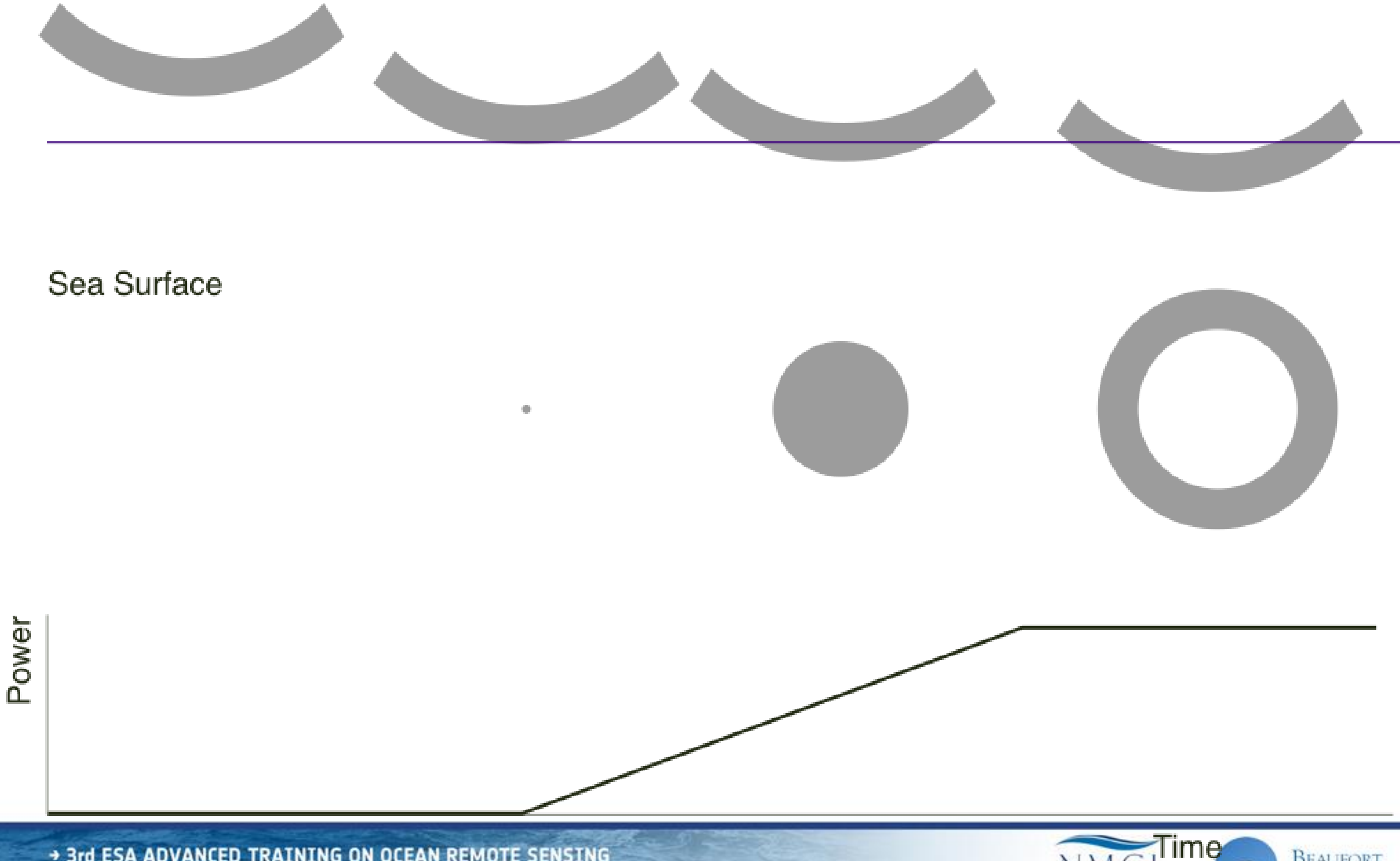
Pulse- vs Beam-Limited

- All the microwave altimeters flown in space to date, including very successful TOPEX/Poseidon, ERS-1 & 2 RA & Envisat RA-2, are pulse-limited except….
- … laser altimeters (like GLAS on ICESAT) are beam-limited
- …and a Delay-Doppler Altimeter "can be seen" as beam-limited in the along-track direction
- To understand the basics of altimetry we will focus on the pulse limited design

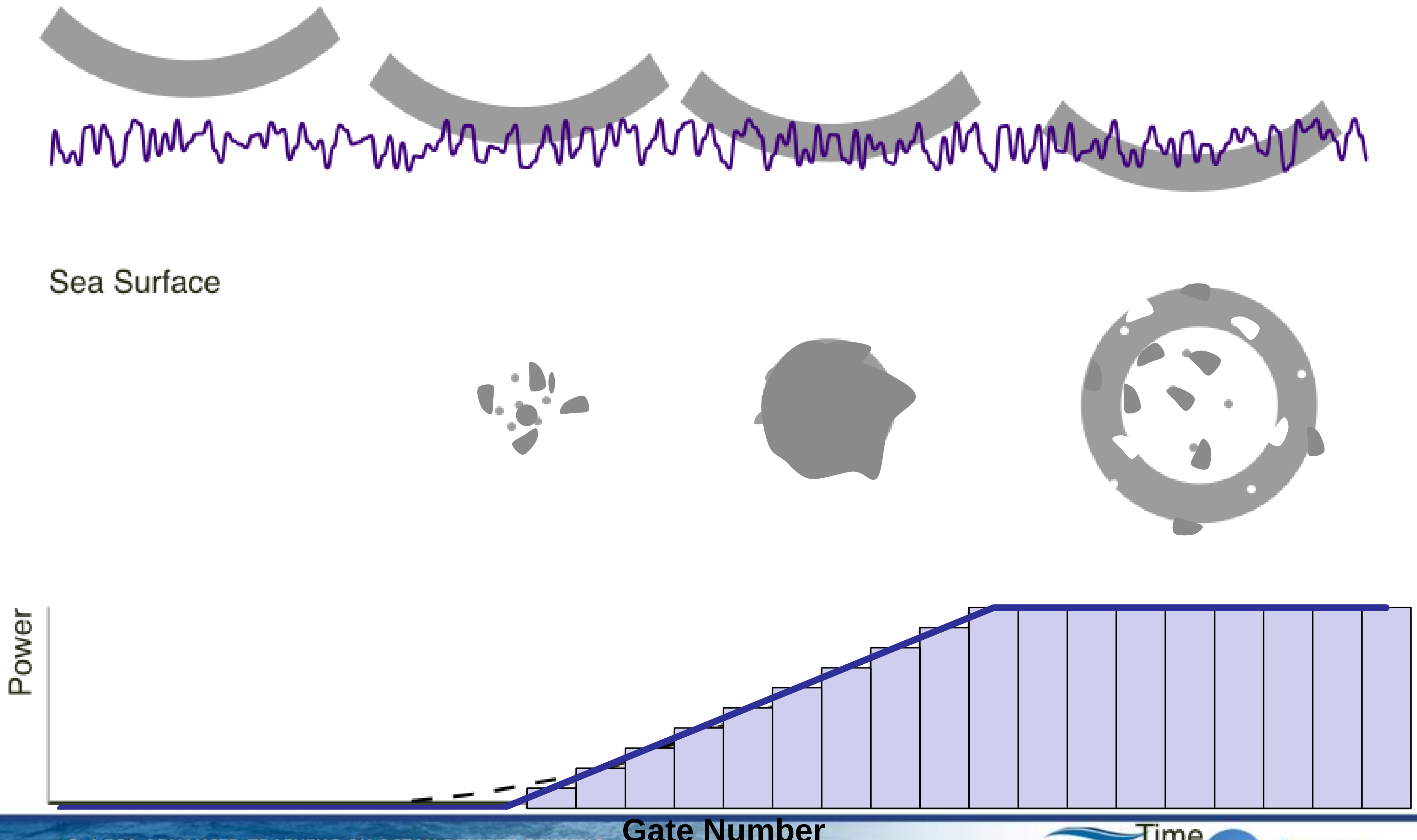
Basics of pulse-limited altimeter theory

- We send out a thin shell of radar energy which is reflected back from the sea surface
- The power in the returned signal is detected by a number of gates (bins) each at a slightly different time

Shell of energy from the pulse



If we add waves …



The area illuminated or ‘effective footprint’

- The total area illuminated is related to the significant wave height noted as SWH [or Hs] (SWH ≈ 4 × std of the height distribution)
- The formula is

$$\frac{pR_0\left(ct + 2H_s\right)}{1 + R_0 / R_E}$$

Where

c is the speed of light
/ is the pulse length
H_s significant wave height
R_0 the altitude of the satellite
R_E the radius of the Earth

Diameters of the effective footprint

H_s (m)	ERS-2/1, ENVISAT Effective footprint (km) (800 km altitude)	TOPEX, Jason-1/2 Effective footprint (km) (1335 km altitude)
0	1.6	2.0
1	2.9	3.6
3	4.4	5.5
5	5.6	6.9
10	7.7	9.6
15	9.4	11.7
20	10.8	13.4

From Chelton et al (1989)

The Brown Model

- Assume that the sea surface is a perfectly conducting rough mirror which reflects only at specular points, i.e. those points where the radar beam is reflected directly back to the satellite

The Brown Model - II

- Under these assumptions the return power is given by a three fold convolution

$$P_r(t) = P_{FS}(t) * P_{PT}(t) * P_H(-z)$$

Where

$P_r(t)$ is the returned power

$P_{FS}(t)$ is the flat surface response

$P_{PT}(t)$ is the point target response

$P_H(-z)$ is the pdf of specular points on the sea surface

The Flat Surface Response Function

- The Flat surface response function is the response you would get from reflecting the radar pulse from a flat surface.
- It looks like

$$P_{FS}(t) = U\left(t - t_0\right) \cdot G(t)$$

Where

$U(t)$ is the Heaviside function

$U(t) = 0$ for $t < 0$; $U(t) = 1$ otherwise

$G(t)$ is the two way antenna gain pattern

The Point Target Response Function

- The point target response (PTR) function is the shape of the transmitted pulse
- Its true shape is given by

$$P_{PT}(t) = \left[\frac{\sin\left(\pi t / \tau\right)}{\pi t / \tau} \right]^2$$

- For the Brown model we approximate this with a Gaussian.

The Brown Model - III

$$P_r(t) = P_{FS}(0)\,\eta P_T \sqrt{2\pi}\,\frac{\sigma_p}{2}\left[1 + erf\left\{\frac{(t - t_0)}{\sqrt{2}\sigma_c}\right\}\right] \quad \text{for } t < t_0$$

$$P_r(t) = P_{FS}(t - t_0)\,\eta P_T \sqrt{2\pi}\,\frac{\sigma_p}{2}\left[1 + erf\left\{\frac{(t - t_0)}{\sqrt{2}\sigma_c}\right\}\right] \quad \text{for } t \geq t_0$$

$$\sigma_c = \sqrt{\sigma_p^2 + \frac{4\sigma_s^2}{c^2}} \qquad\qquad \sigma_s \approx \frac{SWH}{4}$$

$$P_{FS}(t) = \frac{G_0^2 \lambda_R^2 c \sigma_0}{4(4\pi)^2 L_p h^3} \exp\left\{-\frac{4}{\gamma}\sin^2 \xi - \frac{4ct}{\gamma h}\cos 2\xi\right\} I_0\left(\frac{4}{\gamma}\sqrt{\frac{ct}{h}}\sin 2\xi\right)$$

where

$$erf(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2}\, dx$$

(compare this with the Normal cumulative distribution function)

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{\frac{-x^2}{2}}\, dx$$

$$\Phi(x) = \frac{1}{2}\left[1 + erf\left(\frac{x}{\sqrt{2}}\right)\right]$$

I_0() is a modified Bessel function of the first kind

What are we measuring?

- *SWH* - **significant wave height**
- t_0 - the **time** for the radar signal to reach the Earth and return to the satellite
  - we then convert into **range** and finally into **height** – see in the next slides
- σ_0 - the **radar backscatter coefficient**
  - note this is set by the **roughness** at **scales comparable with radar wavelength**, i.e. cm, therefore it is (in some way) related to wind
- sometimes **mispointing angle** ξ can be also estimated from the waveforms

The Brown Model – measured parameters

$$P_r(t) = P_{FS}(0)\,\eta P_T \sqrt{2\pi}\,\frac{\sigma_p}{2}\left[1 + erf\left\{\frac{(t - t_0)}{\sqrt{2}\sigma_c}\right\}\right] \quad \text{for } t < t_0$$

$$P_r(t) = P_{FS}(t - t_0)\,\eta P_T \sqrt{2\pi}\,\frac{\sigma_p}{2}\left[1 + erf\left\{\frac{(t - t_0)}{\sqrt{2}\sigma_c}\right\}\right] \quad \text{for } t \geq t_0$$

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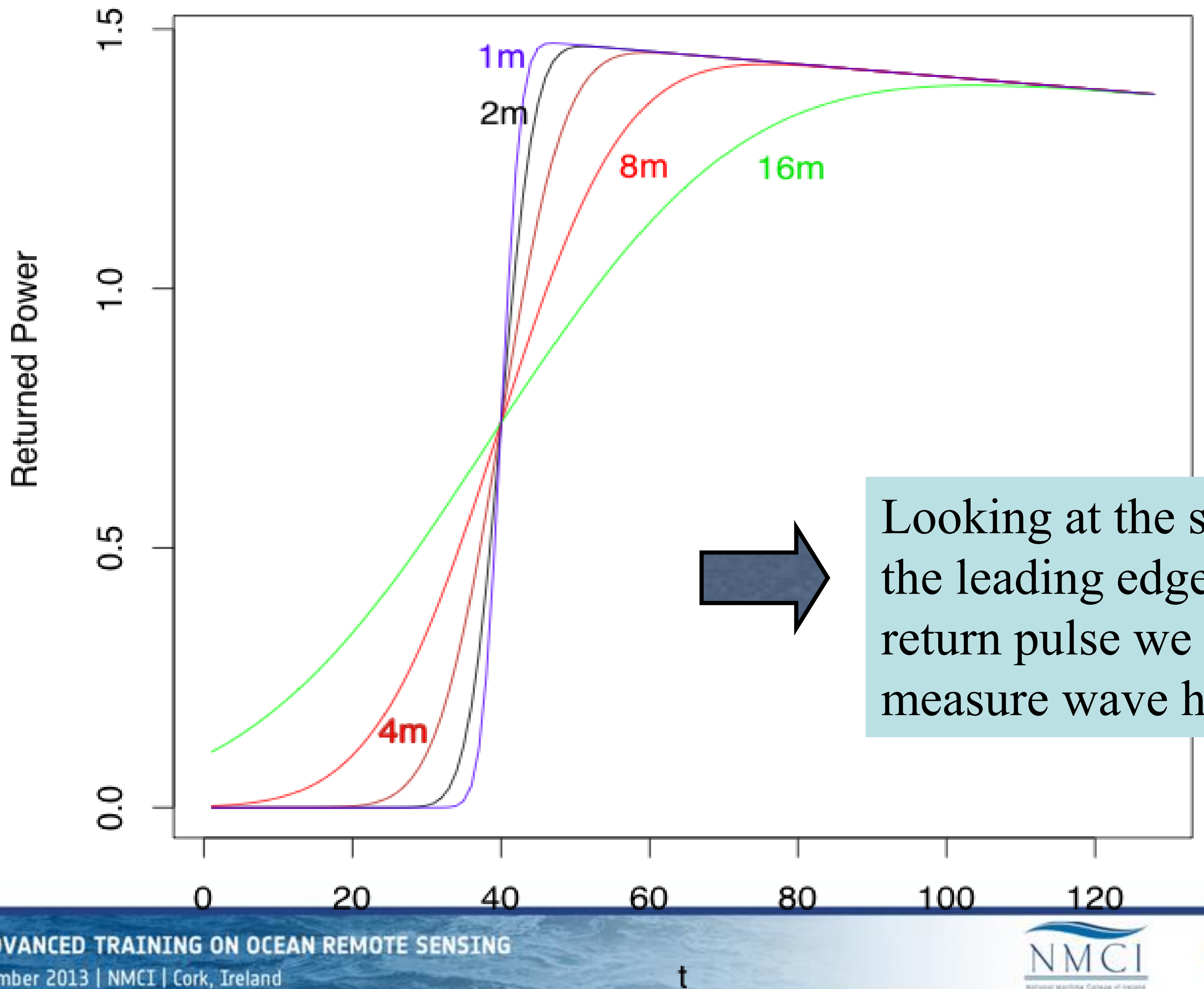
What are the other parameters?

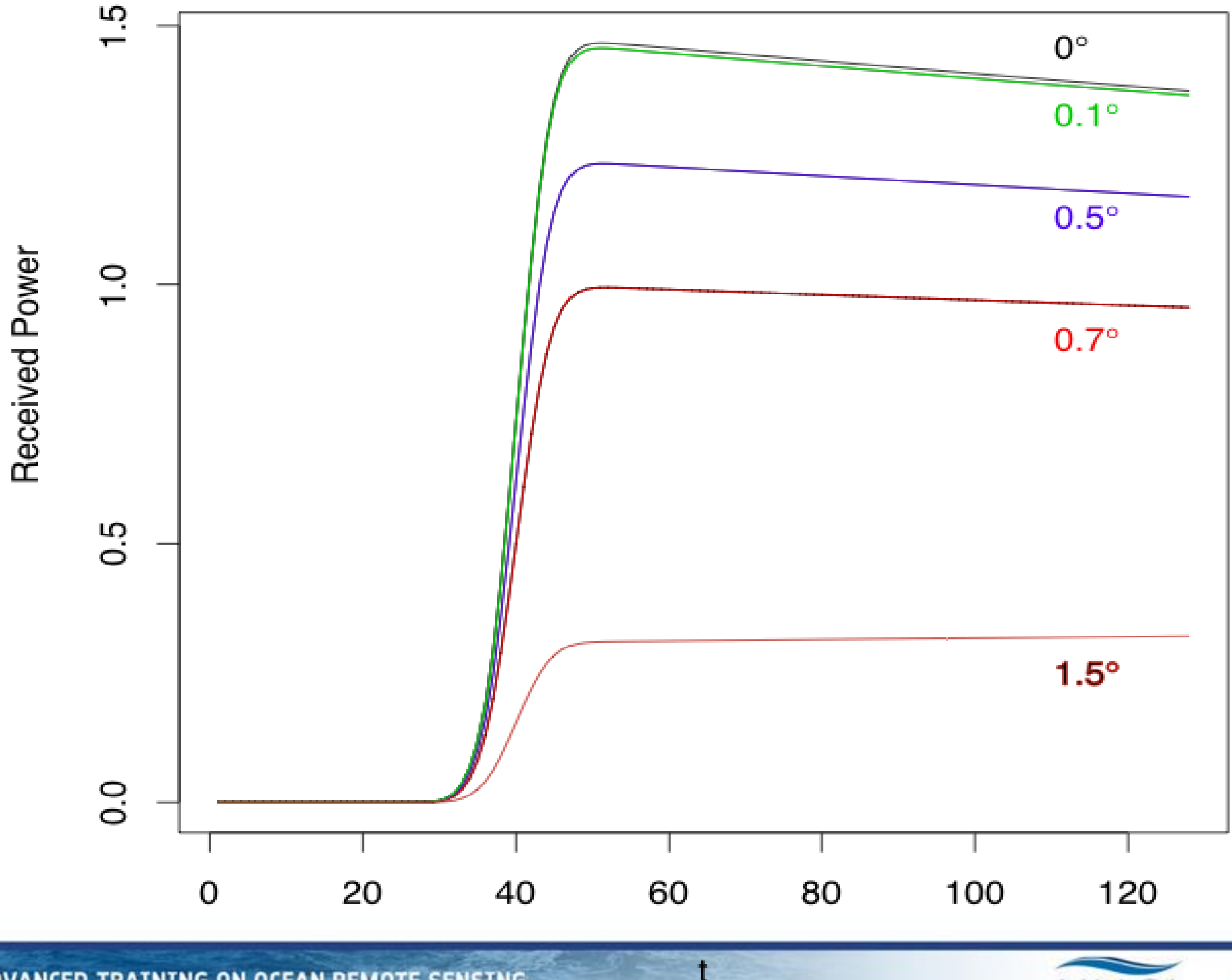
- λ_R is the radar wavelength
- L_p is the two way propagation loss
- h is the satellite altitude (nominal)
- G_0 is the antenna gain
- γ is the antenna beam width
- σ_p is the pulse width
- η is the pulse compression ratio
- P_T is the peak power
- ξ (as we said) is the mispointing angle

Theoretical waveforms – effect of SWH



Looking at the slope of the leading edge of the return pulse we can measure wave height!

The effect of mispointing



Noise on the altimeter

- If we simply use the altimeter as a detector we will still have a signal - known as the thermal noise.
- The noise on the signal is known as fading noise
- It is sometimes assumed to be constant, sometimes its mean is measured
- For most altimeters the noise on the signal is independent in each gate and has a negative exponential distribution.

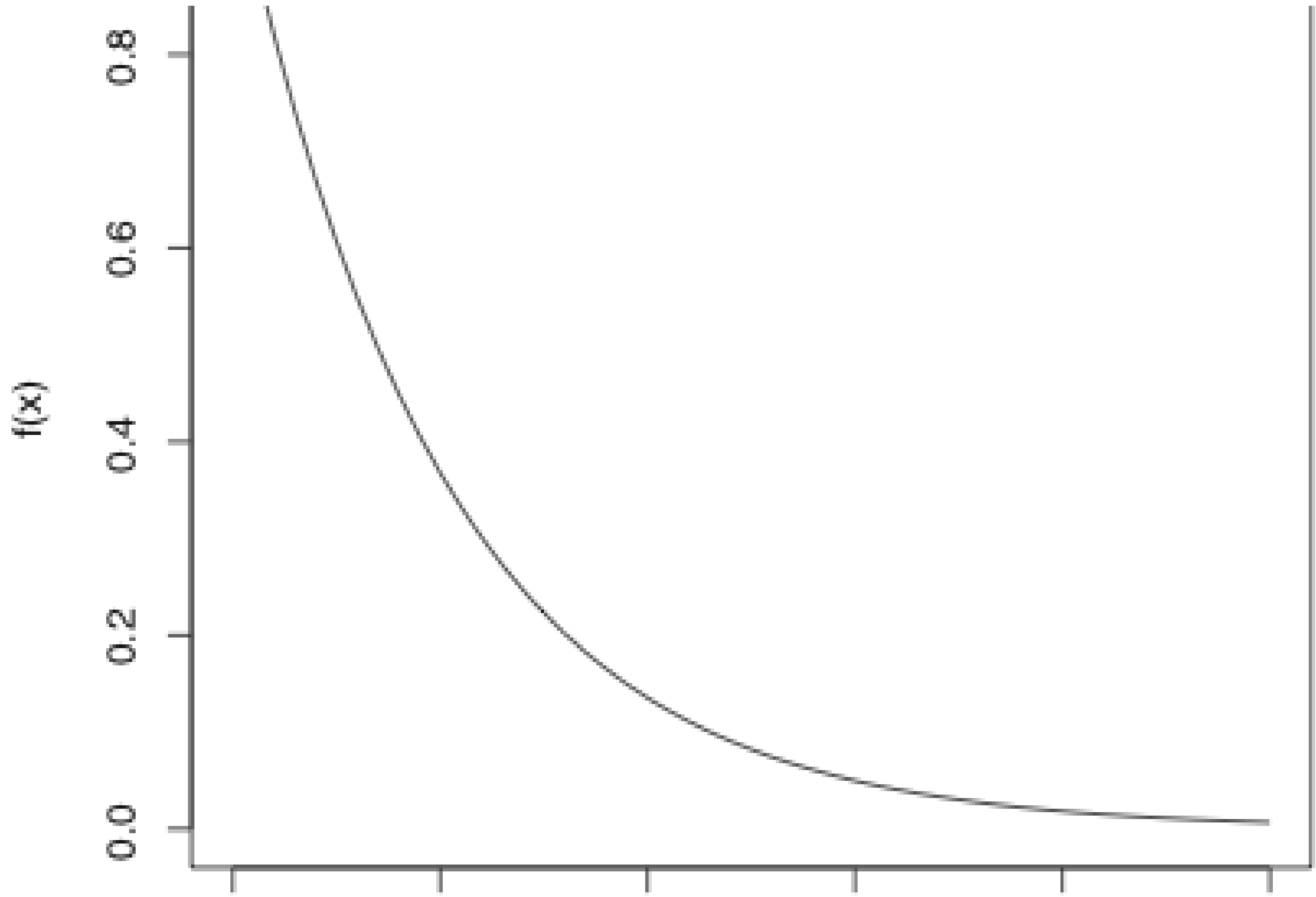
Exponential distribution

- pdf

$$f(x) = \frac{1}{q} e^{-\frac{x}{q}} \qquad 0 < x < \infty$$

- Mean = θ
- Variance = θ^2

Exponential pdf



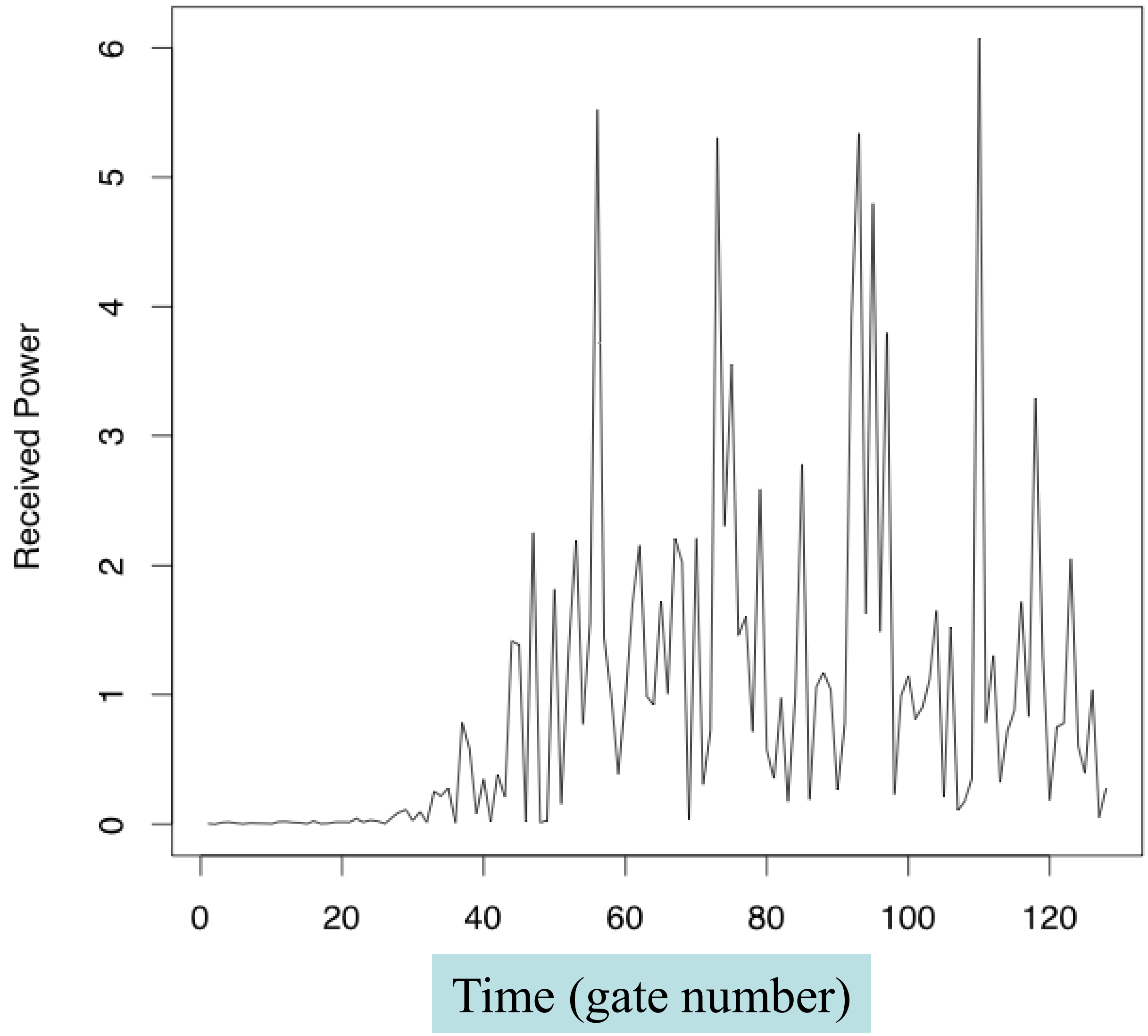
Averaging the noise

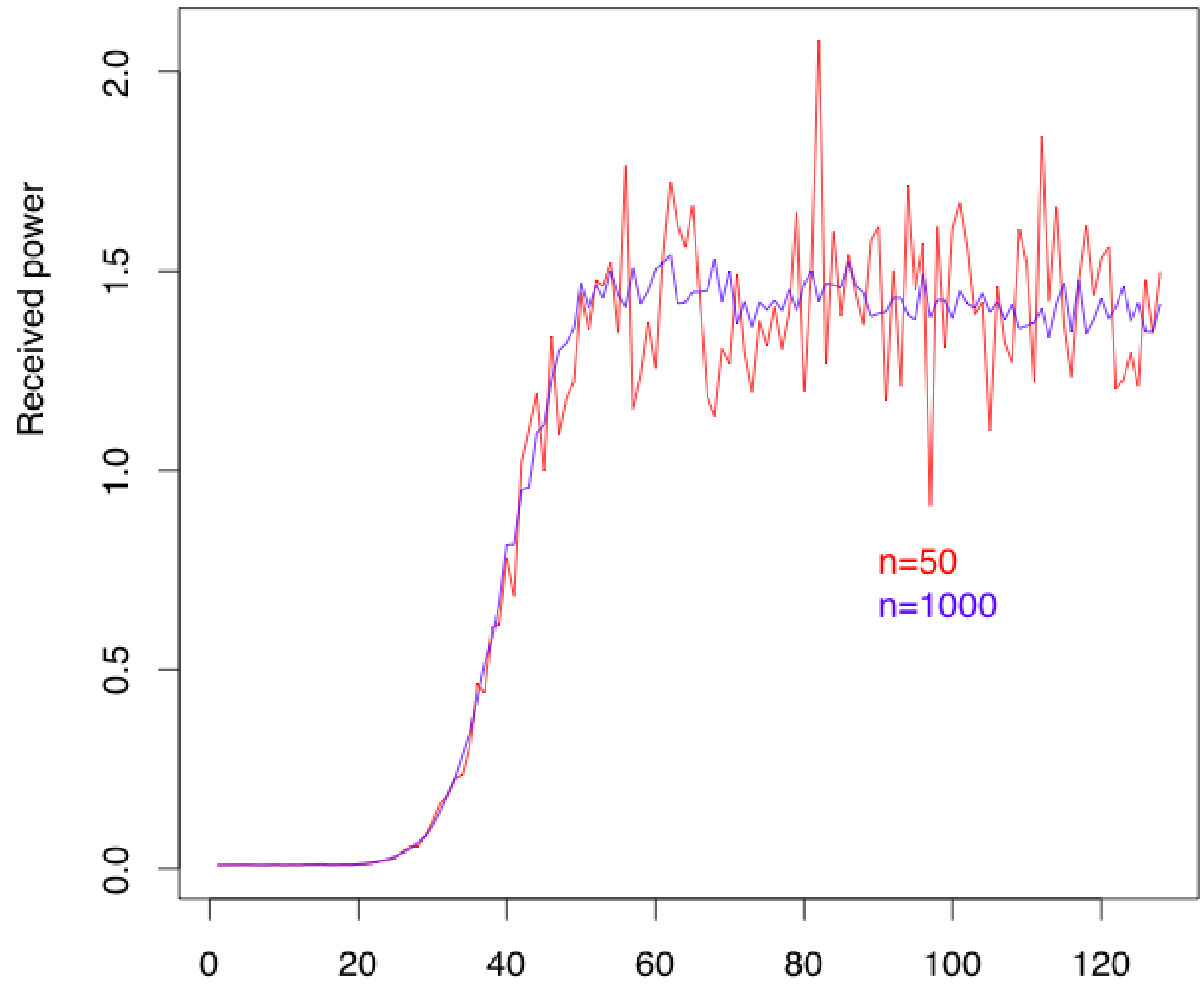
- For a negative exponential distribution the variance is equal to the square of the mean. Thus **the individual pulses are very noisy!**

⇨ **We need a lot of averaging to achieve good Signal to Noise Ratio**

- The pulse repetition frequency is thousands per second
  - 1020 for ERS-1/2, 1800 for Jason & Envisat, 4500 for Topex
- Usually data are transmitted to the ground at ~20Hz and then averaged to ~1 Hz

A single pulse

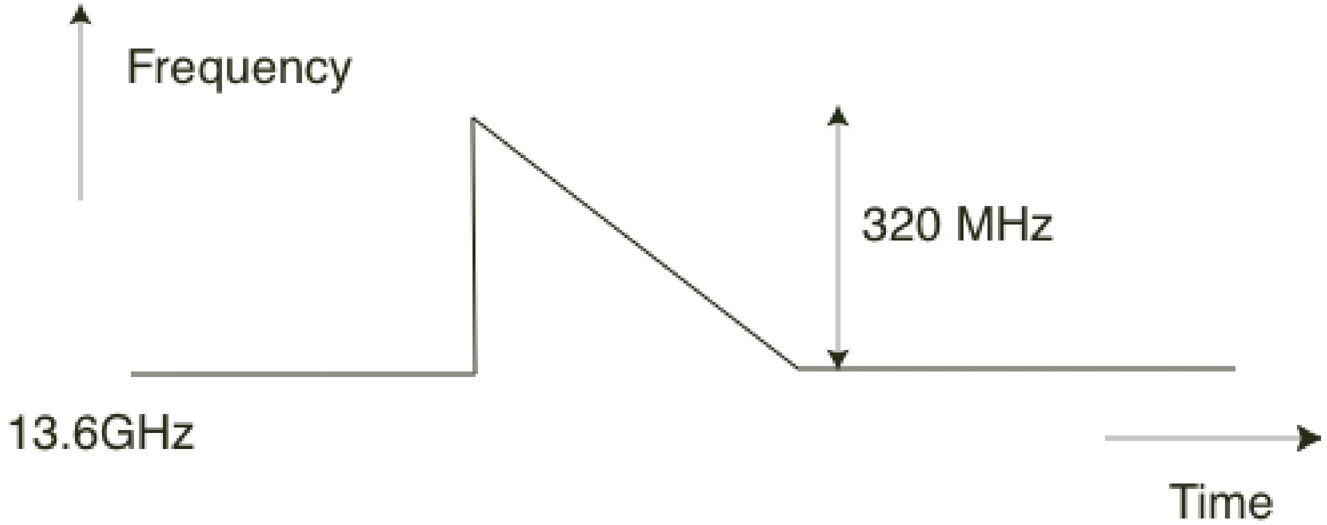




Time (gate number)

How altimeters really work

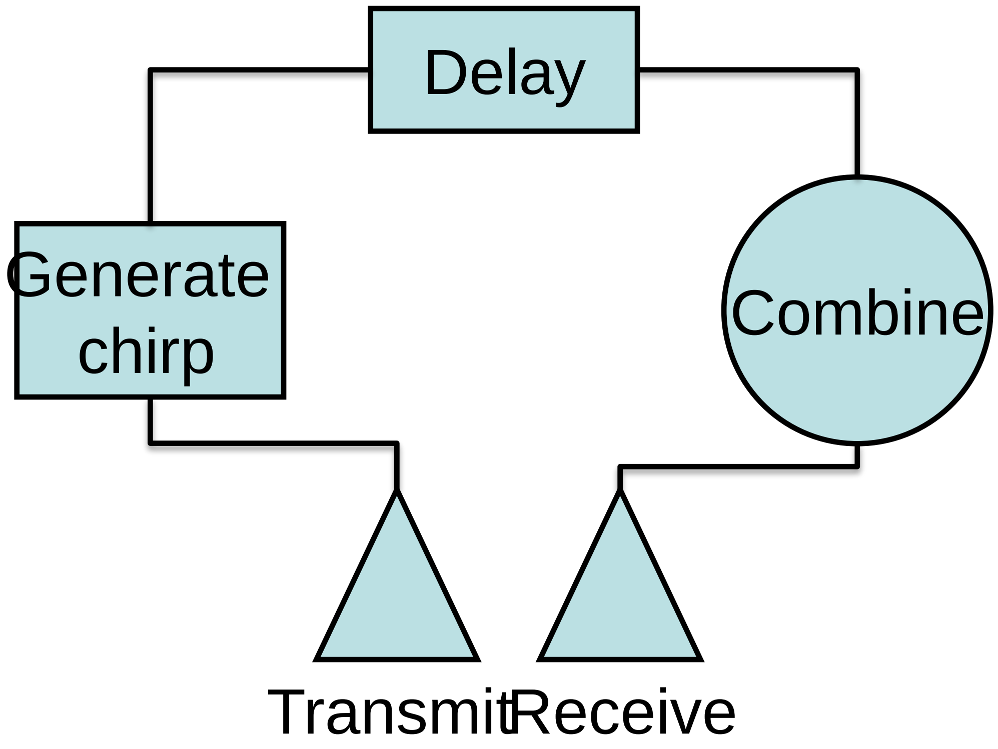
- It is very difficult (if not impossible) to generate a single-frequency pulse of length 3 ns
- It **is** possible to do something very similar in the frequency domain using a chirp: modulating the frequency of the carrier wave in a linear way



- The equivalent pulse width = 1/chirp bandwidth

Full chirp deramp - 1

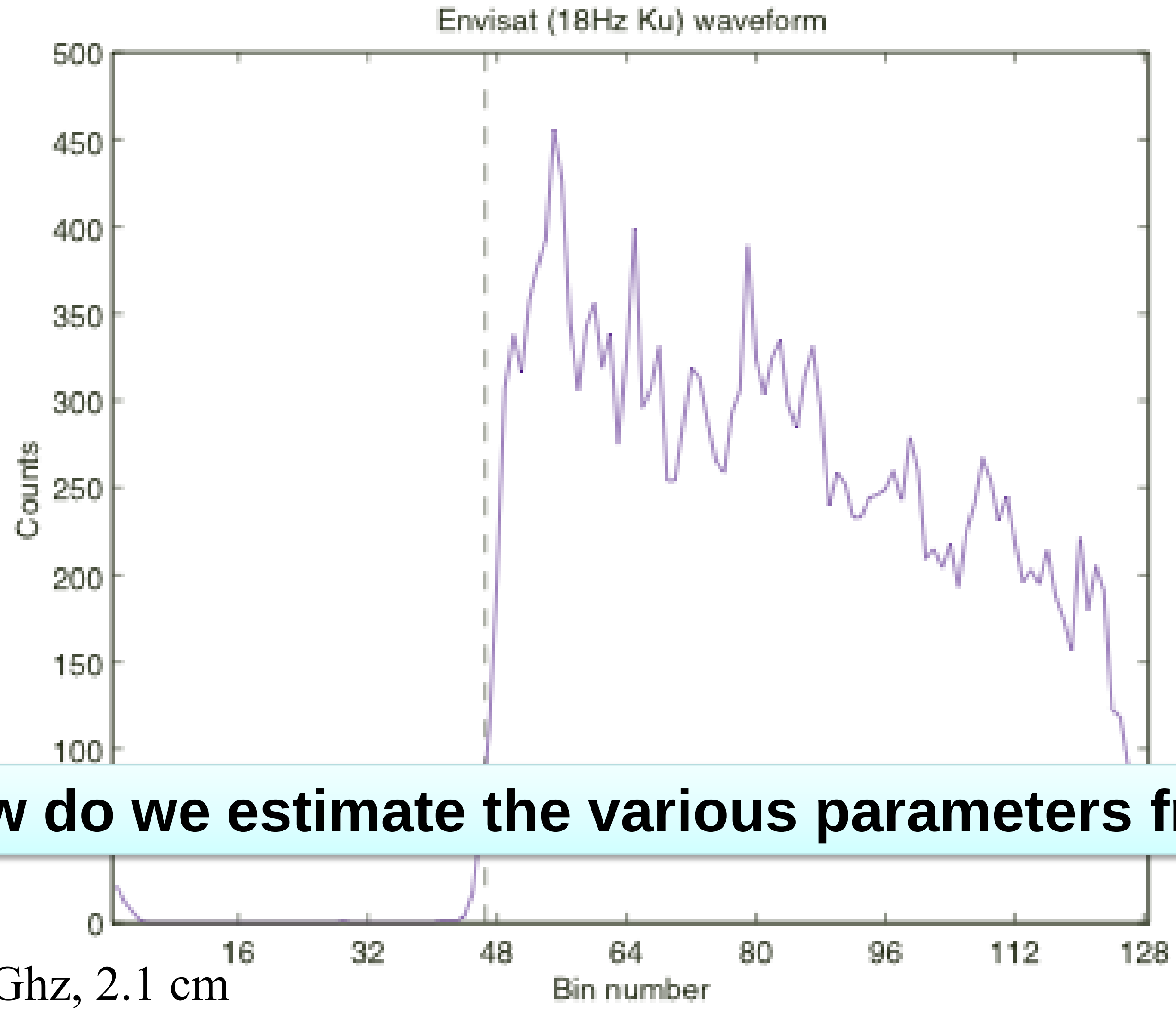
- A chirp is generated
- Two copies are taken
- The first is transmitted
- The second is delayed so it can be matched with the reflected pulse



Full Chirp Deramp - 2

- The two chirps are mixed.
- A point above the sea surface gives returns at frequency lower than would be expected and vice versa
- So a ‘Brown’ return is received but with frequency rather than time along the x axis

A real waveform - from the RA-2 altimeter on ESA's Envisat



How do we estimate the various parameters from this?

Ku band, 13.5 Ghz, 2.1 cm

“Retracking” of the waveforms

= fitting the waveforms with a waveform model, therefore estimating the parameters

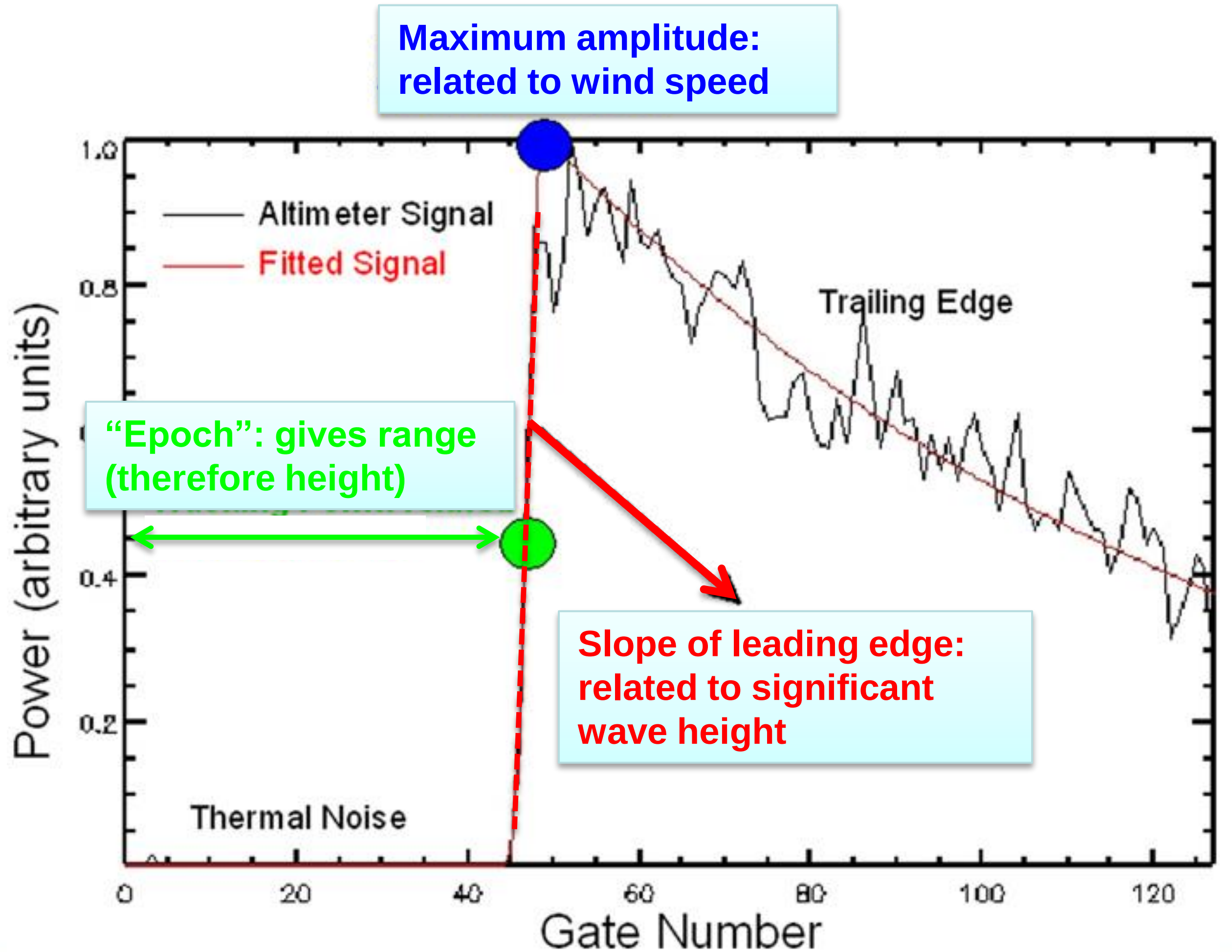
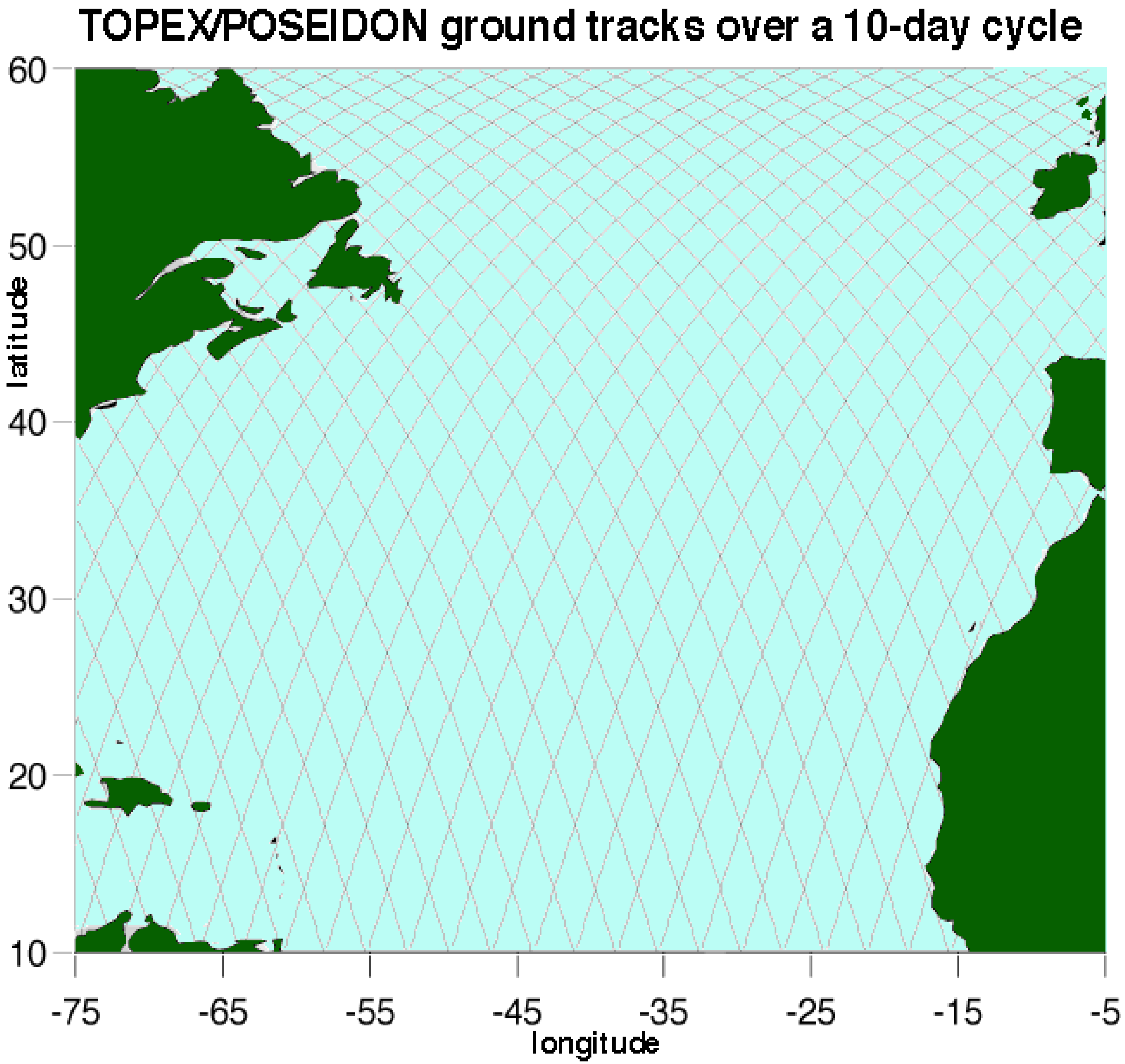


Figure from J Gomez-Enri et al. (2009)

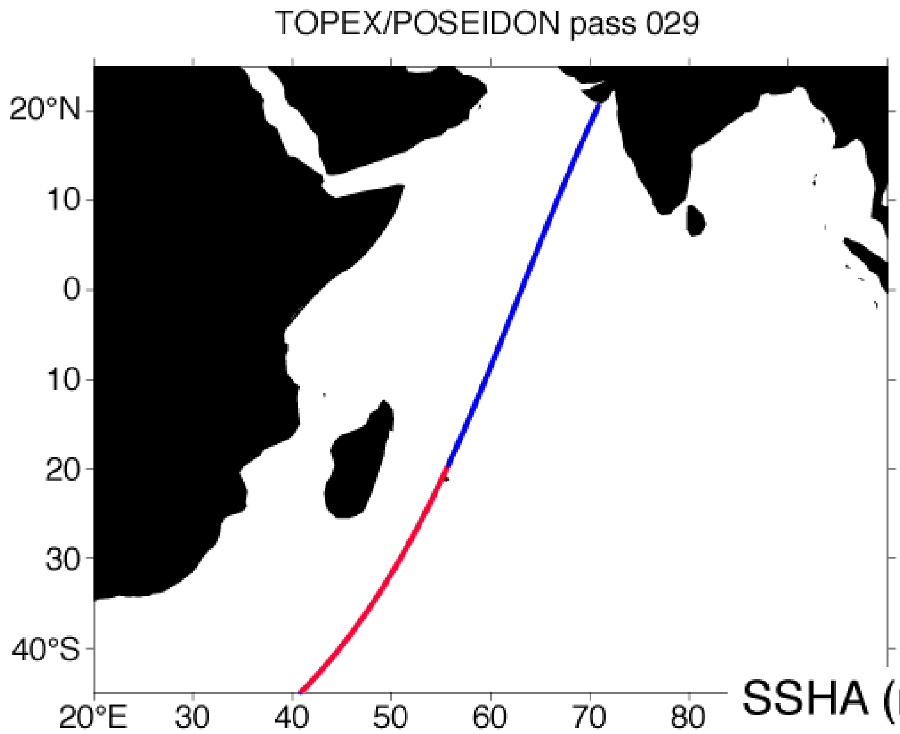
Altimeters flown in space

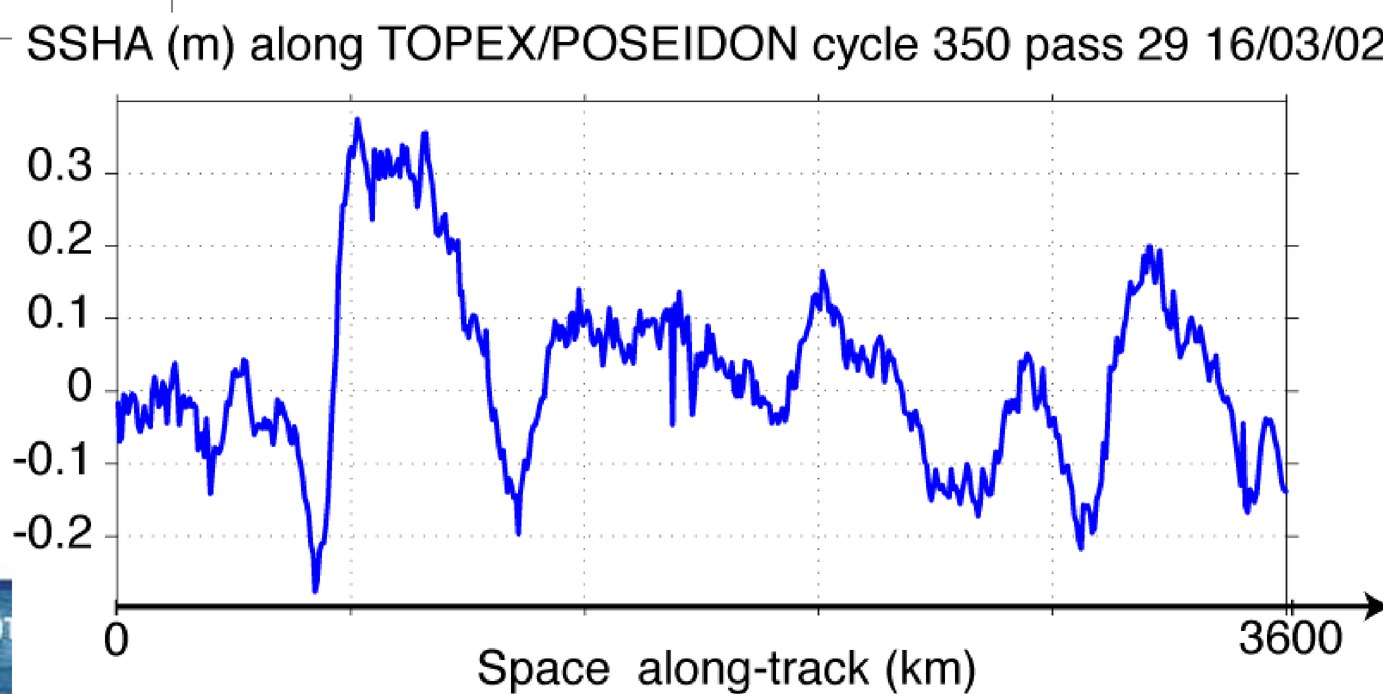
Height	inclination	accuracy	repeat period
GEOS-3 (04/75 – 12/78)			
845 km	115 deg	0.5 m	-
Seasat (06/78 – 09/78)			
800 km	108 deg	0.10 m	3 days
Geosat (03/85 – 09/89)			
785.5 km	108.1 deg	0.10 m	17.5 days
ERS-1 (07/91 – 03/2000); ERS-2 (04/95 – 09/2011)			
785 km	98.5 deg	0.05 m	35 days
TOPEX/Poseidon (09/92 – 10/2005); Jason-1 (12/01 – 06/2013); Jason-2 (06/08 – present)			
1336 km	66 deg	0.02 m	9.92 days
Geosat follow-on (GFO) (02/98 – 09/2008)			
800 km	108 deg	0.10 m	17.5 days
Envisat (03/02 – 04/12)			
785 km	98.5 deg	0.03 m	35 days
CryoSat-2 (04/10 – present) [delay-Doppler]			
717 km	92 deg	0.05 m	369 days (30d sub-cycle)
SARAL/AltiKa (02/13 – present) [Ka-band]			
785 km	98.5 deg	0.02 m	35 days

1-D (along-track) measurement

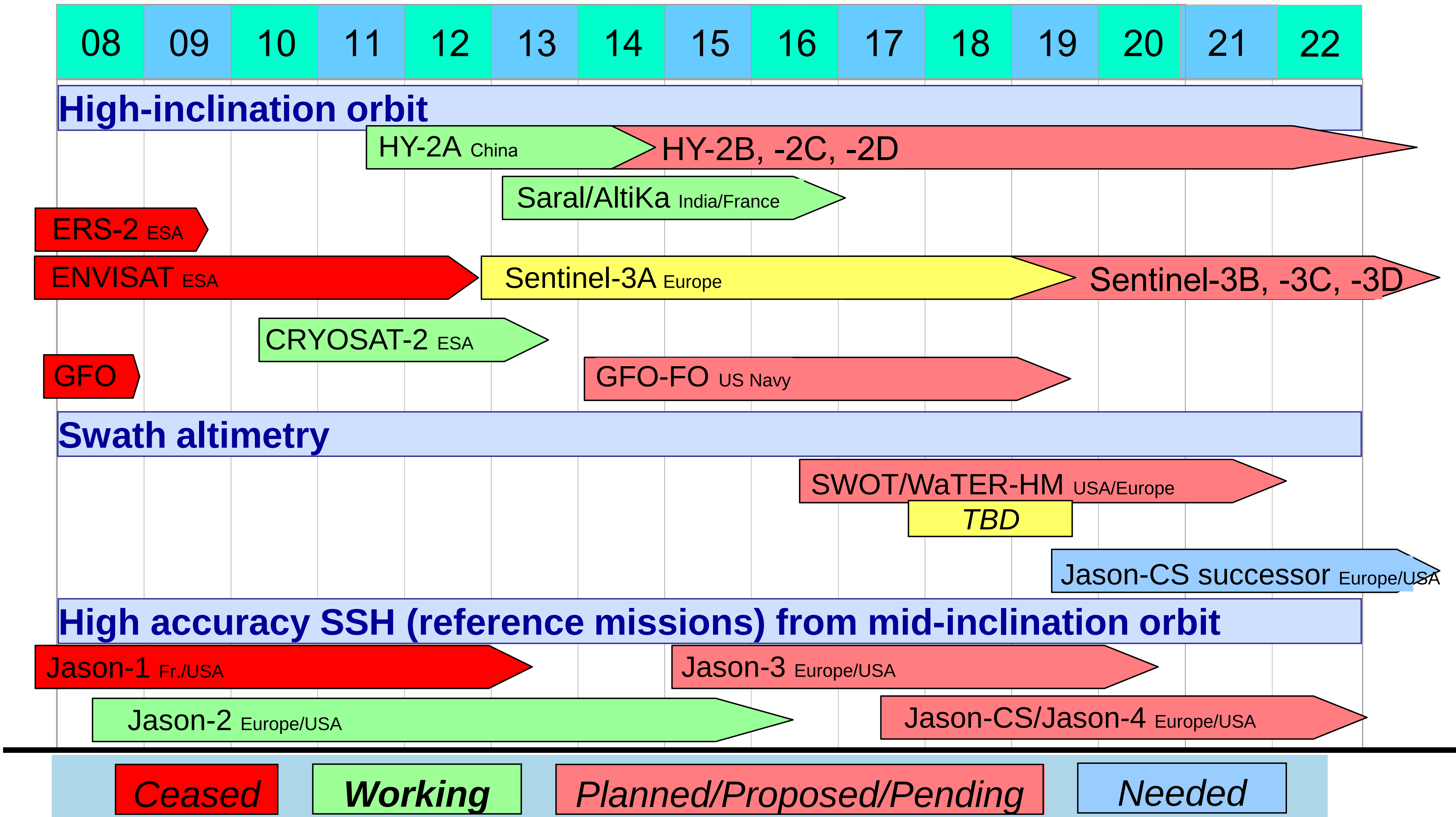


Example: Sea Surface Height along the ground track of a satellite altimeter





Radar Altimeters: Now and Then

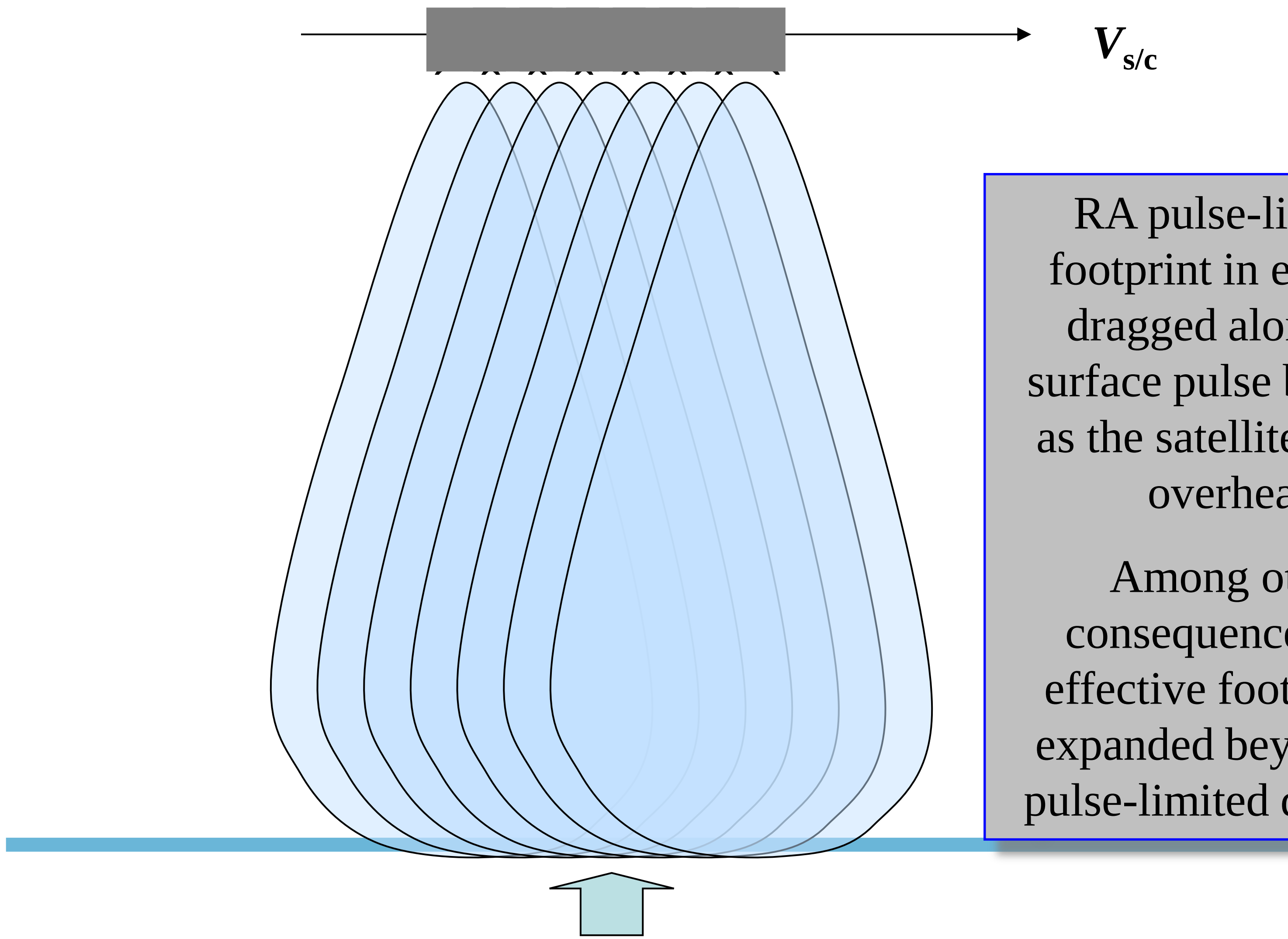


Adapted from CNES, 2009, with acknowledgement

Cryosat-2

- ESA mission; launched 8 April 2010
- LEO, non sun-synchronous
  - 369 days repeat (30d sub-cycle)
  - Mean altitude: 717 km
  - Inclination: 92°
- Prime payload: SIRAL
  - SAR/Interferometric Radar Altimeter **(delay/Doppler)**
  - Modes: Low-Res / SAR / SARIn
- Ku-band only; no radiometer
- Design life:
  - 6 months commissioning + 3 years

Conventional altimeter footprint scan

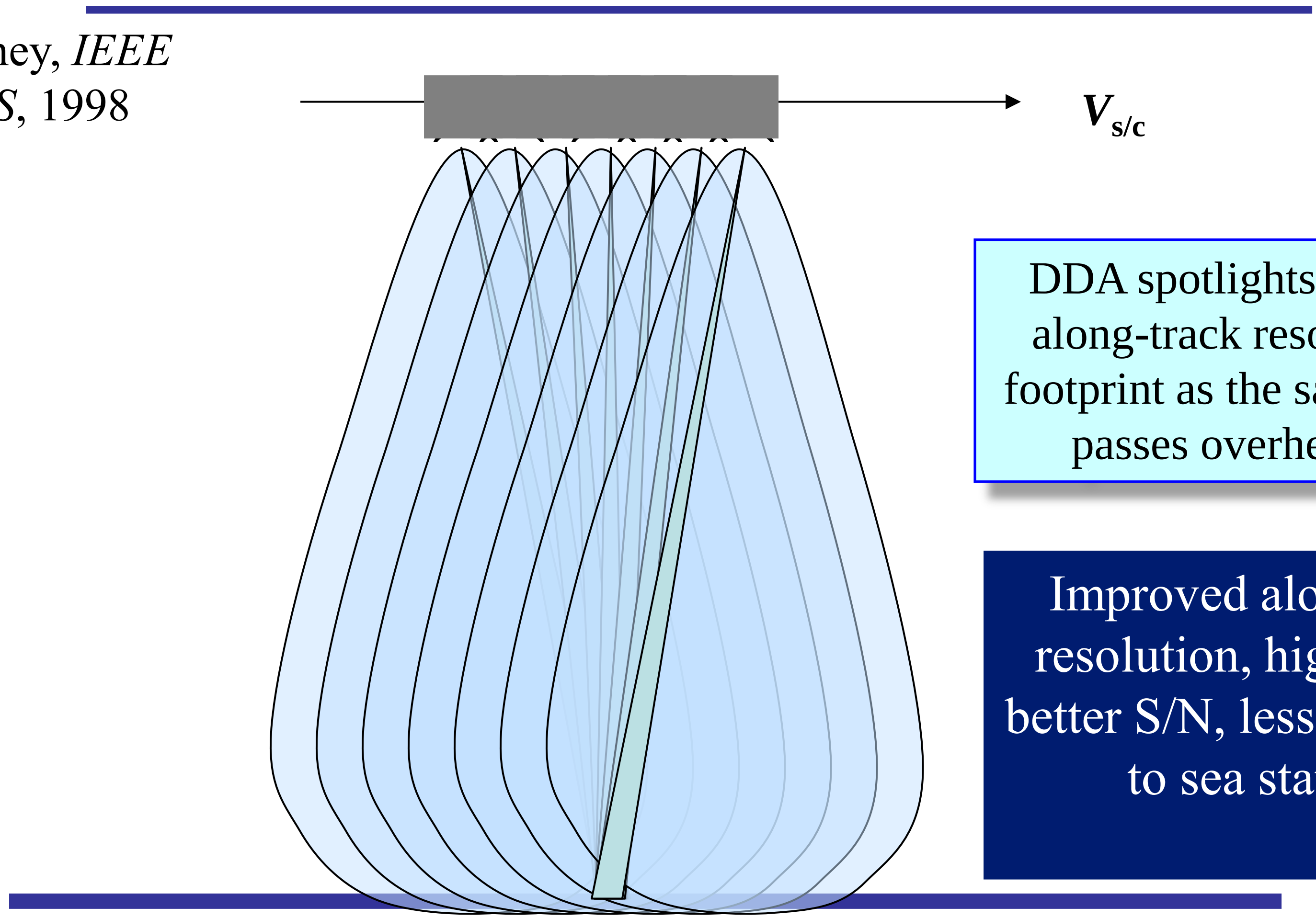


RA pulse-limited footprint in effect is dragged along the surface pulse by pulse as the satellite passes overhead

Among other consequences, the effective footprint is expanded beyond the pulse-limited diameter

Delay-Doppler Altimetry (aka SAR altimetry)

R.K. Raney, *IEEE TGARS*, 1998



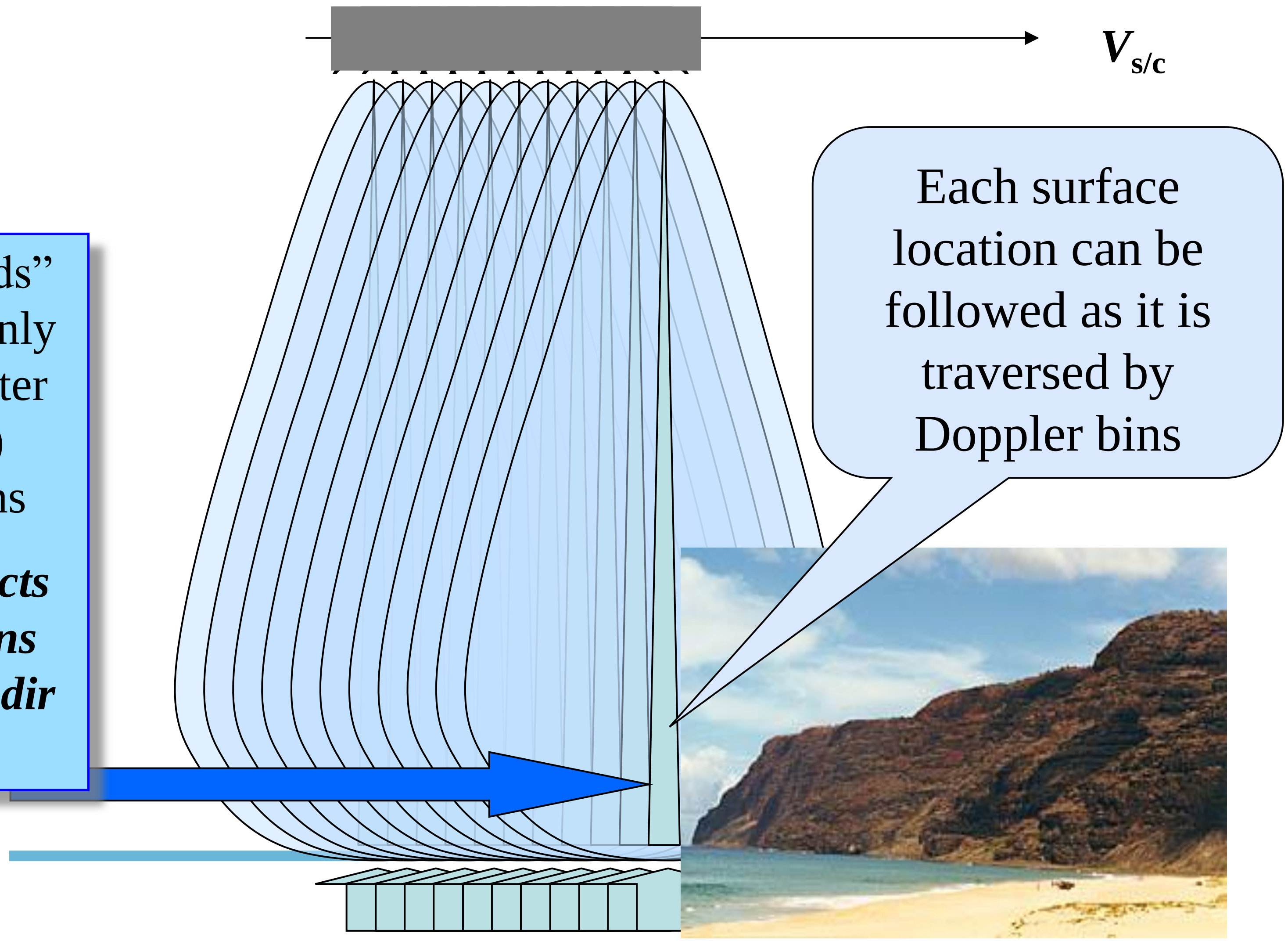
DDA spotlights each along-track resolved footprint as the satellite passes overhead

Improved along-track resolution, higher PRF, better S/N, less sensitivity to sea state,…

DDA (SAR-mode) Footprint Characteristic

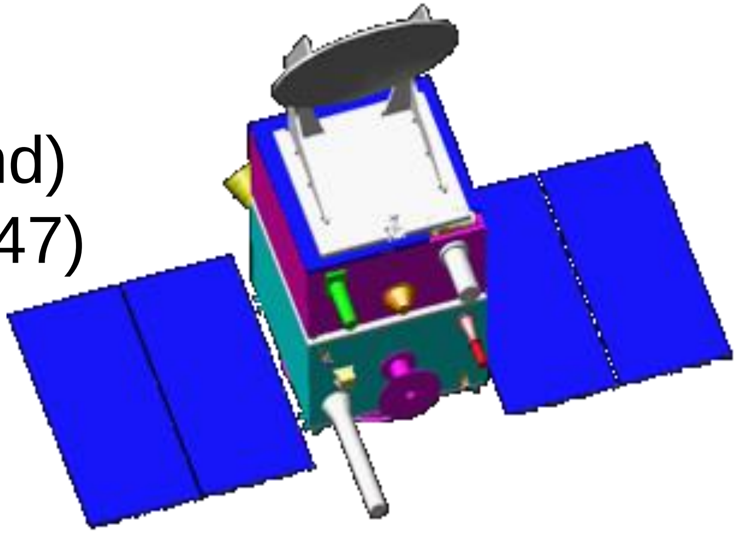
Tracker "reads" waveforms only from the center (1, 2, or 3) Doppler bins

Result? ***Rejects all reflections from non-nadir sources***

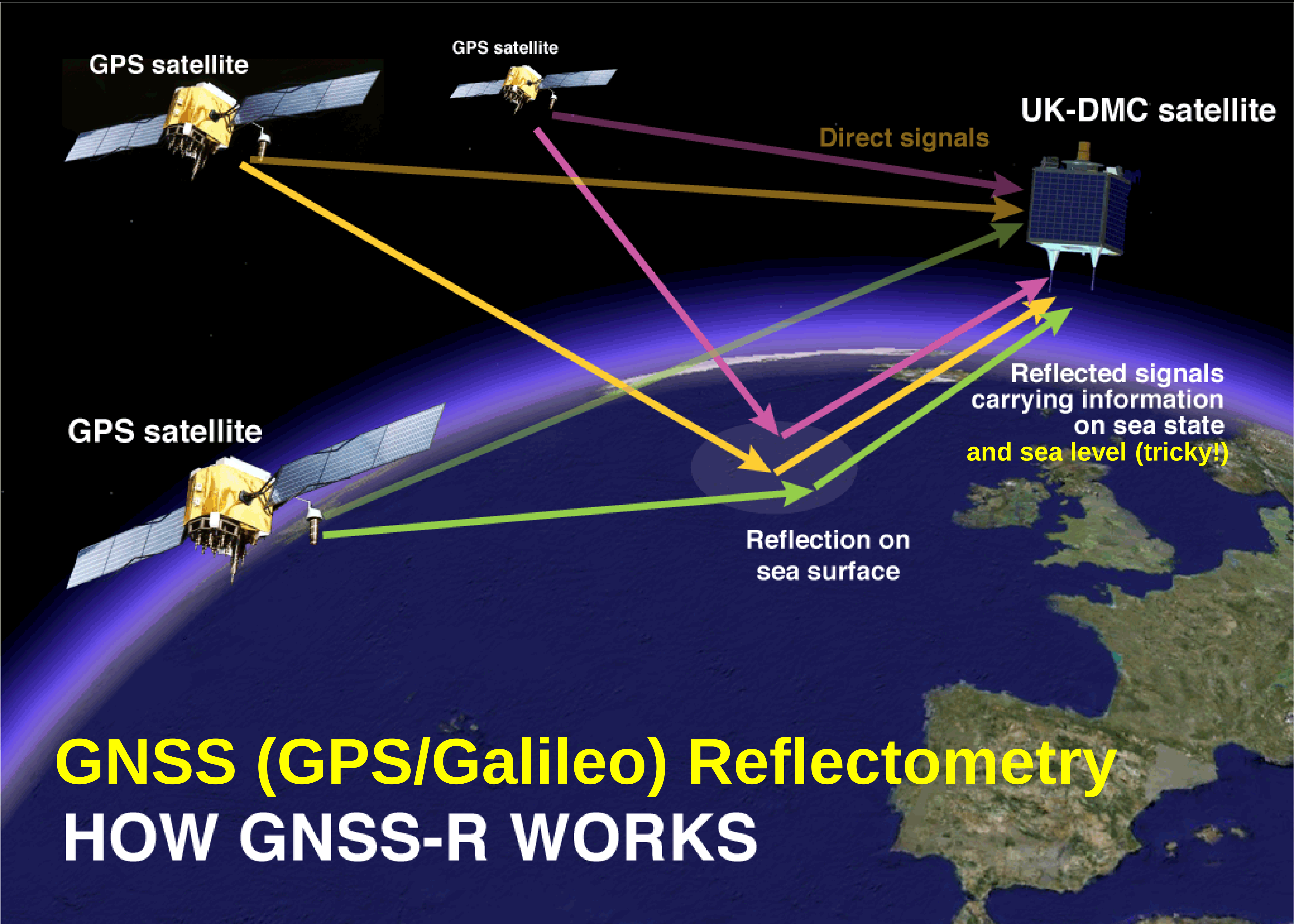


SARAL / AltiKa

- Satellite: Indian Space Research Organization (ISRO)
  - carrying **AltiKa** altimeter by CNES
  - Ka-band 0.84 cm (viz 2.2 cm at Ku-band)
  - Bandwidth (480 MHz) => 0.31 ρ (viz 0.47)
  - Otherwise "conventional" RA
  - PRF ~ 4 kHz (viz 2 kHz at Ku-band)
  - Full waveform mode



- payload includes dual-frequency radiometer
- Sun-synchronous, 35-day repeat cycle (same as ERS/Envisat)
- Navigation and control: DEM and DORIS
- Launched February 2013


GPS satellite
GPS satellite
UK-DMC satellite
Direct signals
Reflected signals
carrying information
on sea state
and sea level (tricky!)
GPS satellite
Reflection on
sea surface
GNSS (GPS/Galileo) Reflectometry
HOW GNSS-R WORKS