THE EARTH’S GRAVITATIONAL FIELD RECOVERY BASED ON AN OPTIMAL COMBINATION OF GRACE AND GOCE SATELLITE DATA

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ABSTRACT
Modelling the global gravitational field of the Earth in terms of spherical harmonic coefficients has been performed by a stand-alone inversion of a 4-month set of the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) Satellite Gravity Gradiometry (SGG) data and a 9-month set of the GOCE kinematic orbits, as well as by a combined inversion of the aforementioned data sets and 1-year sets of the Gravity Recovery and Climate Experiment (GRACE) K-Band Ranging (KBR) data and its kinematic orbits. It is shown, in particular, that an incorporation of the GOCE data may lead to a dramatic improvement of the GRACE-based gravity field models. The added value of the GOCE data is primarily attributed to the diagonal components of the SGG gravitational tensor.

1. INTRODUCTION
The Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) is the first core mission within the Earth Explorer Program of the European Space Agency (ESA). The main objective of the mission, which was successfully launched on March the 17th 2009, is to provide a high-accuracy and high-resolution global model of the Earth’s static gravity field and of the geoid. To this end, the mission acquires Satellite Gravity Gradiometry (SGG) data and high-low Satellite-to-Satellite Tracking (SST) data. The latter are used, in particular, to derive a precise kinematic orbit of the satellite. The data are provided with the sampling interval of 1 s. It is currently expected that the GOCE mission will remain operational at least until the end of 2012 (see http://www.esa.int/esaLP/LPgoce.html).

At the Delft University of Technology, there is a long history of developing innovative algorithms and software for processing satellite gravimetry data. In particular, a new variant of the acceleration approach was proposed [1] for gravity field modelling on the basis of satellite orbit data; the developed methodology was later applied to produce the global gravity field model DEOS_CHAMP-01C_70 [2] from a 1-year set of data acquired by the CHAllenging Minisatellite Payload (CHAMP) satellite mission. Furthermore, the developed technique was generalized [3] to process the K-Band Ranging (KBR) data from the Gravity Recovery and Climate Experiment (GRACE) satellite mission. This allowed the Mass Transport model DMT-1 to be produced, which consists of a time series of monthly solutions describing temporal gravity field variations [4]. Finally, a number of innovative algorithms were developed in order to process efficiently the SGG data from the GOCE satellite mission [5-7, 9].

The presented study is the first attempt to apply the developed algorithms and software to process real GOCE data, with a focus on a global static model of the Earth’s gravity field. The primary goals of the study were as follows:
1. To demonstrate the operability of the developed algorithms and software in the context of real data processing;
2. To confirm the declared performance of the GOCE satellite mission and, possibly, reveal its weak points that have not been noticed so far;
3. To investigate the added value of the GOCE data sets in the presence of the data collected by other satellite gravimetry missions (in the first instance, the GRACE mission).

2. THEORY
In general, processing satellite gravimetry data can be split into three major steps:
1. Pre-processing of the original measurements in order to transfer them into data residuals that contain non-trivial information about the Earth’s gravity field;
2. Inversion of data residuals into residual spherical harmonic coefficients (data combination takes place at this stage); and
3. Compilation and analysis of the resulting gravity field model.

2.1. Processing of the GOCE SGG data

The provided GOCE SGG data are already cleaned from tidal and other nuisance signals. Therefore, pre-processing of the SGG data reduces, in essence, to the subtraction of the gravity gradient signal forecasted on the basis of a reference gravity field model. Thought the study, for stand-alone processing of the GOCE SGG data we employed the EIGEN-5C model [8] and EIGEN-5S (GRACE-only) model [8] for the GRACE and GOCE combined data processing as the reference ones.

Inversion of the GOCE SGG data is a much more complicated task. At this stage, one has to find the vectors \( \mathbf{x} \) composed of unknown parameters (corrections to reference spherical harmonic coefficients) by solving a system of normal equations:

\[
\mathbf{x} = (\mathbf{A}^T \mathbf{C}_d^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{C}_d^{-1} \mathbf{d},
\]

where \( \mathbf{d} \) is the vector of residual SGG data; \( \mathbf{A} \) is the design matrix, which relates the vectors \( \mathbf{d} \) and \( \mathbf{x} \); and \( \mathbf{C}_d \) is the covariance matrix of noise in the SGG data. In applications not related to satellite gravimetry, data noise is usually assumed to be non-correlated and stationary, which leads to a matrix \( \mathbf{C}_d \) that is proportional to a unit one. As soon as the noise standard deviation is known, incorporation of such a matrix into an inversion procedure can be done with ease. Unfortunately, satellite gravimetry data, including the GOCE SGG measurements, are contaminated by coloured (i.e. frequency-dependent) noise, so that the matrix \( \mathbf{C}_d \) has a Toeplitz structure with non-zero off-diagonal elements. To handle such noise, we built beforehand its stochastic model in the form of an Auto-Regressive Moving-Average (ARMA) process [9]. Building an ARMA model is usually accompanied with an estimation of the noise Power Spectral Density (PSD). Strictly speaking, this is not necessary, but allows us to understand the noise properties much better. To obtain a realization of data noise, it is necessary to subtract a set of observations forecasted on the basis of the true gravity field model from the data. Since the true gravity field is not known, we employ an iterative approach: the model obtained after the data inversion is used to re-compute data noise and its stochastic properties at the next iteration. In practice, two iterations of this kind are always sufficient: noise properties estimated after the second iteration do not differ from those obtained after the first iteration.

In order to solve the system of linear equations Eq. 1 efficiently, we make use of the method of Pre-Conditioned Conjugate Gradients (PCCG), so that neither matrix \( \mathbf{A} \) nor matrix \( \mathbf{C}_d^{-1} \) has to be found explicitly. It is sufficient to implement the multiplication of these matrices and of the matrix \( \mathbf{A}^T \) to an arbitrary vector. Usage of tailored algorithms allows matrices \( \mathbf{A} \) and \( \mathbf{A}^T \) to be handled with a high numerical efficiency [6]. As far as the matrix \( \mathbf{C}_d^{-1} \) is concerned, availability of an ARMA model of data noise also allows an efficient and accurate numerical procedure to be built [1,10] in order to handle coloured noise properly. To reach a rapid convergence of the PCCG scheme, we make use of a dedicated block-diagonal preconditioning matrix [5].

At the last stage, the computed corrections are added to the reference gravity field model. To analyze the obtained results, the difference between them and a state-of-the-art gravity field model is considered as an estimation of errors in the models produced. The obtained differences are quantified either in terms of the RMS geoid height errors per degree or as a spatial distribution of geoid height errors. The EGM2008 model [11] is used as the state-of-the-art model throughout the study.

2.2. Processing of satellite orbit data

The primary difference between the GOCE SGG data and satellite orbit data is their link to gravity field parameters. The SGG measurements are “local”, i.e. they describe the properties of the gravity field only in the vicinity of the observation point. This is a very beneficial property because it not only facilitates an analysis of information contents in the data, but also allows efficient inversion schemes to be built.

On the contrary, the satellite orbit depends on the Earth’s gravitation not only at the current point, but also at the points crossed by the satellite in the past. In order to make such data “local” as well, we use the satellite orbits to derive the satellite accelerations, for which a three-point differentiation scheme is used:

\[
\mathbf{a}(t) = \frac{\mathbf{x}(t - \Delta t) - 2\mathbf{x}(t) + \mathbf{x}(t + \Delta t)}{(\Delta t)^2},
\]

where \( \mathbf{x}(t) \) is the satellite position vector at time \( t \); \( \Delta t \) is the data sampling interval, and \( \mathbf{a}(t) \) is the derived acceleration. It can be shown that an acceleration obtained with the three-point differentiation scheme can be interpreted as the acceleration averaged with weight within the differentiation time interval [1].

The residual satellite accelerations of either GRACE satellite are obtained as the difference between those based on the kinematic orbit of the satellite and those derived from a purely dynamic (reference) orbit. The latter one is produced on the basis of the adopted reference force model, which includes the static gravity field signal, as well as tidal and other time-varying signals. In particular, gravity field variations due to
atmosphere and ocean variability are taken into account, for which purpose the AOD1B product (release 04) [12] is used. The force model includes also non-gravitational satellite accelerations, which are defined as calibrated measurements of the on-board satellite accelerometer(s). The dynamic orbit is computed by fitting the kinematic orbit of the satellite; the adjusted parameters being the initial state vector, as well as the accelerometer calibration parameters. The computations of the dynamic orbit (orbit integrations) are performed with the PANDA software developed at the GNSS Research and Engineering Center of Wuhan University.

The residual satellite accelerations of the GOCE satellite are obtained as the difference between accelerations derived from the kinematic orbit by the three-point differentiation scheme and those evaluated (and the averaged) based on the force model at the satellite’s location. At this stage, the force model in this part of the data processing does not include gravity field variations due to atmosphere and ocean variability. This leads to a minor inconsistency with respect to the GRACE SST data processing.

The rest of the GRACE and GOCE SST data processing scheme is very similar to that developed for the SGG data processing.

2.3. Processing of GRACE KBR data

The developed procedure for the GRACE KBR data processing is somewhat similar to that designed for processing satellite orbit data. The three-point differentiation scheme is written in the context of the KBR data as follows:

\[
\bar{\omega}_i = \frac{\cos \theta_{i-1} \cdot \rho_{i-1} - 2\rho_i + \cos \theta_{i+1} \cdot \rho_{i+1}}{(\Delta t)^2},
\]

where \(\rho_{i-1}, \rho_i, \) and \(\rho_{i+1}\) are three successively measured inter-satellite ranges at the epochs \(i-1, i,\) and \(i+1,\) respectively; \(\theta_i\) is the angle between the lines-of-sight at the epochs \(i-1\) and \(i,\) and \(\theta_{i+1}\) is the angle between the lines-of-sight at the epochs \(i\) and \(i+1.\) Following [3], we refer to the result of this three-point scheme as “range combination”. One can show that a range combination can be interpreted as the projection of the averaged inter-satellite acceleration vector onto the inter-satellite line-of-sight at the epoch \(i.\) Again, an “average acceleration” should be understood as the acceleration obtained by averaging with a weight within the differentiation time interval.

The rest of the procedure is identical to that developed for processing satellite orbit data. Further details are described in [3] and [4].

3. RESULTS

3.1. Stand-alone processing of GOCE kinematic orbits

First of all, we performed gravity field modelling on the basis of GOCE orbit data, using the provided kinematic orbits of this satellite in a 9-month interval (August 2009 – April 2010). EGM96 [13] was employed as the reference gravitational field. Two variants of input were used: (i) the full set of satellite positions provided with the 1-s sampling rate and (ii) a sub-set of it characterized by the 30-s sampling (obtained by picking up every 30-th observation from the original data set). The PSD of noise in the satellite accelerations estimated on the basis of the 30-s subset of data is shown in Fig. 1 (top), whereas its approximation with the ARMA-model is shown in Fig. 1 (bottom). The behaviour of this noise is similar to that we observed earlier in case of the CHAMP mission [2], which can be considered as a confirmation of the proper quality of the GOCE GPS data and the kinematic orbits produced on their basis.

![Figure 1. Estimated PSDs of noise in the GOCE satellite accelerations derived from the kinematic orbit of the satellite (top) and approximation of those PSDs with ARMA models (bottom). All three components (radial, along-track, and cross-track) are presented.](image-url)
The quality of the obtained gravity field model is analyzed in Fig. 2. The models DEOS-CHAMP-01C [2] (produced from a 1-year set of CHAMP orbit data with the same data processing methodology) and EGM96 [13] are also included as a reference. One can see that the model based on the GOCE data is definitely more accurate than the EGM96 model and is similar to the DEOS-CHAMP-01C model at degrees 10-20. At higher degrees, the GOCE-based model shows even a higher accuracy, which can be explained by a much lower altitude of the GOCE satellite. A relatively large difference between the obtained GOCE-based model and the EGM2008 (used as the ground-truth) at degrees below 10 still has to be explained.

We also tried to perform gravity field modelling on the basis of the full set of GOCE positions provided with a 1-s sampling. Unfortunately, the results obtained so far (not shown) are not satisfactory. The noise PSD shows in that case a much higher level than in case of the 30-s sampling rate. Also, the quality of the resulting gravity field model is lower. The reasons for this behaviour still have to be understood. One of possible causes is data gaps, which may have a destructive effect on the produced gravity field models in case of a short sampling interval, if the data weighting in the frequency domain is not applied [1] or, presumably, based on an insufficiently accurate stochastic model of data noise.

The 30-s subset of GOCE orbit data was used in the rest of the presented study.

3.2. Stand-alone processing of GOCE SGG data

At this stage, we have used the officially provided 4-month set (November 2009 – February 2010) of the GOCE SGG data. In the first instance, all the four high-precision components (V_{xx}, V_{yy}, V_{zz}, and V_{xz}) in the gradiometer reference frame were taken into consideration.

An accurate stochastic description of the GOCE SGG data plays a key role in gravity modelling on the basis of this data. The estimated PSD of this noise is shown in Fig. 3. One can see that the level of noise in all four high-precision components is close to $10^{-2}$ E in the measurement frequency range (5 to 100 mHz), which is consistent with other estimations. Other remarkable features are high noise level at low frequencies and prominent peaks at the frequencies of 1, 2, … cycles-per-revolution (cpr), which is consistent with findings of the closed loop simulations [15]. A strong variability of noise in the frequency domain implies that a frequency-dependent data weighting is a critical element of the SGG data inversion. The ARMA models based on the obtained noise realizations are shown in Fig. 4, whereas the accuracy of the resulting gravity field model is presented in Fig. 5 (green curve). Unfortunately, the obtained results cannot be considered as satisfactory (particularly, at the degrees below 30). We explain this by strong and highly variable data noise at low frequencies, so that the compiled ARMA models could not describe it appropriately (Fig. 4).
This problem can be solved on the basis of the fact that noise at low frequencies exceeds the non-trivial signal by several orders of magnitude. Therefore, one can safely suppress it without losing signal contents. To that end, we have approximated each component of the SGG data with an empirical analytic function $n(t)$:

$$n(t) = A + Bt + Ct^2 + D \sin \left( \frac{2\pi t}{T_{\text{rev}}} \right) + E \cos \left( \frac{2\pi t}{T_{\text{rev}}} \right) + Ft \sin \left( \frac{2\pi t}{T_{\text{rev}}} \right) + Gt \cos \left( \frac{2\pi t}{T_{\text{rev}}} \right),$$

where $T_{\text{rev}}$ is the revolution period (approximately 1.5 hr), and $A, B, C, D, E, F, G$ are constant coefficients obtained for each orbital revolution by means of a least-squares adjustment. The obtained analytic approximations are subtracted from the data. The PSD of the resulting data sets are shown in Fig. 6, whereas the ARMA models produced on their basis are presented in Fig. 7. One can see that the subtraction of the estimated analytic function has indeed dramatically reduced low-frequency noise in the data. On the other hand, the exploited ARMA models still presume a large noise level at low frequencies (and, therefore, assign low weights to the data there). We find this approach more appropriate than to force the ARMA models to follow blindly noise at low frequencies, since the applied data pre-processing procedure may reduce not only noise, but also signal, so that an assumption of a very low noise level at low frequencies may not be fair.
The accuracy of the resulting gravity field model is presented in Fig. 5 (blue curve). One can see that the data pre-processing procedure described above may significantly improve the quality of gravity field modelling. Thus, we recommend to suppress in this way the low-frequency noise in the SGG data in general (at least, until its causes are fully identified and eliminated in an explicit way).

3.3. Combined processing of GOCE and GRACE data

We have also performed a combined inversion of the GOCE and GRACE data. To that end, the GOCE data were complemented with three 1-year sets of GRACE data: two sets of orbit data (one set per satellite) and a set of GRACE KBR data, all covering the time interval from January to December 2006. EIGEN5S [8] was employed as the reference gravitational field. Three ways to combine the GOCE data with the GRACE data sets were considered: (i) using GOCE orbit data only; (ii) using GOCE SGG data only; and (iii) using all the GOCE data (both orbit and SGG). In addition, gravity field modelling on the basis of the aforementioned GRACE data sets alone was considered.

A comparison of the four obtained models (Fig. 8) allows one to conclude that an incorporation of the SGG data may lead to a significant improvement of the GRACE-based gravity field models. Even if the GOCE SGG data are not used, the errors above spherical harmonic degree 120 reduce about two times. This can be explained, on one hand, by a poor sensitivity of the GRACE mission in the cross-track direction (so that near-sectorial coefficients cannot be determined accurately) and, on the other hand, by a very low orbit of the GOCE mission (so that even relatively inaccurate orbit data deliver more information about nearly-sectorial coefficients than any GRACE data). A combination of the GRACE and GOCE SGG data improves the model quality even further. At the same time, the GOCE orbit data do not lead to any improvement if both GRACE data and GOCE SGG data are already used. In other words, the combination of the last two types of data contains (at least, in the context of the conducted study) more information about gravity field at all degrees and orders than the GOCE orbit data.

An improvement of a GRACE-based model achievable when GOCE data are included becomes even more visible when the accuracy is analyzed in terms of a map of geoid height errors (Fig. 9). In this way, the RMS geoid height error (in case of the maximum spherical harmonic degree 180) reduces from 144 cm to 14 cm.
A comparison of the combined GRACE and GOCE combined model produced in this study with the GOCO01S model by Pail and colleagues [16] is shown in Fig. 10. One can see that both models seem to show a similar accuracy. A somewhat higher accuracy of the GOCO01S model at degrees below approximately 60 can be explained by a longer set of the GRACE data used by Pail and colleagues [16] (7 years vs. only 1 year used in this study). A coincidence of the curves above degree 60 might be interpreted as an indication that the remaining differences with respect to the state-of-the-art EGM2008 model reflect not the accuracy of the produced models but the accuracy of the EGM2008 itself.

Finally, we addressed the issue of the relative added value of 4 high-precision SGG components. By considering different combinations of the SGG components in the context of the GRACE and GOCE combined inversion, we came to the conclusion that all three diagonal SGG components ($V_{xx}$, $V_{yy}$, and $V_{zz}$) provide a valuable contribution and, therefore, should definitely be used in gravity field modelling. The added value of the only high-precision off-diagonal SGG component, $V_{xz}$, is less obvious. Surprisingly, incorporation of the $V_{xz}$ component does not lead to any visible improvement of the results. Even worse, a comparison of geoid height errors obtained with and without it shows that the usage of this component may worsen the solution (notice an increased noise level in the oceanic areas and a spurious anomaly near Galapagos Islands in the top plot in Fig. 11).

Another remarkable feature in both plots in Fig. 11 is a much larger difference with respect to the EGM2008 model in hardly accessible continental areas (equatorial Africa, equatorial South America, Himalayas, Antarctic) than in the rest of the globe. Since there are no reasons for the GOCE mission to show a degraded performance in those areas, the only possible interpretation of these differences is a relatively low accuracy of the EGM2008 model there. In other words, the observed differences in those areas can be considered as the improvement of the Earth gravity field achieved after the processing of the considered combination of the GRACE and GOCE data. Since several years of GRACE were already taken into consideration in compiling the EGM2008 model, it is fair to say that the observed improvement can be mostly attributed to the GOCE mission (more specifically, to the SGG data collected by this mission). It is also worth mentioning that the presence of inaccuracies in the EGM2008 model, which are visible in Fig. 11, is fully consistent with our assumption made above that the compiled plots of RMS geoid height errors per degree show not only errors in the produced models, but also errors in the EGM2008 model.

Figure 10. Accuracy of two combined GRACE and GOCE combined gravity field models. In green: the model produced in the course of the presented study (data used: 1-year sets of GRACE satellite orbit data; 1-year set of GRACE KBR data; 9-month set of GOCE satellite orbit data; and 4-month set of GOCE SGG data). In blue: the GOCO01S model (data used: 7-year sets of GRACE orbit data; 7-year set of GRACE KBR data; 2-month set of the GOCE data). The accuracy of all the models is shown in terms of RMS geoid height error per degree. Accuracy of the EGM96 model is shown as a reference (in red). All the presented errors have been estimated as the difference with respect to the state-of-the-art EGM2008 model.

Figure 11. Accuracy of two gravity field model derived GOCE SGG data. Top: all four accurate SGG components are used ($V_{xx}$, $V_{yy}$, $V_{zz}$, and $V_{xz}$); bottom: only three diagonal SGG components are used ($V_{xx}$, $V_{yy}$, $V_{zz}$).
The accuracy of the models is shown in terms of geoid height errors up to maximum degree and order 180. The presented errors have been estimated as the difference with respect to the state-of-the-art EGM2008 model.

4. CONCLUSIONS AND RECOMMENDATIONS
The conclusions of the presented study can be summarized as follows:

1. Orbit data from the GOCE mission lead to a higher accuracy of gravity field models than similar data from other mission (e.g., CHAMP) thanks to an extraordinary low altitude of the former mission.
2. Elimination of the low-frequency contents from the GOCE SGG data prior to gravity field modelling is highly recommended.
3. Incorporation of GOCE SGG data may lead to a dramatic improvement of GRACE-based gravity field models.
4. No added value of GOCE orbit data has been seen so far when GRACE orbit data, GRACE KBR data, and GOCE SGG data are already taken into consideration.
5. The only high-precision off-diagonal SGG component, $V_{xz}$, did not lead in the conducted study to an improvement of resulting gravity field models. On the contrary, some degradation of the model quality was observed.
6. Though a special attention was paid to ensure a consistency of reference force models used throughout the study, some minor inconsistencies were still present (e.g., related to the atmosphere and ocean variability signal, which is accounted for in processing the GRACE satellite orbit data and GRACE KBR data, but not in processing GOCE SST data). Though it is unlikely that those inconsistencies have an effect on the gravity field models produced, we find it important to fully eliminate them in the course of future improvements.
7. Further research efforts are needed in order fully incorporate GOCE orbit data into gravity field modeling.

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6. REFERENCES


