A KALMAN FILTER BASED MTINSAR METHODOLOGY FOR DERIVING 3D SURFACE DISPLACEMENT EVOLUTIONS


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ABSTRACT

Here we present a novel approach of deriving three-dimensional surface displacement from multi-sensor, multi-track and multi-temporal interferograms. The interferogram variance, e.g., temporal coverage and radar look direction, are eliminated by Kalman filter, which observation and state models have been designed for our InSAR measurements. 21 SAR images over Los Angeles from ENVISAT ascending orbit, ENVISAT descending orbit and ALOS ascending orbit are exploited to validate our approach.

Keywords: InSAR, multi-sensor, multi-track, Kalman filter, three-dimension

1. INTRODUCTION

Multi-temporal InSAR (MTInSAR) [1, 2] have been used widely for studying earth surface deformations related to many geophysical processes. However, MTInSAR techniques have been able to measure one-dimensional (1D) surface deformations in the direction of the line-of-sight (LOS) of the radar. As surface deformations are usually three-dimensional, one-dimensional observation apparently cannot always fully reflect the actual deformations [3]. In addition, the temporal resolution of MTInSAR measurements is limited by the satellite orbit repeat period.

The number of SAR satellites has been increasing rapidly in recent years. It is therefore very desirable to combine the observations from the different SAR satellites and orbits to derive more comprehensive surface deformation measurements.

We present a novel new MTInSAR approach for exploiting multi-sensor, multi-track and multi-temporal interferograms to infer three-dimensional (3D) surface displacements. The proposed approach is based on Kalman filter that has been widely used for modeling various dynamic processes [4]. First, the 1D LOS measurements are estimated from multi-sensor, multi-track and multi-temporal interferograms. The observation model and state models of the Kalman filter are then constructed by considering the imaging geometry and temporal correlation. The 3D surface displacement at all the acquisition times can be estimated based on the models and a weighting scheme that reflects the noise levels of the observations and the deformations.

The accuracy of the measurements in the north-south directions is low due to the polar orbits of the current SAR satellites [3, 5]. In order to ensure the accuracy of the results in the up and east-west directions, we assume that the deformation in the north-south direction is negligible in the case study carried out for the Los Angeles area. The experiment uses 21 SAR acquisitions from ENVISAT ascending and descending orbits and PALSAR ascending orbits. The results are compared with GPS measurements in the area [6, 7].

2. METHODOLOGY

Let us assume a set of $M$ unwrapping differential interferograms be generated from $N+1$ images. The interferograms are generated from the image pairs...
with small baseline for reducing possible errors caused by spatial decorrelation and topographic inaccuracy. For the interferogram formed by SAR acquisitions at time $t_A$ and $t_B$, the phase is a combination of surface deformation in the LOS direction, topographic artifacts and other errors. Note that the LOS deformation $d_{los}$ can be written as follows:

$$d_{los} = [a \ b \ c] \cdot [d_u \ d_e \ d_n]^T$$  \hspace{1cm} (1)

where $d_u$, $d_e$ and $d_n$ are the three-dimensional accumulative deformations occurring in the up-down, east-west and north-south directions, respectively. $a$, $b$ and $c$ are projection vectors of three-dimensional to LOS, which are determined by imaging geometry. $\alpha$ represents the azimuth angle, i.e. the clockwise deviate angle from the North to the satellite flight direction.

For single interferogram, displacement models (e.g. linear model) are usually applied to estimate low-pass (LP) displacement and topographic error:

$$\phi(t_i) = v_{los} \cdot (t_i - t_0) = (a \cdot v_u + b \cdot v_e + c \cdot v_n) \cdot (t_i - t_0)$$  \hspace{1cm} (2)

where $t_0$ is reference time and $\phi(t_0) = 0$. Then, the unknown parameters become:

$$X^T = [v_u \ v_e \ v_n \ \Delta z]$$  \hspace{1cm} (3)

If the aforementioned $M$ interferograms are grouped by $M'$ tracks (obviously $M \geq M'$), a linear system with $M$ observations and four unknowns can be formed as:

$$\delta \phi = [\Lambda \Pi, \epsilon] X$$  \hspace{1cm} (4)

where $A$ is an $(M \times N)$ design matrix corresponding to the unknown deformation values at a series of time. $\epsilon^T$ is a $(M \times 1)$ matrix to the topographic artifacts. $\Pi$ is an $(N \times 3)$ matrix corresponding to the projection vectors. In theory, the unknowns of 3D LP displacements and topographic error in Equation (5) can be easily resolved by least square adjustment when $M \geq 4$ and $M' \geq 3$.

More complicated displacement model, e.g. cubic and seasonal model, can be adopted if adequate interferograms are provided.

### 2.1. Kalman Filter Model

On the basis of above analysis, we find it is the uncoincidences between the acquisition times of interferograms hampering the immediately integration of these interferograms. However, Kalman filter is a notably approach which can neglect the gaps by treating the observations as a set of dynamic data.

Similarly, the LOS displacements derived from the $M$ unwrapping differential interferograms of $M'$ tracks are served as our Kalman filter observations. For simply clarifying the problem, we assume the $i$-th interferogram is generated by the image pair which the master and slave are acquired at time of $t_0$ and $t_i, i = 1, 2, \ldots, M$. We formulate the correspondingly observation and state models for Kalman filter:

$$L(i) = H(i)X(i) + V(i)$$  \hspace{1cm} (5)

$$X(i) = F(i / i - 1)X(i - 1) + \Gamma(i - 1)W(i - 1)$$  \hspace{1cm} (6)

where

$$X(i)^T = [d_u(i) \ d_e(i) \ d_n(i) \ v_u(i) \ v_e(i) \ v_n(i)]$$
are the unknowns formed by three-dimensional surface displacement vectors and three-dimensional instantaneous rate vectors at the time of $t_i$.

$$H(i) = \begin{bmatrix} a(i) & b(i) & c(i) & 0 & 0 & 0 \end{bmatrix}$$

is observation distribution matrix corresponding to the projection vectors of $i$-th interferogram.

$F(i / i-1)$ is state transition matrix, and $I_3$ is $3 \times 3$ identity matrix, $\Delta t$ is time interval between the times of $t_{i-1}$ and $t_i$.

$V(i)$ are observation noises, i.e., the noises contained in the $i$-th interferogram which are induced by spatial-temporal decorrelation and atmospheric artifacts.

$W(i)^T = \begin{bmatrix} w_u(i) & w_v(i) & w_a(i) \end{bmatrix}$ are state noises, which are regarded as three-dimensional acceleration vectors at the time of $t_i$, with $\Gamma(i / i-1)^T$ being state noises distribution matrix.

In such a way, inferring three-dimensional surface displacements $\begin{bmatrix} d_u(i) & d_v(i) & d_a(i) \end{bmatrix}$ from LOS measurements $L(i)$ become a common dynamical problem for the Kalman filter [4]:

$$\begin{align*}
\hat{X}(i / i-1) &= F(i / i-1)\hat{X}(i-1) \\
D_{\hat{X}}(i / i-1) &= F(i / i-1)D_{\hat{X}}(i-1)F(i / i-1)^T \\
&+ \Gamma(i / i-1)D_{\nu}(i-1)\Gamma(i / i-1)^T \\
J(i) &= D_{\hat{X}}(i / i-1)H(i)^T \cdot \\
&\left[ H(i)D_{\hat{X}}(i / i-1)H(i)^T + D_{\nu}(i) \right]^{-1} \\
\hat{X}(i) &= \hat{X}(i / i-1) \\
&+ J(i) \left[ L(i) - H(i)\hat{X}(i / i-1) \right] \\
D_{\hat{X}}(i) &= \left[ E - J(i)H(i) \right]D_{\hat{X}}(i / i-1) \\
\end{align*}$$

where $J(i)$ is filtering gain matrix. $\hat{X}(i / i-1)$ and $\hat{X}(i)$ are the predicted and update values for the unknowns, with correspondingly variances $D_{\hat{X}}(i / i-1)$ and $D_{\hat{X}}(i)$, respectively.

### 2.2. Processing algorithm

In the realistic scenario, it is virtually important to provide possible precise initial values in InSAR application as our filtering epochs are great limited by the satellite repeat frequency. However, the 3D instantaneous rate vectors and correspondingly variances are usually difficult to exactly identify without any priori information. A practical strategy is using the estimations from the 3D LP displacements resolving as the approximation of initial rate states.

We also need to consider the availabilities of interferograms which are not involved in the Kalman filter models constructing. In SBAS methodology, only interferograms with small baselines are exploited in order to reduce spatial decorrelation, topographic errors and unwrapping errors [2]. An analogous strategy is adopted in our Kalman filter approach.

Moreover, some discontinuities would like to be presented in the observation sequence, which are
caused by the interval between the acquisition times of the first images from different tracks. As the repeat periods of different orbits and different platforms are generally diverse from each other, e.g. 35 days for ENVISAT and 46 days for ALOS, it is practically impossible to get synchronization of their acquisition time. The state model can be used to fill these “gaps” by predicting the state of subsequential image acquisition time from the estimation of preceding image acquisition time.

As far as surface displacement estimations in the north-south direction are concerned, we remark that it is also a challenge to promise high precision north-south displacement results from Kalman filter approach. Especially in the case of only-right look interferograms being available, any reliable estimation is not expected in this direction [5]. Sometimes when the north-south deformations are not significant, an assumption of zero to the north-south deformation is practical in order to guarantee high precisions of the results in the other two directions, i.e. the up-down and east-west vectors.

3. RESULTS
We have 21 SAR acquisitions on the area of Los Angeles from Nov. 15, 2006 to Nov. 18, 2007, including 5 ASAR images from ENVISAT ascending orbit, 9 ASAR images from ENVISAT descending orbit and 7 PALSAR images from ALOS ascending orbit. 18 SB interferograms are generated following the processing algorithm (Fig. 1). As only-right look interferograms are available in this study, assumption of neglect to the north-south displacement vectors is made to guarantee the accuracy of other two vectors.

In Fig. 2, we present the estimated up-down and east-west accumulated displacement during Nov. 22, 2006 and Nov. 18, 2007, which indicate both the suffered seasonal vertical surface deformations and accumulated horizontal motions [6,7]. The area of ELSC site is used as the reference point (see the square in Fig. 2). Another 5 GPS sites (BLSA, CCCS, HOLP, LBC1, SNHS) are exploited to validate InSAR results, which locations have

![Fig. 1. Spatial-temporal baselines of the 18 actual interferograms for the Los Angeles area.](image)
been marked as triangles in Fig. 2. Good conformances have been found with both up-down and east-west deformations for all the GPS sites (Fig. 3).

Fig. 2. Deformations of Los Angeles in the up (left) and East (right) directions during Nov. 22, 2006 and Nov. 18, 2007, derived from Kalman filter approach. The square symbols represent the location of the reference point, and the triangle symbols are the 5 SCIGN sites used for comparison in Fig. 3.
Fig. 3. Comparisons of InSAR results (triangles with error bars) and SCIGN GPS time series measurements (crosses). The up and Eastern components are given in the left and right panels, respectively. The RMSEs between the InSAR and GPS results are shown for each of the points in the plots.

4. CONCLUSIONS
In this paper, we describe a novel approach for resolving three-dimensional surface displacement from multi-sensor, multi-track and multi-temporal InSAR LOS measurements. Our approach is based on Kalman filter, which allow the integration of the interferograms without considering the discrepancies of satellite tracks and temporal coverages, and production of a time series of 3D displacements at each used SAR image acquisition times. In such a way, we can fully utilize the available interferograms and significant increase the temporal monitoring frequency.

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REFERENCES


