A NEW GLOBAL CRUSTAL MODEL BASED ON GOCE DATA GRIDS

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1st GOCE SOLID EARTH workshop
16-17 October 2012
Faculty ITC, University of Twente
Enschede, The Netherlands
CRUSTAL MODELS

SEISMIC MODEL

Moho model from Meier et al. 2007; unit [km].

It is provided with its error standard deviation map (mean = 3 km, max = 8 km)

SEISMIC AND GRAVITY

Moho model from Sjöberg and Bagherbandi 2011; unit [km].
CRUST2.0 PROBLEMS

Low resolution implies sharp variations into the model both in density and depth of the layers that are not physical;

CRUST2.0 Moho depth; unit [km].

CRUST2.0 density contrast at the Moho; unit [kg/m$^3$].

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CRUST2.0 PROBLEMS

The gravitational effect of the CRUST2.0 model has been computed by means of numerical integration on a 0.5° x 0.5° grid in terms of $T_{rr}$ at GOCE satellite altitude.

The CRUST2.0 effect is one order of magnitude larger than those observed by GOCE:

- std of 1015 mE for CRUST2.0
- 231 mE for GOCE

GOCE CAN BE THE SOLUTION!

Difference between the second radial derivative of the gravitational potential computed from CRUST2.0 and observed by GOCE; unit [mE].
The main idea behind the space-wise approach is to estimate the spherical harmonic coefficients of the geo-potential model by exploiting the spatial correlation of the Earth gravitational field.

Intermediate results that can be used for local applications:

- filtered data (potential and gravity gradients) along the orbit
- grid values at mean satellite altitude
THE GOCE SPACE-WISE SOLUTION

Space-wise solution

GOCE data grids

400 Monte Carlo samples

Second radial derivative [mE]

Potential [m^2 s^{-2}]

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INVERSION METHOD

The inversion method is based on a linearized expression (Strang van Hees 2000; Sjoberg 2011), that relates the coefficients of the anomalous potential to the coefficients of a functional defined as the product between the **Moho undulation** \( \delta D(\phi, \lambda) \) with respect to a reference spherical Moho of radius \( R_M \) and its density contrast \( \Delta \rho(\phi, \lambda) \):

\[
\delta \omega(\phi, \lambda) = \delta D(\phi, \lambda) \cdot \Delta \rho(\phi, \lambda)
\]

\[
\delta \omega_{nm} = \rho_E \left( R_E - \bar{D} \right) \beta_n \delta T_{nm}
\]

\[
\beta_n = -\frac{(2n+1)}{\left(1 - \frac{\bar{D}}{R_E}\right)^{n+3}}
\]

\[
(1 \pm x)^n \approx 1 \pm nx
\]

\[
\bar{\beta}_n = -\frac{(2n+1)}{3}
\]
The inversion method is based on a linearized expression (Strang van Hees 2000; Sjoberg 2011), that relates the coefficients of the anomalous potential to the coefficients of a functional defined as the product between the **Moho undulation** $\delta D(\varphi, \lambda)$ with respect to a reference spherical Moho of radius $R_M$ and its density contrast $\Delta \rho(\varphi, \lambda)$:

$$\delta \omega(\varphi, \lambda) = \delta D(\varphi, \lambda) \cdot \Delta \rho(\varphi, \lambda)$$

- $\Delta \rho(\varphi, \lambda)$ should not depend on the radial coordinate

$$\delta \omega_{nm} = \rho_E \left( R_E - \bar{D} \right) \beta_n \delta T_{nm}$$

- $\delta T_{nm}$ are coefficients of the potential due to Moho undulation

$$\beta_n = -\frac{(2n+1)}{n+3} (1 \pm x)^n \approx 1 \pm nx$$

- $\bar{\beta}_n = -\frac{(2n+1)}{3}$

$\beta_n$ depends on the linearization point
The first problem is related to the two layer hypothesis: as seen before in order to apply the method the density contrast should depend only on \((\varphi, \lambda)\).

\[ \Delta \rho(\varphi, \lambda) = 480 \text{ kg/m}^3 \]

\[
\rho(D = 30) - \rho(D = 25) = 80 \text{ kg/m}^3
\]
A possible solution can be:
- choose as **linearization depth** the maximum depth of the Moho;
- neglect the radial density variation of the mantle;

\[
\delta D = I \left[ T_{rr} - T_{rr\bar{D}}^{\text{top}} \left( \rho_{\text{Crust}} \right) - T_{rr\bar{D}}^{\text{top}} \left( \rho_{\text{Mantle}} \right) \right]
\]

\( I = \text{“Inversion operator”} \)
TWO LAYER HYPOTHESIS (POSSIBLE SOLUTION)

A possible solution can be:

- choose as **linearization depth the maximum depth** of the Moho;

- neglect the radial density variation of the mantle;

\[
\delta D = I \left[ T_{rr} - T_{rrD}^{top} \left( \rho_{\text{Crust}} \right) - T_{rr\bar{D}}^{top} \left( \rho_{\text{Mantle}} \right) \right]
\]

\( I \) = “Inversion operator”

\[
\delta D = I \left[ T_{rr} - T_{rrD}^{top} \left( \rho_{\text{Crust}} \right) - T_{rr\bar{D}}^{top} \left( \rho_{\text{Mantle}} \right) + T_{rr50}^{\delta D} \left( \rho_{\text{Crust}} - \bar{\rho}_{\text{Crust}} \right) \right]
\]

Iterative procedure!
RETRIEVAL OF THE MEAN MOHO DEPTH

\[ \delta \omega_{nm} = \rho_E \left( R_E - \bar{D} \right) \beta_n \delta T_{nm} \]

\( \delta T_{nm} \) are coefficients of the gravitational potential due to Moho undulation

In principle the effect of the whole Earth masses (apart from the unknown Moho) should be removed from the observations.

Actually it is not possible because this mass distribution is not known with sufficient accuracy.
RETRIEVAL OF THE MEAN MOHO DEPTH

\[ \delta \omega_{nm} = \rho_E \left( R_E - \overline{D} \right) \beta_n \delta T_{nm} \]

\( \delta T_{nm} \) are coefficients of the gravitational potential due to Moho undulation

We remove from the GOCE observations the normal potential and the residual effect of the assumed known layers (i.e. topography, bathymetry, sediments).

This implies that the residual signal is zero mean

The mean Moho is not retrievable from gravitational anomalies
**RETRIEVAL OF THE MEAN MOHO DEPTH (POSSIBLE SOLUTION)**

\[
\frac{\omega(\varphi, \lambda) + \mu_\omega}{\Delta \rho(\varphi, \lambda)} + \bar{D} = D(\varphi, \lambda)_{\text{obs}}
\]

This equation can be solved by using as observations:

1) A single seismic Moho depth value (with no error…)

2) A set of seismic profiles or local Moho models with the corresponding errors (using a least square adjustment)

3) A global Moho depth model e.g. CRUST2.0 with its own error (using a least square adjustment)

Gravimetric solution weakly combined with seismic data

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TRANSFER FUNCTION ERRORS

Numerical Integration of $\delta T_{rr}$ at satellite altitude

Inversion according to the considered procedure and spherical harmonic synthesis of the Moho
**Transfer Function Errors**

$$\beta_n = -\frac{(2n+1)}{(1 - \frac{\bar{D}}{R_E})^{n+3}}$$

$$\tilde{\beta}_n = -\frac{(2n+1)}{3}$$

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Transfer Function Errors

- Smoothing effect due to the high frequency in the polar area (~30% of the signal)
- Smoothing effect due to the linearized model used (~2% of the signal)

Close loop error; unit [km].

Error std < 1 km!
GOCE OBSERVATION ERRORS

GOCE observation errors can be obtained by propagating the GOCE grid errors into the estimated Moho undulation errors. This is not done by a formal covariance propagation but by a Monte Carlo solution propagating the available grid error samples through the whole chain of the inversion algorithm.

Error degree medians estimated by the close loop test applying regularization at high degrees.

The Moho can be estimated up to degree 210 (by applying a proper regularization)

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GOCE OBSERVATION ERRORS

GOCE propagated error; unit [km].

Error std < 100 m! (polar caps excluded)
INVERSION METHOD (SOLUTION)

With these adjustments the inversion method would allow us to:

1) Model crustal densities with radial variation;

2) Estimate the mean Moho from seismic information;

3) Retrieve the Moho undulation up to degree 210 (better than 1°x1°) from GOCE grid with a commission error smaller than 1 km;

Note that most of the final error is not due to the approximation in the transfer function or to GOCE observations but to the modeling of crustal densities.
INPUT DATA

TOPOGRAPHY
OCEAN
ICE

SRTM

1°x1° LASKE ET AL. 1997

CRUST

50 LAYERS

UPPER MANTLE

SIMMONS ET AL. 2010
For each province a function relating density to depth has been computed (from Christensen and Mooney 1995 for the continental crust and from Carlson and Raskin 1984 for the oceanic crust)

Oceanic ridge are separately modeled from the rest of the oceanic crust.

Otherwise their gravitational signal would produce a wrong deeper (more than 20 km) Moho.
The effect of the upper layers has been estimated in terms of $T_{rr}$ signal at satellite altitude (downloadable from GOCE WPS, see poster…)

The $T_{rr}$ topography effect has been modeled and removed at satellite altitude in the GOCE observations.

The $T_{rr}$ bathymetry effect has been modeled and removed at satellite altitude in the GOCE observations.
FIRST SOLUTION

mean value of Moho = 22.439 km
std. value of Moho = 9.935 km

ocean mean Moho = 17.060 km
cont. mean Moho = 31.379 km

ocean mean Moho C20 = 13.095 km
cont. mean Moho C20 = 33.134 km

Estimated Moho; unit [km].
FIRST SOLUTION

\[
\frac{\omega(\varphi, \lambda) + h_i \omega_i(\varphi, \lambda) + \mu_\omega}{\rho_M(\varphi, \lambda) - h_i \rho_C(\varphi, \lambda) \chi_i(\varphi, \lambda)} + \overline{D} = D(\varphi, \lambda)_{obs}
\]

1 scale factor \( h_i \) for each patch \( i \)

It requires to compute the gravitational effect of each patch \( i \) and to solve a least square adjustment with Tikhonov regularization.

Considered error standard deviation of the CRUST2.0 model; unit [km].
First solution

\[
\frac{\omega(\varphi, \lambda) + h_i \omega_i(\varphi, \lambda) + \mu_\omega}{\rho_M(\varphi, \lambda) - h_i \rho_C(\varphi, \lambda) \chi_i(\varphi, \lambda)} + \bar{D} = D(\varphi, \lambda)_{obs}
\]

1 scale factor \(h_i\) for each patch \(i\)

It requires to compute the gravitational effect of each patch \(i\) and to solve a least square adjustment with Tikhonov regularization.

Density scale factor; unit [%].
**Estimated Moho; unit [km].**

**Difference between the two solutions; unit [km].**
mean value of Moho = 20.796 km
std. value of Moho = 11.315 km

mean value of Moho OLD = 22.439 km
std. value of Moho OLD = 9.935 km

ocean mean Moho = 13.931 km
cont. mean Moho = 32.906 km

ocean mean Moho OLD = 17.060 km
cont. mean Moho OLD = 31.379 km

ocean mean Moho C20 = 12.944 km
cont. mean Moho C20 = 35.445 km

Estimated Moho; unit [km].
CONCLUSION

GOCE space-wise grids can be used to constrain the relation between density and Moho depth with very high accuracy (std <1 km ).
In other words given a crustal density model the presented inversion method can estimate the Moho depth.

The computed Moho global model is well consistent with the actual gravity field, thus overcoming the main limitation of CRUST2.0.

The final model is an improvement of the CRUST2.0 because the shallowest layers are updated with more accurate data at higher resolution, while the remaining crust is modeled according to the same reference paper but with a higher number of layers. The solution is constrained with GOCE data and partially includes CRUST2.0 seismic information (where judged reliable).
FUTURE WORKS

Conclude the error analysis of the Moho model by simulating Monte Carlo samples describing the variability of the crustal density which is the main cause of errors.

Improving the modeling of the crustal densities by:

1) Refining the geological provinces according to GOCE signal

2) Collecting additional geophysical/geological data at local scale (especially in ridge and subduction regions)

Evaluating the model with independent (from CRUST2.0) data