Multi-baseline PolInSAR

Basic concepts and methods

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Why multiple baselines?

Multiple baselines $\Leftrightarrow$ Illumination from multiple points of view

Multi-baseline (MB) systems:

- Multiple pass systems:
  
  * airborne and spaceborne SARs

- Multiple antenna systems:
  
  * ground based Radars

MB campaigns involve:

- Higher costs:
  
  * airborne: $\approx x \ 1$
  
  * ground based: $\approx x \ N$

- More sophisticated processing:
  
  * see single vs multi-baseline InSAR...
MB systems offers one important advantage: more equations

⇒ Increased robustness against disturbances (temporal decorrelation…)
   and/or Relaxation of hypotheses required in the single baseline case
Why multiple baselines?

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⇒ More unknowns are available to characterize the vertical structure of the scene
**MB systems offers one important advantage: more equations**

- Increased robustness against disturbances (temporal decorrelation…)
- and/or Relaxation of hypotheses required in the single baseline case
- More unknowns are available to characterize the vertical structure of the scene

**Why multiple baselines?**

**MB** PolInSAR provide access to the 3D distribution of the polarimetric properties of the scene

MB allow to pass from model based inversion to full Tomographic reconstruction

\[ N=2 \quad N=3 \quad N \text{ is large} \]

- *Top Height, Extinction*
- *Mean, Std, Skewness*
- *Backscattered Power*
- *Backscattered Power*
- *Polarimetry (alpha, entropy, …)*

\[ z \quad z \quad z \]
Outline

Introduction to SAR Tomography
• Basic Concepts
• Tomographic Scene Reconstruction
• Polarimetry and Tomography: Examples
• Phase Calibration

Optimization Methods
• Multi-layer Optimization
• Multi-baseline Coherence Optimization

Ground-volume Decomposition
• Problem Statement
• SKP Structure
• SKP Decomposition
• Regions of Physical Validity
• Boundary Solutions
• Case Studies

Conclusions
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Basic Concepts

Multiple baselines $\Leftrightarrow$ Illumination from multiple points of view

By collecting several baselines it is possible to synthesize an antenna along the cross range direction as well.

3D focusing is possible in the coordinate system: slant range, azimuth, cross range.
Resolution is determined by pulse bandwidth along the slant range direction, and by the lengths of the synthetic apertures in the azimuth and cross range directions. The SAR resolution cell is split into \textbf{multiple layers}, according to baseline aperture.

\[
\Delta r = \frac{c}{2B} \quad \Delta v = \frac{\lambda r}{2A_v} \quad \Delta x = \frac{\lambda r}{2A_x}
\]

- $B$: pulse bandwidth
- $A_v$: baseline aperture
- $A_x$: azimuth aperture
- $\lambda$: carrier wavelength

For most systems: $\Delta v \gg \Delta r, \Delta x$.

\[
\Delta z \approx \Delta v \cdot \sin(\theta)
\]
Tomographic Scene Reconstruction

Assuming typical airborne or spaceborne MB geometries, SAR Tomography can be formulated according to one simple principle:

Each focused SLC SAR image is obtained as the Fourier Transform of the scene complex reflectivity along the cross-range coordinate

\[ y_n(r, x) = \int s(r, x, v) \exp \left( -j \frac{4\pi}{\lambda r} b_n v \right) dv \]

- \( y_n(r, x) \): SLC pixel in the \( n \)-th image
- \( s(r, x, v) \): average complex reflectivity of the scene within the SAR 2D resolution cell at \( (r, x) \)
- \( b_n \): normal baseline for the \( n \)-th image
- \( \lambda \): carrier wavelength

\( \Rightarrow \) The cross-range distribution of the complex reflectivity can be retrieved through Fourier-based techniques
Performances are often limited by baseline sparseness and aperture
⇒ SAR Tomography is commonly rephrased as a Spectral Estimation problem, based on the analysis of the data covariance matrix among different tracks

Remark: it is customary to normalize $R$ such that entries on the main diagonal are unitary
⇔ $R$ is the matrix of the interferometric coherences for all baselines

$$\{R\}_{nm} = \frac{E[y_n y^*_m]}{\sqrt{E[y_n^2] E[y_m^2]}} = \gamma_{nm}$$
Spectral Estimators:

- **Beamforming:**
  
  inverse Fourier Transform; coarse spatial resolution; radiometrically consistent

  \[ \hat{S}(v) = a^H(v) \hat{R}a(v) \quad a(v) = \begin{bmatrix}
  \exp(j \frac{4\pi}{\lambda r} b_1 v) \\
  \exp(j \frac{4\pi}{\lambda r} b_2 v) \\
  \cdots \\
  \exp(j \frac{4\pi}{\lambda r} b_n v)
\end{bmatrix} \]

- **Capon Spectral Estimator:**

  spatial resolution is greatly enhanced, at the expense of radiometric accuracy:

  \[ \hat{S}(v) = \frac{1}{a^H(v) \hat{R}^{-1} a(v)} \]

- **Methods based on the analysis of the eigenstructure of R (MUSIC, ESPRIT…):**

  determination of the dominant scatterering centers; mostly suited for urban scenarios

- **Methods based on sectorial information (Truncated SVD, PCT…):**

  optimal basis choice (e.g.: Legendre), depending on a-priori info about the scene vertical extent

- **Model based methods (NLS, COMET…):**

  model based; high radiometric accuracy; high computational burden; possible model mismatches

- **Compressive sensing:**

  localization of few scattering centers via L1 norm minimization; mostly suited for urban scenarios
Example: Tomographic reconstruction of a forest scenario

Simulated Backscattered Power Distribution (BPD)

Contributions from volume backscattering
Contributions from ground backscattering
Contributions from ground-trunk interactions

|\[ |s(r,x,z)|^2 \] |

Simulated

Beamforming

Capon Spectrum

COMET - 2 Layers
Polarimetry and Tomography: Examples

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<th>BioSAR 2007 - ESA</th>
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Tomographic reconstruction of an azimuth cut:

- Reflectivity (HH) – Average on 9 tracks

The analyzed profile is almost totally forested, except for the dark areas

**HH:**
- Dominant phase center is ground locked
- Vegetation is barely visible

**Similar conclusions for VV**

**HV:**
- Dominant phase center is ground locked
- Vegetation is much more visible
Examples from BioSAR 2007

Ground Phase Center Height – Full Pol Tomography

Volume Phase Center Height – Full Pol Tomography

Histogram

\[ \sigma_{SAR-LIDAR} \approx 1 \text{ m} \]

Remarks:
Phase center estimation is carried out through parametric estimation (COMET)
Full Pol Tomography is implemented by assuming that ground and volume phase center height is invariant with polarization
Examples from BioSAR 2007

Ground Phase Center Height from HH

Volume Phase Center Height from HH

Volume Phase Center Height [m]

Full Pol Tomography

HH Tomography

Full Pol Tomography
Remark: many open areas are sensed as noise in HV, consistently with the Small Perturbation Model
Remark: slightly higher volume phase center, consistent with the hypothesis of a higher extinction coefficient in VV
### Campaigns

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<td>Sethi- ONERA</td>
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<tr>
<td>Period</td>
<td>August 2009</td>
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<tr>
<td>Site (among others)</td>
<td>Paracou, French Guyana</td>
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<tr>
<td>Scene</td>
<td>Tropical forest estimated 150 species per hectare Dominant families: Lecythidaceae, Leguminoseae, Chrysobalanaceae, Euphorbiaceae.</td>
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<td>Slant range resolution</td>
<td>≈1 m</td>
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<td>Vertical resolution</td>
<td>15 m</td>
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Tomographic reconstruction of two azimuth cuts:

Method: coherent focusing

All panels have been re-interpolated such that the ground level corresponds to 0 m

HH
Visible contribution from the ground level beneath the forest
Vegetation is well visible

HV
Poor contributions from the ground level beneath the forest
Vegetation is well visible
Tomographic reconstruction of radar scattering from four different heights
Method: coherent focusing

Polarization: HH

The strongest dependence on terrain topography is found at the ground level
The most uniform tomographic layer is found at about 15-20 m above the ground
Highest layers exhibit a dependence on terrain topography, similarly to the ground layer

Tomographic data exhibit a more complex dependence of terrain topography than traditional SAR data.
A closer look...

Examples from TropiSAR
This resolution cell gathers contributions from terrain only.

=> Signal intensity in this cell is affected by terrain slope the same way as in traditional SAR images of bare surfaces.
This cell is completely within the volume layer, independently on volume orientation w.r.t. the Radar LOS.
=> Signal intensity in this cell is independent of terrain slope

This resolution cell gathers contributions from terrain only.
=> Signal intensity in this cell is affected by terrain slope the same way as in traditional SAR images of bare surfaces
This cell is completely within the volume layer, independently on volume orientation w.r.t. the Radar LOS.
=> Signal intensity in this cell is independent of terrain slope.

The scattering volume within cells at the boundaries of the vegetation layer depends on volume orientation w.r.t. the Radar LOS.
=> Signal intensity in this cell is affected by terrain slope in a similar way as the cell corresponding to the ground layer.

This resolution cell gathers contributions from terrain only.
=> Signal intensity in this cell is affected by terrain slope the same way as in traditional SAR images of bare surfaces.
Co-polar signature at the ground layer reveals ground-trunk double bounce interactions dominate the signal from flat areas despite the presence of a 40 m dense tropical forest.
TropiSCAT - ESA - 2011

- a static ground-based radar observing a tropical forest
  - Located in French Guyana - same site as TropiSAR
  - Team members from ONERA, CNES, CESBIO, POLIMI
  - Automatic and systematic acquisition
  - Fully polarimetric (HH, HV, VH and VV)
  - Tomographic capability (to have a vertical discrimination of backscattering mechanisms)
  - Coupled with geophysical parameters measurements (provided by INRA - National Institute for Agronomic Research)

- GOAL: provide continuous observations (15 mn sampling) over a time span of one year
Examples from TropiSCAT
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MB PolInSAR
Vertical resolution ≈ 1 ÷ 15 m
N ≈ 6 ÷ 50

Vertical resolution ≈ 10 ÷ 30 m
N ≈ 6 ÷ 15

Vertical resolution >> 30 m
N ≥ 2

Single Baseline PolInSAR
N = 2
Multi-layer Optimization

The analysis so far has been limited to the comparison of Tomographic results from different polarizations. Further information can be extracted by *jointly* exploiting baseline and polarization diversity.

Multi-layer optimization techniques do this by finding the *optimum* polarization for *each* layer:

Two benefits:
- Enhanced classification capabilities
- Tomographic resolution is improved
Multi-layer optimization techniques extend single-pol Spectral Estimators by considering the data covariance matrix among all tracks and all polarizations.

Data vector [3Nx1]: \[ y = \begin{bmatrix} y_{MB}(w_1) \\ y_{MB}(w_2) \\ y_{MB}(w_3) \end{bmatrix} \]

Data covariance matrix [3Nx3N]: \[ W = E[yy^H] = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \]
Multi-layer Optimization

Multi-layer optimization techniques extend single-pol Spectral Estimators by considering the data covariance matrix among all tracks and all polarizations. In most cases, the extension from single-pol to multi-pol is simply obtained through an eigenvalue problem.

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<th>Capon Estimator</th>
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<tbody>
<tr>
<td>$\hat{S}(v) = a^H(v)\hat{R}a(v)$</td>
<td>$\hat{S}(v) = (a^H(v)\hat{R}^{-1}a(v))^{-1}$</td>
</tr>
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<td>$\hat{S}_{MP}(v) = \max_k \left{ k^H(B^H(v)\hat{W}B(v))k \right}$</td>
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<tr>
<td>$\iff (B^H(v)\hat{W}B(v))k_{opt} = \lambda_{\max} k_{opt}$</td>
<td>$\iff (B^H(v)\hat{W}^{-1}B(v))k_{opt} = \lambda_{\min} k_{opt}$</td>
</tr>
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</table>
Example: Separation of two closely spaced scattering centers

Simulated Backscattered Power Distribution (BPD)

Fourier Spectrum

Capon Spectrum

Resolution Enhancement – Capon Spectrum

Double Bounce Scattering (Dihedral)
Single Bounce Scattering (Trihedral)

Target polarization diversity: $\text{acos}(\langle \mathbf{k}_1, \mathbf{k}_2 \rangle)$ [deg]
A real world example: imaging of a truck under the foliage

From: Y. Huang, L. Ferro-Famil, A. Reigber, “Under Foliage Object Imaging Using SAR Tomography and Polarimetric Spectral Estimators,” Eusar 2010 – Courtesy of the authors

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<th>System</th>
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<tbody>
<tr>
<td>Site</td>
<td>Dornstetten, Germany</td>
</tr>
<tr>
<td>Tomographic tracks</td>
<td>21 – Fully Polarimetric</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>L-Band</td>
</tr>
<tr>
<td>Vertical resolution</td>
<td>2 m</td>
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Multi-baseline Coherence Optimization

Coherence optimization enhances InSAR capabilities by allowing the analysis of multiple targets with different polarimetric responses within the same resolution cell.

Example: resolving three closely spaced point scatterers

The interferometric coherences associated with the three points alone are obtained by optimizing w.r.t. the projection vector:

\[
\gamma(S_k) = \gamma(w^{(k)}_{opt}, w^{(k)}_{opt})
\]

\[
w^{(k)}_{opt} = \arg \max \{ \gamma(w, w) \}
\]

where:

\[
\gamma(w_i, w_j) = \frac{E[y_n(w_i)y^*_m(w_j)]}{\sqrt{E[y_n(w_i)^2]E[y_m(w_j)^2]}}
\]
Multi-baseline Coherence Optimization

Coherence optimization enhances InSAR capabilities by allowing the analysis of multiple targets with different polarimetric responses within the same resolution cell. MB coherence optimization methods simultaneously optimize coherences in several baselines. Thus, they are expected to deliver more robust estimates:

Two approaches are considered:

Multiple Scattering Mechanisms (MSM)
A distinct SM is assigned to each track.

\[
\max \left\{ \sum_{n=1}^{N} \sum_{m=1}^{N} |\gamma_{nm}(w_n, w_m)| \right\} : \angle w_n^H w_m = 0
\]

- Fit for SMs that might have different polarimetric signatures in different tracks
- Robust to miscalibration

Equalized Scattering Mechanism (ESM)
Enforces equal polarimetric signatures of scatterers along all baselines

\[
\max \left\{ \sum_{n=1}^{N} \sum_{m=1}^{N} |\gamma_{nm}(w, w)| \right\}
\]

- Implies data stationarity
- Leads to lower coherence magnitudes,
- Processes all available information by enforcing more constraints, and thus more accurately
Multi-baseline Coherence Optimization

A real world example: MB coherence optimization


Coherence between passages 1 and 2 associated with the dominant SM

Remarks:
SB optimized coherences achieve higher values than MB

Relevant contrast improvement of MB over SB, particularly over forested areas.
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Problem Statement

Decompose the data covariance matrix into ground-only and volume-only contributions

\[ y = \begin{bmatrix} y_{MB}(w_1) \\ y_{MB}(w_2) \\ y_{MB}(w_3) \end{bmatrix} \]

\[ W = E[yy^H] = W_g + W_v \]
Ground-volume decomposition implies:

- **Separation of Structural Properties**
  
  => Separated Tomographic Imaging of Ground-only and Volume-only Contributions

- **Separation of Polarimetric Properties**
  
  => Evaluation of the Ground to Volume Backscattered Power Ratio for each polarization
SKP Structure

Without loss of generality, the received signal can be assumed to be contributed by \( K \) distinct Scattering Mechanisms (SMs), representing ground, volume, ground-trunk scattering, or other

\[
y_n(w_i) = \sum_{k=1}^{K} s_k(n; w_i) \quad s_k(n, w_i) : \text{contribution of the } k\text{-th SM in Track } n, \text{Polarization } w_i
\]

Three fundamental hypotheses will be retained:

H1): **Statistical independence** among different SMs

H2): **Invariance of the interferometric coherences** of each SM w.r.t. polarization

\[ \Rightarrow \] negligible variation of the EM properties of each SM (subsurface penetration, volume extinction,…) w.r.t. polarization

H3): **Invariance of the polarimetric signature** of each SM on the choice of the track

\[ \Rightarrow \] events like floods, fires, frosts, are expected not to occur during the acquisition campaign

\[
E[y_n(w_i)y_m^*(w_j)] = \sum_{k=1}^{K} c_k(w_i, w_j) \cdot \gamma_k(n, m)
\]

\( c_k(w_i, w_j) : \text{polarimetric correlation of the } k\text{-th SM in polarizations } w_i, w_j \)

\( \gamma_k(n,m) : \text{interferometric coherence of the } k\text{-th SM in the } nm\text{-th interferogram} \)

\[
c_k(w_i, w_j) = E[s_k(n; w_i)s_k^*(m; w_j)]
\]

\[
\gamma_k(n,m) = \frac{E[s_k(n; w_i)s_k^*(m; w_j)]}{\sqrt{E[s_k(n; w_i)^2]E[s_k(m; w_j)^2]}}
\]
The same result is expressed in matrix form as a Sum of Kronecker Products (SKP)

\[
E[y_n(w_i)y_m^*(w_j)] = \sum_{k=1}^{K} c_k(w_i, w_j) \cdot \gamma_k(n, m) \iff W = E[yy^H] = \sum_{k=1}^{K} C_k \otimes R_k
\]

Each SM is represented by a Kronecker Product (KP) of two matrices:

**Polarimetric Signature,** \(C_k:\)
- polarimetric covariance matrix of the \(k\)-th SM alone \([3 \times 3]\)
- \(\iff\) Electromagnetic properties of the \(k\)-th SM

\[
C_k = \begin{bmatrix}
    c_k(w_1, w_1) & c_k(w_1, w_2) & c_k(w_1, w_3) \\
    c_k(w_2, w_1) & c_k(w_2, w_2) & c_k(w_2, w_3) \\
    c_k(w_3, w_1) & c_k(w_3, w_2) & c_k(w_3, w_3)
\end{bmatrix}
\]

**Structure Matrix,** \(R_k:\)
- matrix of the interferometric coherences of the \(k\)-th SM alone \([N \times N]\)
- \(\iff\) Backscattered power distribution of the \(k\)-th SM

\[
R_k = \begin{bmatrix}
    \gamma_k(1,1) & \gamma_k(1,2) & \cdots & \gamma_k(1,N) \\
    \gamma_k(2,1) & \gamma_k(2,2) & \cdots & \gamma_k(2,N) \\
    \vdots & \vdots & \ddots & \vdots \\
    \gamma_k(N,1) & \gamma_k(N,2) & \cdots & \gamma_k(N,N)
\end{bmatrix}
\]

\(R_k, C_k\) are (semi)positive definite by definition.
SKP Decomposition

SKP Decomposition = *fast* technique for the decomposition of *any* matrix into a SKP

\[ W \rightsquigarrow \text{SKP Dec} \rightsquigarrow \text{Two sets of matrices } U_p, V_p \text{ such that: } W = \sum_{p=1}^{K} U_p \otimes V_p \]

**Theorem:**

Let \( W \) be contributed by \( K \) SMs according to H1,H2,H3, i.e.: \[ W = \sum_{k=1}^{K} C_k \otimes R_k \]

*then*, the matrices \( U_k, V_k \) are related to the matrices \( C_k, R_k \) via a linear, invertible transformation defined by **exactly** \( K(K-1) \) real numbers.

**Corollary:**

*If* only ground and volume scattering occurs, i.e: \[ W = C_g \otimes R_g + C_v \otimes R_v \]

*then*, there exist two real numbers \((a,b)\) such that: \[ C_g = (a-b)^{-1}((1-b)U_1 - bU_2) \]
\[ R_g = aV_1 + (1-a)V_2 \]
\[ C_v = (a-b)^{-1}(-(1-a)U_1 + aU_2) \]
\[ R_v = bV_1 + (1-b)V_2 \]
Region of Physical Validity

How to find \((a,b)\)?

- Select values of \((a,b)\) that give rise to (semi) positive definite \(C_g, C_v, R_g, R_v\)

\(\Leftrightarrow\) Region of Physical Validity (RPV): all solutions within this region are physical validity of the solution

- Explore all the solutions within the RPV and pick the best one according to some criterion
Physically valid ground and volume coherence between passages 1 and 2

The RPV is formed by two branches, spanned by the parameters \( (a,b) \)

Single-baseline (N=2):

The union of branches \( a, b \) results in the same region of physical validity as in PolInSAR

\[ \Rightarrow \] Consistency with single-baseline methods!

Multi-Baseline (N>2):

The positive definitiveness constraint results in the regions of physical validity to shrink from the outer boundaries towards the true ground and volume coherences

\[ \Rightarrow \] The higher the number of tracks, the easier it is to pick the correct solution
Boundary Solutions

By definition, the points at the outer or inner boundaries of the two branches correspond to the case where one of the four matrices $C_g, C_v, R_g, R_v$ is singular. Each of the boundary solutions has a specific physical interpretation.

Branch $a \iff$ 
- **ground** structure matrix $R_g$ and **volume** polarimetric signature $C_v$

Branch $b \iff$ 
- **volume** structure matrix $R_v$ and **ground** polarimetric signature $C_g$
Case Studies

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Vegetation is barely visible

Similar conclusions for VV

HV:
Dominant phase center is ground locked
Vegetation is much more visible
Model validation: $\mathbf{W} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v$

Methodology:
- evaluation of the error between the sample covariance matrix and its best L2 approximation with $K = \{1, 2, 3, 4\}$ KPs

Remark: the best L2 approximation is obtained simply by taking the dominant $K$ terms of the SKP decomposition

\[
\|\hat{\mathbf{W}}_K - \mathbf{W}\|_F = \min_{\mathbf{W}_K} \|\hat{\mathbf{W}} - \mathbf{W}_K\|_F
\]

\[
\text{error} = \frac{\|\hat{\mathbf{W}} - \mathbf{W}_K\|_F}{\|\hat{\mathbf{W}}\|_F}
\]

\[
\|\hat{\mathbf{W}} - \hat{\mathbf{W}}_2\|_F < 0.1 \cdot \|\hat{\mathbf{W}}\|_F
\]

$> 90\%$ of the information can be represented by the sum of just two KPs
Inner boundary solutions

Significant contributions from the ground level.

\(\Rightarrow\) Volumetric scattering at the ground level

Consistent with:
- Backscattering from understorey or lower tree branches
- Multiple interactions of volumetric scatterers with the ground

Residual volume contributions visible above the ground
Intermediate solutions

Volumetric contributions from the ground level are partly rejected

Backscattering contributions from the whole volume structure are emphasized

**Improved volume rejection**

**LIDAR Terrain Height**
**LIDAR Forest Height**
Intermediate solutions

Improved ground rejection

Backscattering contributions from the whole volume structure are emphasized

Case Studies: BioSAR 2007

LIDAR Terrain Height
LIDAR Forest Height
Intermediate solutions

Improved volume rejection

Ground contributions rejected

Contributions from the lower canopy are partly rejected

Backscattering contributions from the upper volume structure are emphasized
Outer boundary solutions

Ground and lower canopy contributions are rejected

Only upper canopy contributions are visible

Volume structure is maximally coherent

Volume top height is nearly invariant to the choice of the solution, therefore constituting a robust indicator of the volume structure

Maximum volume rejection

Ground structure is maximally coherent

LIDAR Terrain Height
LIDAR Forest Height
**Case Studies: BioSAR 2008**

<table>
<thead>
<tr>
<th>Campaign</th>
<th>BioSAR 2008 - ESA</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td>E-SAR - DLR</td>
</tr>
<tr>
<td>Site</td>
<td>Krycklan river catchment, Northern Sweden</td>
</tr>
<tr>
<td>Scene</td>
<td>Boreal forest</td>
</tr>
<tr>
<td>Topography</td>
<td>Hilly</td>
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<tr>
<td>Tomographic Tracks</td>
<td>6 + 6 – Fully Polarimetric (South-West and North-East)</td>
</tr>
<tr>
<td>Carrier Frequency</td>
<td>P-Band and L-Band</td>
</tr>
<tr>
<td>Slant range resolution</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Azimuth resolution</td>
<td>1.6 m</td>
</tr>
<tr>
<td>Vertical resolution (P-Band)</td>
<td>20 m (near range) to &gt;80 m (far range)</td>
</tr>
<tr>
<td>Vertical resolution (L-Band)</td>
<td>6 m (near range) to 25 m (far range)</td>
</tr>
</tbody>
</table>
Tomographic Reconstruction of an azimuth cut:
Polarization: HV
Method: Capon Spectrum

Results are geocoded onto the same ground range, height grid

All panels have been re-interpolated such that the ground level corresponds to 0 m

Loss of resolution from near to far range, especially at P-Band ($\Delta z > 80$ m at far ranges)

Relevant contributions from the ground level below the forest are found at P-Band
Model validation: \( \mathbf{W} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v \)

Methodology:
- evaluation of the error between the sample covariance matrix and its best L2 approximation with \( K = \{1, 2, 3, 4\} \) KPs

Remark: the best L2 approximation is obtained simply by taking the dominant \( K \) terms of the SKP decomposition

\[
\hat{\mathbf{W}}_K = \arg \min_{\mathbf{W}_K} \left\{ \| \hat{\mathbf{W}} - \mathbf{W}_K \|_F \right\}
\]

\[
\text{error} = \| \hat{\mathbf{W}} - \mathbf{W}_K \|_F
\]

For \( P\)-Band:

\[
\left\| \hat{\mathbf{W}} - \hat{\mathbf{W}}_2 \right\|_F < 0.05 \cdot \left\| \mathbf{W} \right\|_F
\]

> 95% of the information can be represented by the sum of just two KPs

For \( L\)-Band:

\[
\left\| \hat{\mathbf{W}} - \hat{\mathbf{W}}_2 \right\|_F < 0.1 \cdot \left\| \mathbf{W} \right\|_F
\]

> 90% of the information can be represented by the sum of just two KPs
Backscattered Power Distribution for Ground Scattering

Outer Boundary Solution

Significant rejection of volume contributions

Better results at P-Band, due to better ground visibility

Some leakage from the volume is present at L-Band in areas with dense forest and steep slopes
Backscattered Power Distribution for Volume Scattering

**Inner Boundary Solution**

\( C_g \) is singular

This solution corresponds to the polarization which is supposed not to be affected by ground contributions.

**P-Band**

Significant contributions from the ground level.

\[ \Leftrightarrow \] Volumetric scattering at the ground level

Consistent with:

- Backscattering from understorey or lower tree branches
- Multiple interactions of volumetric scatterers with the ground
Backscattered Power Distribution for Volume Scattering

Intermediate Solution

By moving from the inner to the outer boundary the contributions from the ground level are gradually rejected

P-Band
Backscattering contributions from the whole volume structure are emphasized

L-Band
Contributions from the lower canopy are partly rejected
Backscattering contributions from the upper volume structure are emphasized

C_g is full rank
Backscattered Power Distribution for Volume Scattering

Outer Boundary Solution

$C_g$ is full rank

Only upper canopy contributions are visible, due to rejection of ground and lower canopy contributions

This phenomenon is more evident at P-Band, due to the coarse vertical resolution

Volume top height is nearly invariant to the choice of the solution, confirming the result of BioSAR 2007
### Case Studies: TropiSAR

<table>
<thead>
<tr>
<th>Campaign</th>
<th>TropiSAR- ESA</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td>Sethi- ONERA</td>
</tr>
<tr>
<td>Period</td>
<td>August 2009</td>
</tr>
<tr>
<td>Site (among others)</td>
<td>Paracou, French Guyana</td>
</tr>
<tr>
<td>Scene</td>
<td>Tropical forest estimated 150 species per hectare Dominant families: Lecythidaceae, Leguminoseae, Chrysobalanaceae, Euphorbiaceae.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tomographic tracks</th>
<th>6 – Fully Polarimetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>P-Band</td>
</tr>
<tr>
<td>Slant range resolution</td>
<td>≈1 m</td>
</tr>
<tr>
<td>Azimuth resolution</td>
<td>≈1 m</td>
</tr>
<tr>
<td>Vertical resolution</td>
<td>15 m</td>
</tr>
</tbody>
</table>
Tomographic Reconstruction of an azimuth cut:

**Case Studies: TropiSAR (courtesy of ONERA)**

HV:
Poor contributions from the ground level beneath the forest
Vegetation is well visible

Visible contribution from the ground level beneath the forest
Vegetation is well visible
Model validation: \( \mathbf{W} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v \)

Methodology:
- evaluation of the error between the sample covariance matrix and its best L2 approximation with \( K = \{1,2,3,4\} \) KPs

Remark: the best L2 approximation is obtained simply by taking the dominant \( K \) terms of the SKP decomposition

\[
\| \hat{\mathbf{W}} - \hat{\mathbf{W}}_2 \|_F < 0.1 \cdot \| \hat{\mathbf{W}} \|_F
\]

\( > 90\% \) of the information can be represented by the sum of just two KPs

\[
\hat{\mathbf{W}}_K = \arg \min_{\mathbf{W}_K} \left\{ \| \hat{\mathbf{W}} - \mathbf{W}_K \|_F \right\}
\]

\[
\text{error} = \frac{\| \hat{\mathbf{W}} - \mathbf{W}_K \|_F}{\| \hat{\mathbf{W}} \|_F}
\]
Inner boundary solutions

Poor contributions from the ground level beneath the forest

Volume structure appears to be evenly distributed

Case Studies: TropiSAR

Strong volume contributions visible above the ground
Intermediate solutions

Backscattering contributions from the upper volume structure are emphasized.

Case Studies: TropiSAR

Improved volume rejection
Outer boundary solutions

Ground and lower canopy contributions are rejected

Only upper canopy contributions are visible

Volume structure is maximally coherent

Volume top height is nearly invariant to the choice of the solution

Case Studies: TropiSAR

Maximum volume rejection

Ground structure is maximally coherent
Introduction to SAR Tomography
  • Basic Concepts
  • Tomographic Scene Reconstruction
  • Polarimetry and Tomography: Examples
  • Phase Calibration

Optimization Methods
  • Multi-layer Optimization
  • Multi-baseline Coherence Optimization

Ground-volume Decomposition
  • Problem Statement
  • SKP Structure
  • SKP Decomposition
  • Regions of Physical Validity
  • Boundary Solutions
  • Case Studies

Conclusions
Conclusions

Multi-baseline Polarimetric SAR Tomography

- expensive (need multiple passes)
- non-trivial processing (accurate phase calibration, advanced Spectral Estimation techniques w.r.t. 2D SAR focusing)

Yet, it allows to see the vertical structure of distributed media (for every polarization)

≣ Natural tool for validation and development of physical models

Joint multi-baseline – multi-polarimetric processing

- Signal space is enlarged => further elements of diversity

≣ Killer application for coarse vertical resolution (i.e.: few baselines) TomSAR campaigns

Where do we go now?

- How to get radiometric accuracy and super-resolution imaging of distributed media?
- How to embed temporal decorrelation models into multi-baseline scenarios?
- 3D target reconstruction in presence of dielectric media (ice/sand).
References

Polarimetric and tomographic phenomenology of forests


Multi-layer optimization

- S. Sauer, L. Ferro-Famil, A. Reigber, E. Pottier, “Multi-aspect POL-InSAR 3D Urban Scene Reconstruction at L-Band”, *Eusar 2008*
- Y. Huang, L. Ferro-Famil, A. Reigber, “Under Foliage Object Imaging Using SAR Tomography and Polarimetric Spectral Estimators”, *Eusar 2010*

Coherence optimization


SKP decomposition: theory, algorithms and physical implications