## Some applications of climate Data Assimilation

- Really two half lectures;
- An example of data assimilation for climate and biogeochemistry;
- An unfinished approach to model ensembles and a lesson on why it's sometimes good to go back to first principles.

## Uncertainties in the relationship between concentrations and emissions

- P. J. Rayner (Univ. Melbourne)
- E. Koffi (LSCE)
- M. Scholze (U. Bristol)
- P. Friedlingstein (LSCE)
- M. Raupach (CSIRO)
- T. Kaminski (FASTOPT)
- J.-L. Dufresne (IPSL)

"It's only a model"

(Monty Python and the Holy Grail)

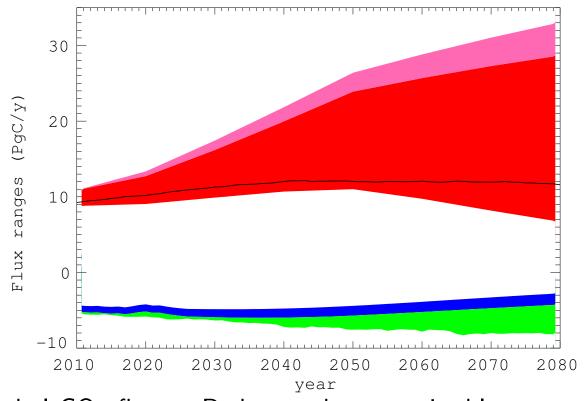
## **Papers**

- This submitted to Philosophical Transactions of the Royal Society;
- Optimization from Koffi et al. (2010) almost submitted to Global Biogeochemical cycles.

#### **Outline**

- Uncertainties in the carbon cycle;
- A simple predictive model and its uncertainty;
- A little on sensitivity;
- Confronting the model with data;
- Conclusions.

#### **Motivation**



Ranges of global  $CO_2$  fluxes. Red = anthropogenic, blue = ocean, green = land, pink = other vulnerabilities from Raupach et al., Tellus, 2010. Uptakes from IPCC-2007 Fig. 10.21. Black line shows emission scenario.

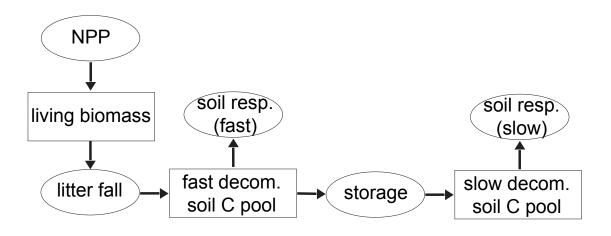
## **Sources of Terrestrial Model Uncertainty**

- Different models include different processes;
- Equivalent processes are described with different equations;
- There are many uncertain parameters in these models.

## **Exploring Parameter Uncertainty**

- Write simple box model of terrestrial carbon cycle
- Climate model → global model → simple model;
- Calculate sensitivities of future uptake to inputs;
- Calculate uncertainty of future uptake as function of uncertainty in input parameters;
- Assimilate current data and study reduced uncertainty on future uptakes.

## Simple Model



$$NetUptake = \begin{array}{c} \textbf{Production} - (1 - \textbf{K}) \times \textbf{LitterDecomposition} \\ -SoilOutgassing \end{array}$$

SoilOutgassing  $\propto$  SoilPool  $\times \omega^{\kappa} \mathbf{Q}_{10}^{T_a/10}$ 

where  $\omega=$  soil moisture and  $T_a=$  air temperature.

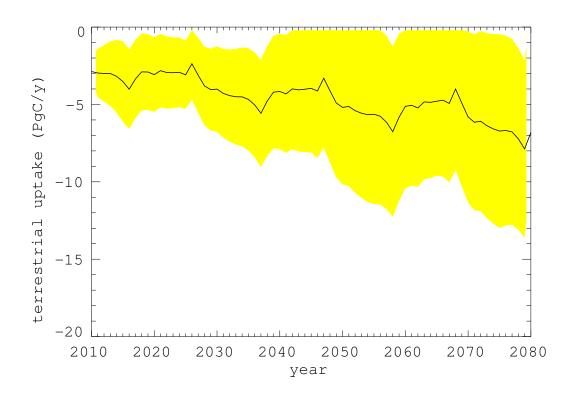
$$\frac{\partial \text{SoilPool}}{\partial t} = \mathbf{K} \times \mathbf{LitterDecomposition} - \text{SoilOutgassing}$$

#### **Technical Details**

- Need derivatives of outputs of simple model and global model with respect to their inputs;
- Simple model can be differentiated by hand;
- Global model differentiated by the software "Transformation of Algorithms in FORTRAN" http://www.fastopt.com.

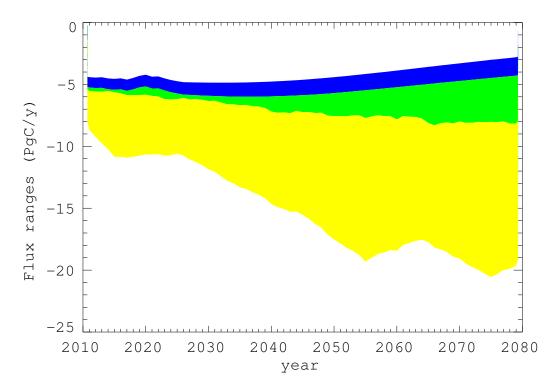
 $\label{eq:Uncertainty} \text{Uncertainty}(\text{uptake}) = J \times \text{Uncertainty}(\text{parameters}) \times J^T$  where J is derivative and T is transpose.

## **Uptake from Prior Model**



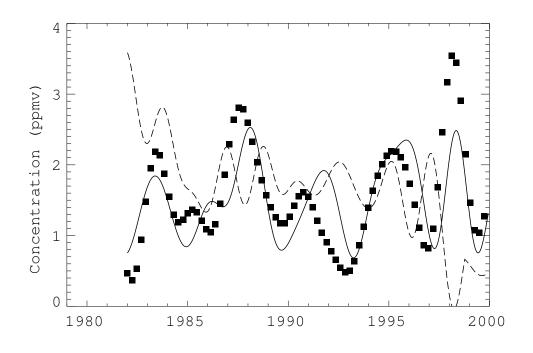
Terrestrial uptake (no climate change) from prior model and its 90% confidence interval. Uptake is anchored at its 2000–2010 value.

## Comparison with Other Uncertainties



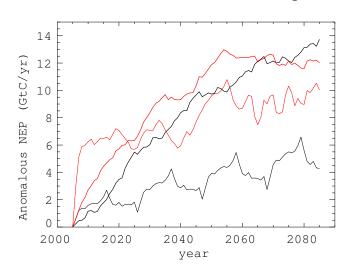
Range of uptakes, blue = ocean, green = land from IPCC models. The yellow band represents the 90% confidence interval of the uncertainty in the simple model.

## Fitting Atmospheric Growth Rate



Smoothed global growth rate, squares = obs, dashed = prior, solid = optimised. Note the great improvement in phasing with the optimisation.

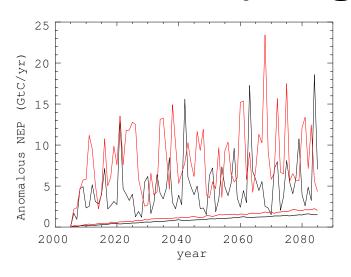
## **Comparing Uptakes**



Decadal mean  $\delta NEP$ , 2000–2090. Black lines = current climate, red = climate change. Thin lines = prior parameters, thick = optimized.

- $\delta \text{NEP} = \text{NEP} \overline{\text{NEP}}(t = 2000 -2010)$
- Unrealistically rapid increase;
- High  $\frac{\partial \text{GPP}}{\partial \text{CO}_2}$ : 0.3PgC/yr/ppm cf FULLBETHY = 0.23 and LPJ = 0.19. ORCHIDEE anyone?

## **Comparing Uncertainties**



Uncertainty in decadal mean  $\delta \text{NEP}$ , 2000–2090. Black lines = current climate, red = climate change. Thin lines = prior parameters, thick = optimized.

- Uncertainty in  $\delta \text{NEP}$  calculated as  $\mathbf{C}(x) = J\mathbf{C}(p)J^T$  where  $J = \frac{\partial x}{\partial p}$  and  $\mathbf{C}$  is covariance;
- Uncertainties completely dominated by climate change;
- Partially reflects small uncertainty on photosynthesis parameters.

## For those who prefer numbers

Case	sum (PgC)	Uncertainty ( $1\sigma$ PgC)
prior no-clim	278	126
prior clim-change	656	1141
optimized no-clim	717	78
optimized clim-change	799	107

Value and uncertainty for integrated  $\delta \rm NEP$  from 2000–2090

#### Climate Feedback Parameter

• 
$$G = \frac{\sum \delta \text{NEP(climate)}}{\sum \delta \text{NEP(noclimate)}};$$

ullet Can calculate  $\frac{\partial G}{\partial p}$  (unpleasant) and hence uncertainty of G;

$$\sigma(G) = \sqrt{\nabla_p G \mathbf{C}(p) \nabla_p G^T}$$

• For C(p) diagonal (prior) this is simple sum.

## **Equations again**

SoilOutgassing 
$$\propto$$
 SoilPool  $\times \omega^{\kappa} \mathbf{Q}_{10}^{T_a/10}$ 

where  $\omega = \text{soil moisture}$  and  $T_a = \text{air temperature}$ .

$$\frac{\partial \text{SoilPool}}{\partial t} = \mathbf{K} \times \mathbf{LitterDecomposition} - \text{SoilOutgassing}$$

- $\sigma(G) = 3.54$  for prior;
- 80% from  $\kappa$  with rest from  $\mathbf{Q}_{10}$  and  $\mathbf{K}$ ;
- $\sigma(G) = 0.04$  posterior.

#### **Conclusions**

- Tangent linear models are fun;
- The carbon-cycle/climate feedback uncertainty is very large even within one model;
- For BETHY the sensitivity of respiration to soil moisture is the biggest contribution to uncertainty;
- The atmospheric record is sufficient to constrain this aspect of model dynamics.

# Using Data Assimilation with Model Ensembles

- Work very much in progress;
- Similar efforts in physical climate.

#### **Transcom**

- How much uncertainty in inversions due to transport?
- Three phases: compare forward models, run known tracers, compare inversions;
- Law et al., Tellus, 1996, Denning et al., Glob. Biogeochem.
   Cyc. 1999, Gurney et al., Nature 2002, Baker et al., Glob. Biogeochem. Cyc., 2006.

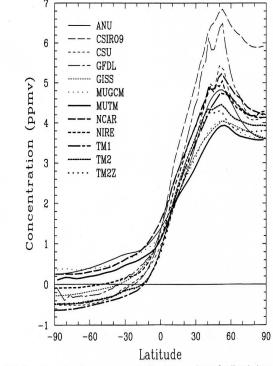
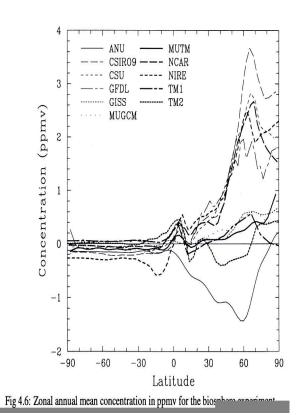


Fig. 3.1: Zonal annual surface mean concentration in ppmv due to fossil emissions.



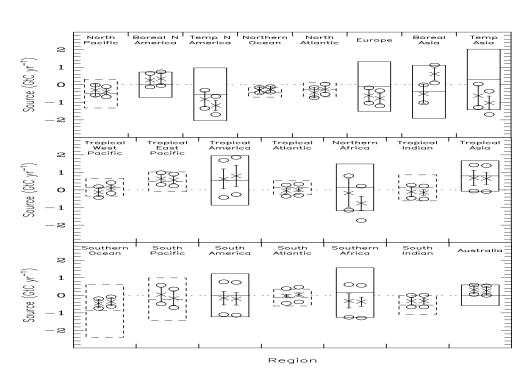
Zonal mean concentration from fossil fuel source

Zonal mean response to annually balanced biosphere source

#### Transcom III

- Run inversions changing only response functions from different models;
- Data and uncertainties, prior and uncertainties and algorithm fixed.
- Annual mean case: 26 response functions, 17 models;
- Seasonal and interannual cases: 268 response functions, 12 models.

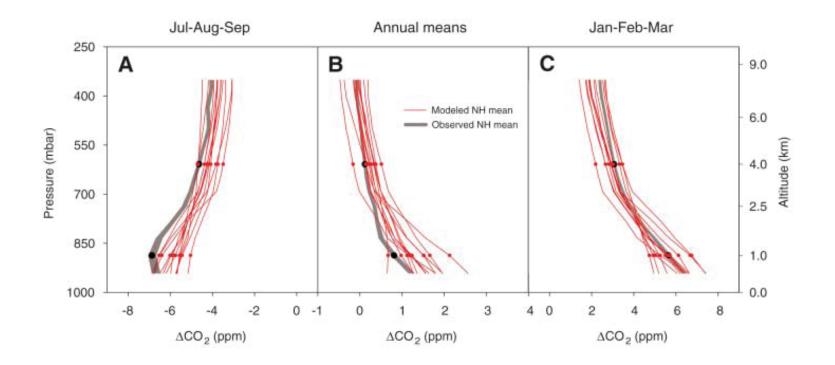
## Gurney et al., Nature, 2002



Inversion results for the control (left bar) and no-biosphere (right bar). Mean fluxes are the 'X'. Positive = source. Prior flux and uncertainty: horizontal bar and boxes (land in green, ocean in blue). Within model

uncertainty = circles, between model uncertainty = length of vertical bars. Regions are shown in their approximate north-south and east-west relationship.

## Impact of Vertical Transport on Inversions



#### **Notes**

- Stephens et al., Science, 2007;
- Compared different models against independent climatology of profiles;
- Did not consider posterior uncertainty;
- Main point that not all models are equal.

## The Approach

- Use statistical techniques to choose among an ensemble;
- Include the choice of model as an extra unknown;
- Produce a PDF among the models;
- Weight means etc by this PDF

## Set-up

- Prior PDF for fluxes and data as for Gurney et al., Nature, 2002, i.e Gaussian;
- Uniform prior distribution for model choice (every model equally likely);
- Relative probability for each model depends on overlap between simulation and data (size of black triangle).

## Making a long story short

- ullet Collection of linear models  ${f H}_1 \dots {f H}_N$ s
- Let  $G(\vec{\mu}, \mathbf{C})$  be Gausian distribution with mean  $\vec{\mu}$  and covariance  $\mathbf{C}$

$$P(\vec{x}, \mathbf{H}) \propto G(\vec{x} - \vec{x}_0, \mathbf{C}(\vec{x}_0) * G(\vec{y} - \mathbf{H}\vec{x}, \mathbf{C}(\vec{y}))$$

$$P(\mathbf{H}) = \int d\vec{x} P(\vec{x}, \mathbf{H})$$

## Skipping the painful algebra

$$P\mathbf{H} \propto \left[\det \mathbf{H}\mathbf{C}(\vec{x}_0)\mathbf{H}^T + \mathbf{C}(\vec{y})\right]^{-0.5} \exp{-\frac{1}{2}(\vec{y} - \mathbf{H}\vec{x}_0)^T} \left[\mathbf{H}\mathbf{C}(\vec{x}_0)\mathbf{H}^T + \mathbf{C}(\vec{y})\right]^{-0.5} \exp{-\frac{1}{2}(\vec{y} - \mathbf{H}\vec{x}_0)^T} \exp{-\frac{1}{2}(\vec{y} - \mathbf{H}\vec{x}_0)^T} \left[\mathbf{H}\mathbf{C}(\vec{x}_0)\mathbf{H}^T + \mathbf{C}(\vec{y})\right]^{-0.5} \exp{-\frac{1}{2}(\vec{y} - \mathbf{H}\vec{x}_0)^T} \exp{-\frac{1}{2}(\vec{y} - \mathbf{H}\vec{x}_$$

$$\sum P(\mathbf{H}_i) = 1$$

- Can be calculated without ever performing an inversion;
- Similar to maximum value of  $P(\vec{x}, \mathbf{H})$ .

## Sample of Tabulated values

Model	$P(\mathbf{H})$
JMA-CDTM.maki	0.66
MATCH.law	0.29
MATCH.bruhwiler	0.02
SKYHI.fan	1e-7

- Unrealistically strong discriminant (7 orders of magnitude)
- Problem over-determined so many obs left to discriminate among models
- Model error  $(\mathbf{C}(\vec{y}))$  should not be independent.

## **Applications and Problems**

- Using data assimilation to improve model structure as well as parameters;
- Choosing among an ensemble of models;
- Unusually dependent on proper formulation of uncertainties.

## **Summary for Today**

- Climate DA is possible and can help us improve climate prediction;
- We can learn a lot by propagating uncertainty into prediction;
- We can extend DA beyond improving a particular model into the domain of model choice but it's not easy in real cases.

## **Overall Summary**

- Data assimilation best thought of as a statistical problem;
- Watch the statistics of inputs and results carefully;
- There's a lot to gain by considering more than just the best guess for unknowns and simulations;
- The basic theory is flexible enough for interesting extensions, like model choice but sometimes you have to go back to first principles.