Wavelet-Based Compressed Sensing for Polarimetric SAR Tomography

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Outline

- SAR Tomography
- Wavelet-Based CS (WCS)
  - Single-Channel WCS
  - Polarimetric WCS
- Experimental Results
- Comments & Conclusions
✓ SAR Tomography
Multibaseline Acquisition

\[
\begin{bmatrix}
    b_1 \\
    b_2 \\
    \vdots \\
    b_M
\end{bmatrix}
= \begin{bmatrix}
    a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,N} \\
    a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,N} \\
    a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,N} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    a_{M,1} & a_{M,2} & a_{M,3} & \cdots & a_{M,N}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    \vdots \\
    x_N
\end{bmatrix}
\]

\[a_{i,k} = e^{-j \frac{4\pi}{\lambda} r_{i,k}}\]

\[
B \in \mathbb{C}^M \\
A \in \mathbb{C}^{M \times N} \\
X \in \mathbb{C}^N
\]

\[B = AX\]
✓ Wavelet-Based CS (WCS)
Overview of WCS Tomography

**Single-Channel WCS**

\[ B = AX \]

(multiple looks)

\[ p \in \mathbb{R}_{\geq 0}^N \]

(power)

**Polarimetric WCS**

\[ B = AX \]

HH

\[ B = AX \]

HV

\[ B = AX \]

VV

(multiple looks)

\[ C^1 \in \mathbb{C}^{3 \times 3} \]

\[ C^2 \in \mathbb{C}^{3 \times 3} \]

\[ \vdots \]

\[ C^N \in \mathbb{C}^{3 \times 3} \]

(coherency matrix)
✓ Single-Channel WCS
Covariance Matrix Model

\[ B = AX \]

\[ C_B = E \{ B \, B^* \} \in \mathbb{C}^{M \times M} \]

\[ C_B = A \, \text{diag}(p) \, A^* \]

\[ K_B = \Phi \, p \]

\( p \in \mathbb{R}_{\geq 0}^N \) (backscattered power)
L1-Norm Minimization

\[
\min_{\tilde{\bar{p}}} \|W\tilde{\bar{p}}\|_1
\]
subject to \[
\|\Phi\tilde{\bar{p}} - \hat{K}_B\|_2 \leq \varepsilon
\]
\[
\tilde{\bar{p}} \in \mathbb{R}_{\geq 0}^N
\]
Retain the 5 Largest Wavelet Coefficients

\[ p \]
\[ \alpha = Wp \]
\[ W^T \alpha_5 \]
Retain the 5 Largest Wavelet Coefficients

$p$

$\alpha = Wp$

$W^T\alpha_5$
✓ Polarimetric WCS
Covariance Matrix Model

form covariance

\[ B_i = AX_i \]

\[ B_j = AX_j \]

\[ Q_{i,j} = E \{ B_i B_j^* \} \in \mathbb{C}^{M \times M} \]

rearrange

\[ D = \begin{bmatrix} \text{vec}(Q_{1,1}) & \text{vec}(Q_{2,1}) & \cdots & \text{vec}(Q_{3,3}) \end{bmatrix} \in \mathbb{C}^{M^2 \times 9} \]

get 2 scattering mechanisms

(truncated SVD)

\[ D_2 = \sum_{i=1}^{2} \sigma_i u_i v_i^* \]

(Tebaldini, 2009)
Covariance Matrix Model

(best rank-2 approximation of $D$)

$$D_2 = \Phi \sum_{s=1}^{2} \theta_s C_s^T = \Phi Z$$

$C_s \in \mathbb{C}^9$
(rearranged 3-by-3 coherency matrix)

$\theta_s \in \mathbb{R}_\geq 0^N$
(cross-range power distribution)

$\Phi \in \mathbb{C}^{M^2 \times N}$
(constructed from steering matrix)

(Tebaldini, 2009)
Covariance Matrix Model

(best rank-2 approximation of $D$)

$$D_2 = \Phi \sum_{s=1}^{2} \theta_s C_s^T = \Phi Z$$

$C_s \in \mathbb{C}^{9 \times 9}$
(rearranged 3-by-3 coherency matrix)

$Z \in \mathbb{C}^{N \times 9}$

(Tebaldini, 2009)
Covariance Matrix Model

\[ Z = \sum_{s=1}^{2} \theta_s C_s^T \]

sparsely represented by wavelets

\[ Z = \sum_{s=1}^{2} W_s^T \alpha_s \beta_s^T V^* \]

lies in a 2-D subspace

\[ Z = W^T \left( \sum_{s=1}^{2} \alpha_s \beta_s^T \right) V^* \]

is a row-sparse matrix that describes \[ Z \in \mathbb{C}^{N \times 9} \]
Mixed-Norm Minimization

row-sparse

best rank-2 approximation of sample covariance matrix

\[
\min_{\tilde{\gamma}} \| \tilde{\gamma} \|_{2,1} \\
\text{subject to } \| \Theta W^T \tilde{\gamma} V^* - \hat{D}_2 \|_F \leq \varepsilon \\
\tilde{\gamma} \in \mathbb{R}^{N \times 2}
\]

wavelets

data-adaptive basis
Mixed-Norm Minimization

$$\min_{\tilde{\gamma}} \| \tilde{\gamma} \|_{2,1}$$

subject to

$$\| \Phi W^T \tilde{\gamma} V^* - \hat{D}_2 \|_F \leq \varepsilon$$

$$\tilde{\gamma} \in \mathbb{R}^{N \times 2}$$

rearranged rows should be PSD (physical validity)
✓ Experimental Results
Test Site: Dornstetten (Germany)

- Fully polarimetric data
- L-band (1.3 GHz)
- 21 parallel passes
- 20 m-baseline
Single-Channel WCS Results

21 passes

10 passes
Span – 21 passes 9-by-9 window

Fourier

Capon

WCS
Span – 10 passes 9-by-9 window

Fourier

Capon

WCS
Polarimetric WCS Results

10 passes

6 passes
VV Channel – 10 passes 20x20 m²

Fourier

Capon

WCS
VV Channel – 6 passes 20x20 m²
HV Channel – 10 passes 20x20 m²

Fourier

Capon

WCS
HV Channel – 6 passes 20x20 m²

Fourier

Capon

WCS
HH Channel – 10 passes 20x20 m²

Fourier

Capon

WCS
HH Channel – 6 passes 20x20 $m^2$
✓ Comments & Conclusions
Comments & Conclusions

Wavelet-Based CS allows for:

- Super-resolution with few baselines
- An increase in the height of ambiguity
- Physical validity

But keep in mind:

- The acquisition geometry is nondeterministic
- Very sparse baselines can reduce the usable swath
- Important to choose an appropriate range of heights
Questions?

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## E-SAR Sensor Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chirp bandwidth</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Chirp duration</td>
<td>5 μs</td>
</tr>
<tr>
<td>L-band center frequency / PRF</td>
<td>1.3 GHz / 400 Hz</td>
</tr>
<tr>
<td>Altitude above ground</td>
<td>3200 m</td>
</tr>
<tr>
<td>Nominal aircraft speed</td>
<td>90 m/s</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.66 - 2.07 m</td>
</tr>
<tr>
<td>Swath-width</td>
<td>3 km</td>
</tr>
</tbody>
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