

Demo: Inverse Modelling/Assimilation

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Overview

BP1: Basic concepts: from remote sensing measurements to surface albedo estimates (1)

BP2: Basic concepts: from remote sensing measurements to surface albedo estimates (2)

TK1: Introduction to inverse modelling

TK2: Intro tangent and adjoint code construction

TK3: Demo: BRF inverse package and Carbon Cycle Data Assimilation System

BP3: Monitoring land surfaces: applications of inverse packages
of both surface BRF and albedo models

Recap:

- Derivative information useful to solve inverse problems
- Derivative information can be efficiently provided by tangent linear and adjoint models
- Basics of derivative coding

Outline

Application to real models

- Demo RPV inversion package
- Demo CCDAS/Network designer
- Some other packages

RPV

- RPV: Rahman, Pinty, Verstraete (1993)
- Delivers bidirectional reflectance factor (BRF)
- Has 3-4 parameters
- recall Bernard's talk !
- Inversion package available for download

Identical twin experiments

Identical twin experiments provide standard test for assimilation/inverse modelling system:

- Select values for control variables x_{true}
- Define an observation system (including uncertainties)
- Generate (cook) corresponding observations with model itself
- Select some start value for control variables x_0
- Run assimilation system (starting from x_0)

How well can the assimilation system recover x_{true} from the cooked observations?

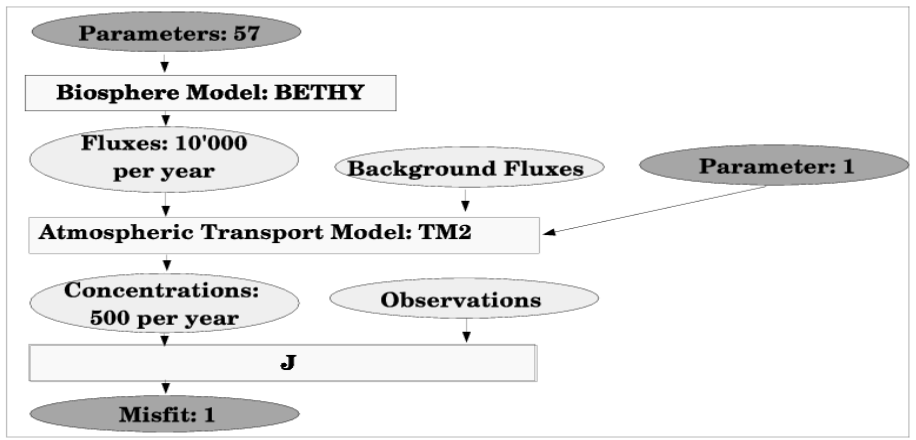
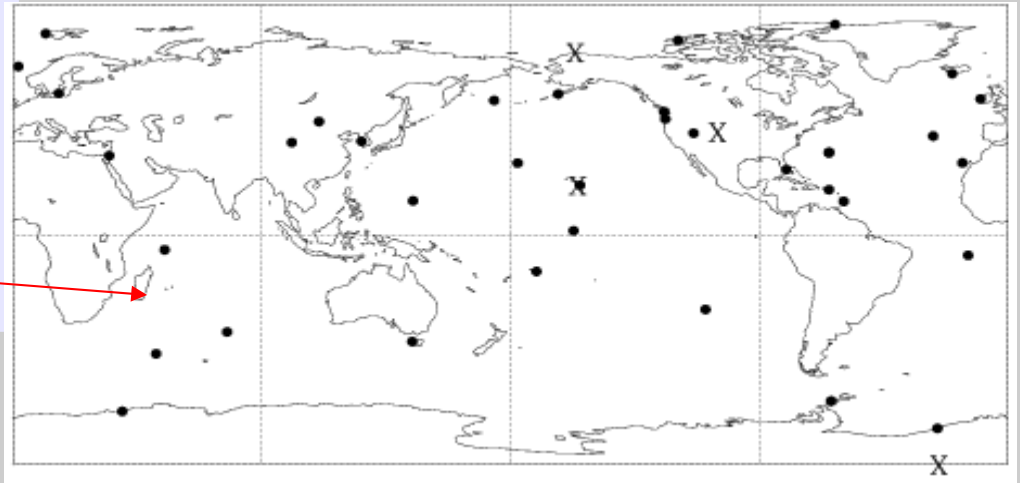
Prior term should be skipped (or given huge uncertainty):

$$J(x) = \frac{1}{2} \left(\sum_{i=1,nd} \left(\frac{M_i(x) - d_i}{\sigma_{d_i}} \right)^2 + \sum_{i=1,np} \left(\frac{x_i - p_i}{\sigma_{p_i}} \right)^2 \right)$$

But this renders the assimilation (inverse problem) harder, because constraints are relaxed

CCDAS Background calibration step

- Terrestrial biosphere model BETHY (Knorr 97) delivers CO2 fluxes to atmosphere
- Uncertainty in process parameters from laboratory measurements
- Global atmospheric network provides additional constraint



covariance of uncertainty in priors for parameters

covariance of uncertainty in measurements + model

priors for parameters

observed concentrations

$$J(x) = \frac{1}{2} [(x - x_0)^T C_x^{-1} (x - x_0) + (c(x) - d)^T C_d^{-1} (c(x) - d)]$$

Posterior Uncertainty, linear case

$$J(\tilde{\mathbf{x}}) = \frac{1}{2} [(\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d})^T \mathbf{C}(d)^{-1} (\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d}) + (\tilde{\mathbf{x}} - \mathbf{x}_0)^T \mathbf{C}(x_0)^{-1} (\tilde{\mathbf{x}} - \mathbf{x}_0)]$$

If the model is linear:

and data + priors have Gaussian PDF, then the posterior PDF is also Gaussian:

$$\rho(x) \sim e^{-J(x)}$$

with mean value:

$$\mathbf{x} = \mathbf{x}_0 + [\mathbf{M}^T \mathbf{C}(d)^{-1} \mathbf{M} + \mathbf{C}(x_0)^{-1}]^{-1} \mathbf{M}^T \mathbf{C}(d)^{-1} (\mathbf{d} - \mathbf{M}\mathbf{x}_0)$$

and uncertainty:

$$\mathbf{C}(x)^{-1} = \mathbf{M}^T \mathbf{C}(d)^{-1} \mathbf{M} + \mathbf{C}(x_0)^{-1}$$

which are related to the Hessian of the cost function:

$$\mathbf{C}(x)^{-1} = \mathbf{H}$$

$$\frac{\partial^2 J}{\partial x_i \partial x_j}$$

For a non-linear model, this is an approximation

Propagating uncertainties forward

Usually, the control variables of the inverse problem are not the quantities we are ultimately after.

Typically our real target quantities are produced by running the model with the optimised parameters either again over the assimilation period (diagnostic run) or into the future or past (prognostic run).

In addition, we would like to propagate the uncertainty in the parameter to the uncertainty in the target quantity.

If we approximate the model by its linearisation, this looks as follows:

x_{po}, C_{po} : Posterior parameters

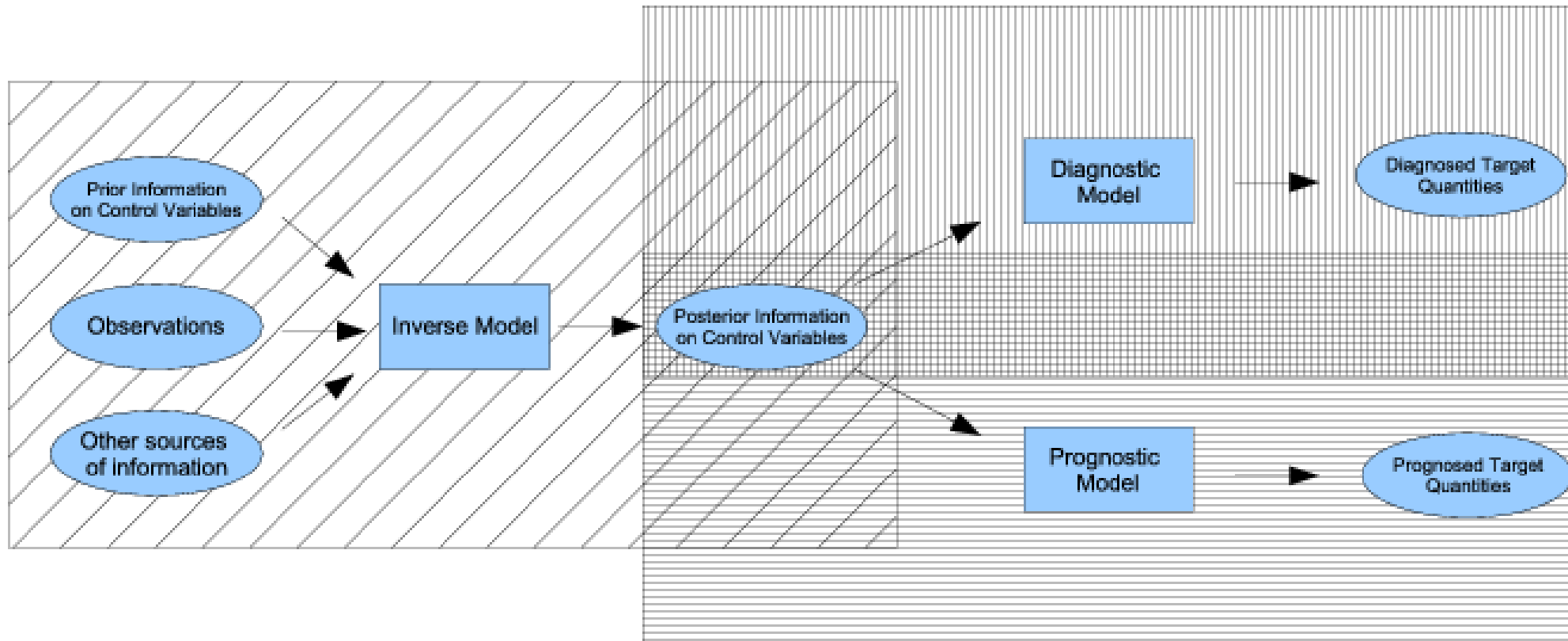
$J(x)$: Cost function

$\frac{d^2 J(x)}{dx^2}$: Hessian

$y(x), \sigma_y$: Target quantity

$$\begin{aligned} C_{po} &\approx \frac{d^2 J(x_{po})^{-1}}{dx^2} \\ \sigma_y^2 &\approx \frac{dy(x_{po})}{dx} C_{po} \frac{dy(x_{po})^T}{dx} \\ &\approx \frac{dy(x_{po})}{dx} \frac{d^2 J(x_{po})^{-1}}{dx^2} \frac{dy(x_{po})^T}{dx} \end{aligned}$$

Scheme for Analysis and Propagation of Uncertainties



Network Design

Formulation of the question:

“What is the best observational network one can build with so many stations, transects, satellites, etc... or so many Euros”

- Need to define a criterion for the quality of a given network
- Need a method for assessing the quality
- Need to run some sort algorithm that searches the space of potential networks for the best one

A bit simpler:

“How good is a given (real or synthetic) observational network?”

Uncertainty for target in 2 steps

x : Parameters

x_{pr} : Priors

C_{pr} : Uncertainties

$M(x)$: Model

d : Observations

C_d : Their uncertainties

σ_{d_i} : Uncorrelated!

$J(x)$: Cost function

$\frac{d^2 J(x)}{dx^2}$: Hessian

x_{po} : Posterior parameters

C_{po} : Posterior uncertainties

$y(x)$: Target quantity

σ_y : Its uncertainty

$$J(x) = \frac{1}{2} (x - x_{pr})^T C_{pr}^{-1} (x - x_{pr}) + \frac{1}{2} \sum_{i=1,nd} \left(\frac{M_i(x) - d_i}{\sigma_{d_i}} \right)^2$$

$$\frac{d^2 J(x)}{dx^2} = C_{pr}^{-1} + \sum_{i=1,nd} \frac{1}{\sigma_{d_i}^2} \frac{d^2}{dx^2} (M_i(x) - d_i)^2$$

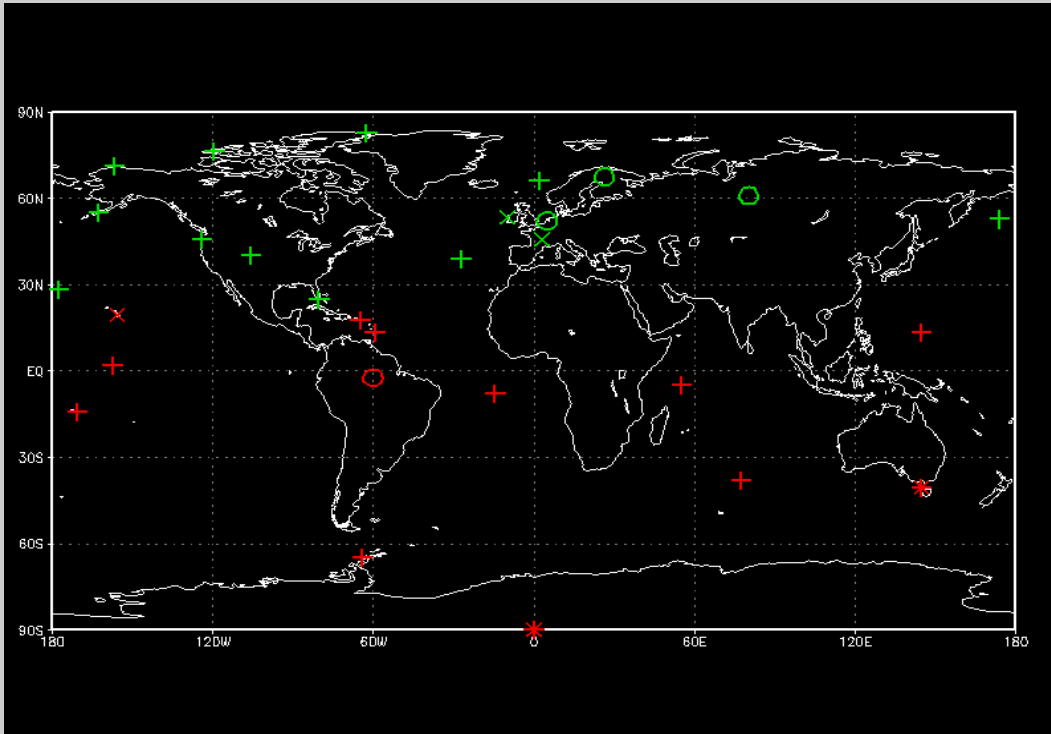
- Hessian independent of x for linear model
- For synthetic data use $d = M(x)$.
- Decomposes nicely, can precompute model contribution

$$C_{po} \approx \frac{d^2 J(x_{po})^{-1}}{dx^2}$$

$$\sigma_y \approx \frac{dy(x_{po})}{dx} C_{po} \frac{dy(x_{po})^T}{dx} \approx \frac{dy(x_{po})}{dx} \frac{d^2 J(x_{po})^{-1}}{dx^2} \frac{dy(x_{po})^T}{dx}$$

Derivative information can be efficiently provided by compiler tool TAF

Sketch of Network Designer



Observations [sigma]

+ Flask [enter]
x Continuous [enter]
o Eddy Flux [enter]

| Compute |

Targets [sigma]

European Uptake []
Global Uptake []

Funding

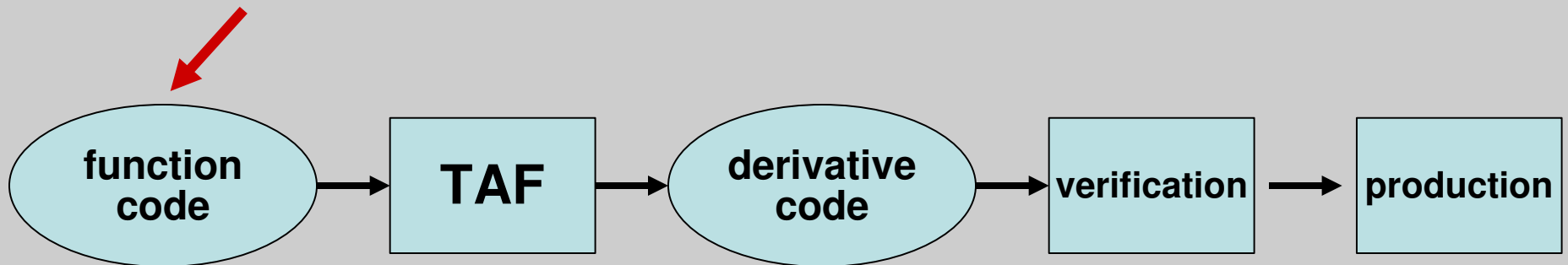
Joint effort !



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Applications: Strategy

- Apply combination of modifying function code and TAF enhancements
- For a code under development, the major effort is to make it TAF compatible once, keeping the updates TAF compatible is usually easy.
- Generated derivative code must always be verified before being used in production
- Optimise generation of derivative code, don't modify generated code by hand
This is essential to automate the process chain from new release of function code to production



Some assimilation/Inverse Modelling Systems

- RPV (<http://fapar.jrc.it/>)
- JRC-TIP: 2 stream inversion package
- Terrestrial assimilation system (<http://CCDAS.org>)
- and the corresponding network design project:(<http://IMECC.CCDAS.org>)
- Assimilation of MERIS data (<http://rs.CCDAS.org>)

- Ocean: MITGCM: Ocean state estimation (<http://MITgcm.org>)
- Tangent and Adjoint of GMAO GCM for quasi-operational NWP

In the pipe:

- Coupled ocean sea-ice assimilation for Arctic (DAMOCLES: FP6)
- Coupled atmosphere-land-ocean-ice around Planet Simulator: CPDAS (CCMAP-QUEST)

More AD applications and papers:

<http://FastOpt.com>

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