

# Effect of statistical analysis parameters

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# Effect of statistical analysis parameters: Definitions

- Optimal interpolation:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{B}\mathbf{H}(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}(\mathbf{y}_o - \mathbf{H}\mathbf{x}_b)$$

$$\begin{aligned}\hat{\mathbf{A}}_{ij} &= \sigma_b^2 \mu_{b_{ij}} & \mu_{b_{ij}} &= (1 + r_{b_{ij}} / L_b) \exp(-r_{b_{ij}} / L_b) \\ \mathbf{R}_{ij} &= \sigma_y^2 \mu_{y_{ij}} & \mu_{y_{ij}} &= (1 + r_{y_{ij}} / L_y) \exp(-r_{y_{ij}} / L_y)\end{aligned}$$

$\mathbf{H}$  is a simple interpolation operator



$$x_{a_i} = x_{b_i} + W[i]_{b_{i1}}((y_{o_1} - x_{b_1}) - C(y_{o_2} - x_{b_2})) + i]_{b_{i2}}((y_{o_2} - x_{b_2}) - C(y_{o_1} - x_{b_1}))]$$

$$C = \frac{(\sigma_b^2 i]_{b_{12}} + \sigma_y^2 i]_{y_{12}})}{(\sigma_b^2 + \sigma_y^2)} \quad W = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_y^2} (1 - C^2)^{-1}$$



# Effect of statistical analysis parameters: Example

- ANALYSIS\_2OBS

Constructs 1-D analysis from 2 idealised obs with specified values and error characteristics

You may change the analysis parameters at the prompt, or press ENTER to retain previous values

- Current value o1 of ob#1    1.00000

- New o1 (between -2 and +2):

- Current position x1 of ob#1    4.00000

- New x1 (between 0 and 10):

- Current value o2 of ob#2    -1.00000

- New o2 (between -2 and +2):



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- New x2 (between 0 and 10):



# Effect of statistical analysis parameters: Example

- Observation error  $s_y$  (used for both obs) 1.00000

- New  $s_y$ :

The observation error correlation depends on the distance between obs

- Current observation error correlation length  $L_y$ : 0.00000

- New  $L_y$  (put  $L_y = 0.0$  for uncorrelated errors):

- Background error  $s_b$  (constant) 1.00000

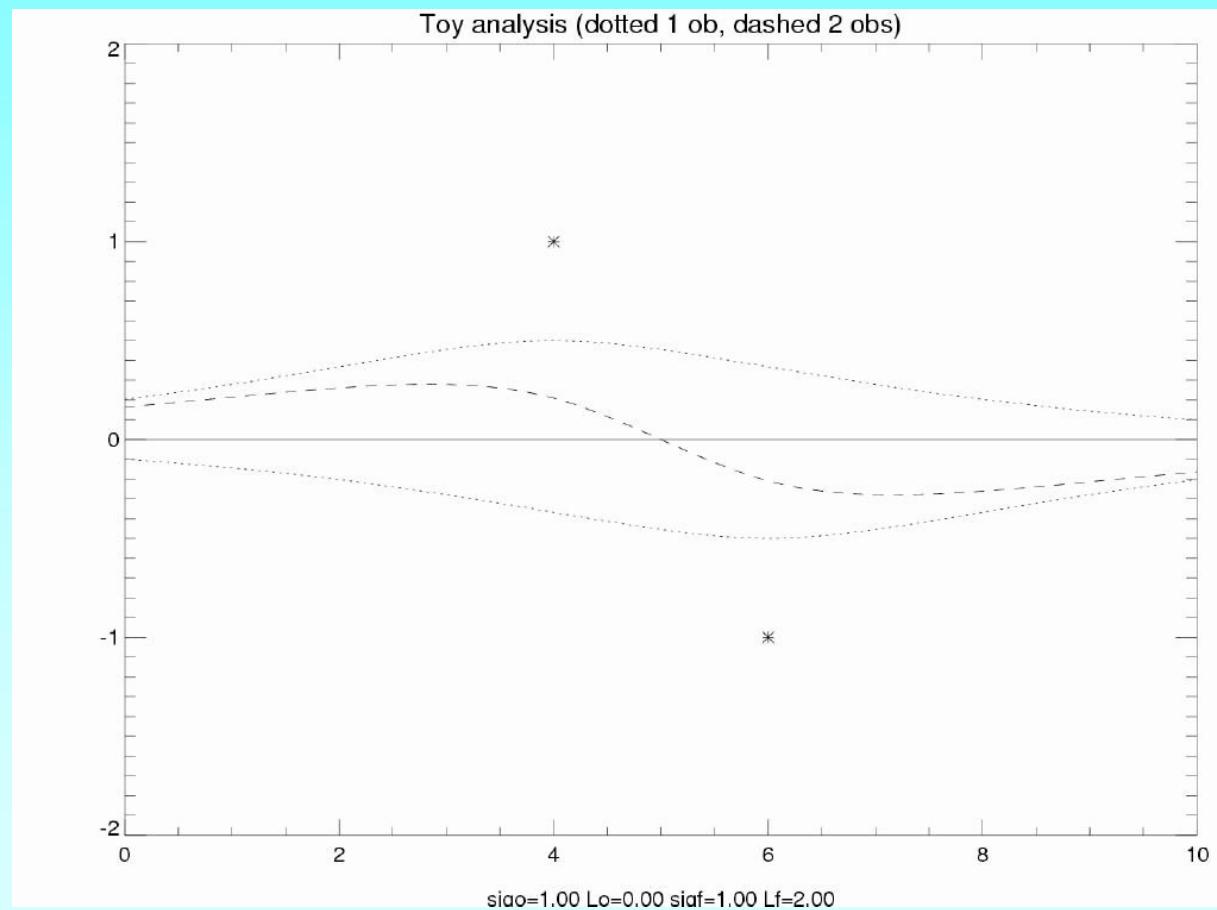
- New  $s_b$ :

- Background error correlation length  $L_b$  2.00000

- New  $L_b$  (put  $L_b = 0.0$  for uncorrelated errors):



# Effect of statistical analysis parameters: Example



# Effect of statistical analysis parameters: Example

- ANALYSIS\_MOBS

Constructs 1-D analysis from multiple idealised obs with specified values and error characteristics

- In this programme, we use two different ob types (A and B).
- The number of obs of each type should be in range 0 to 7.
- Press ENTER to retain previous values
- You may specify two separate observation types (A and B)
- Different observation types have different characteristics



# Effect of statistical analysis parameters: Example

- TYPE-A OBSERVATIONS
- Number of type A obs, noa: 3
- New noa (up to 7; 0 if none):
- Current type A ob locations 5.00000 4.00000 6.00000
- Current type A ob values 1.00000 1.00000 1.00000
- Do you want to change them (y or n)? n
- New type-A ob locations 5.00000 4.00000 6.00000
- New type-A ob values 1.00000 1.00000 1.00000
- Observation error for type A,  $s_{yA}$ : 2.00000
- Observation error correlation length  $L_{yA}$ : 0.00000



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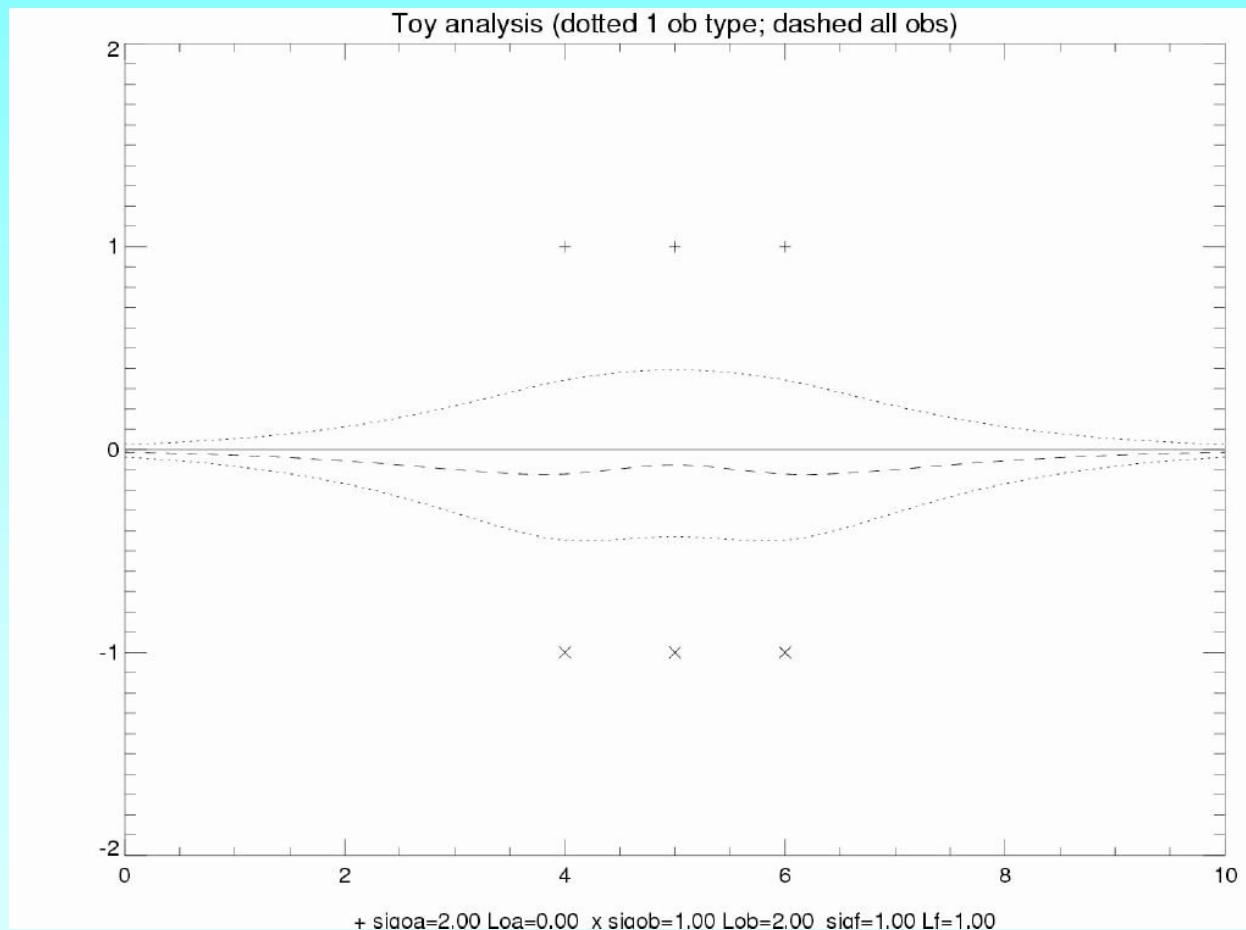


# Effect of statistical analysis parameters: Example

- TYPE-B OBSERVATIONS
  - Number of type B obs, nob: 3
  - New nob (up to 7; 0 if none):
  - Current type-B ob locations 5.00000 6.00000 4.00000
  - Current type-B ob values -1.00000 -1.00000 -1.00000
  - Do you want to change them (y or n)? n
  - New type-B ob locations 5.00000 6.00000 4.00000
  - New type-B ob values -1.00000 -1.00000 -1.00000
  - Observation error for type B,  $s_{yB}$  1.00000
  - Observation error correlation length  $L_{yB}$  2.00000
- BACKGROUND ("FORECAST") ERROR STATISTICS  
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  - Background error  $s_b$  (constant) 1.00000



# Effect of statistical analysis parameters: Example



# Effect of statistical analysis parameters: Example

- ANALYSIS\_SIM - Toy Analysis System
- Constructs 1-D analysis from simulated truth, forecast and observations with specified values and error characteristics
- You may change parameters, or just press ENTER to keep existing values
- The truth field is one of the following:
  - 0 - zero everywhere
  - 1 - pseudo step function
  - 2 - sum of several sinusoidal curves
  - 3 - statistically generated



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- Current truth type: 0



# Effect of statistical analysis parameters: Example

- BACKGROUND FIELD
  - You may use a background field that is shifted relative to the "truth"
  - OR a background equal to truth + random perturbation
  - Current amount of background shift (positive on left):  
**0.00000**
  - New shift:
  - Current background error,  $s_b$ :      **2.00000**
  - New  $s_b$ :
  - Background error correlation length  $L_y$       **2.00000**
  - New  $L_y$  (put  $L_y = 0$  for uncorrelated errors):



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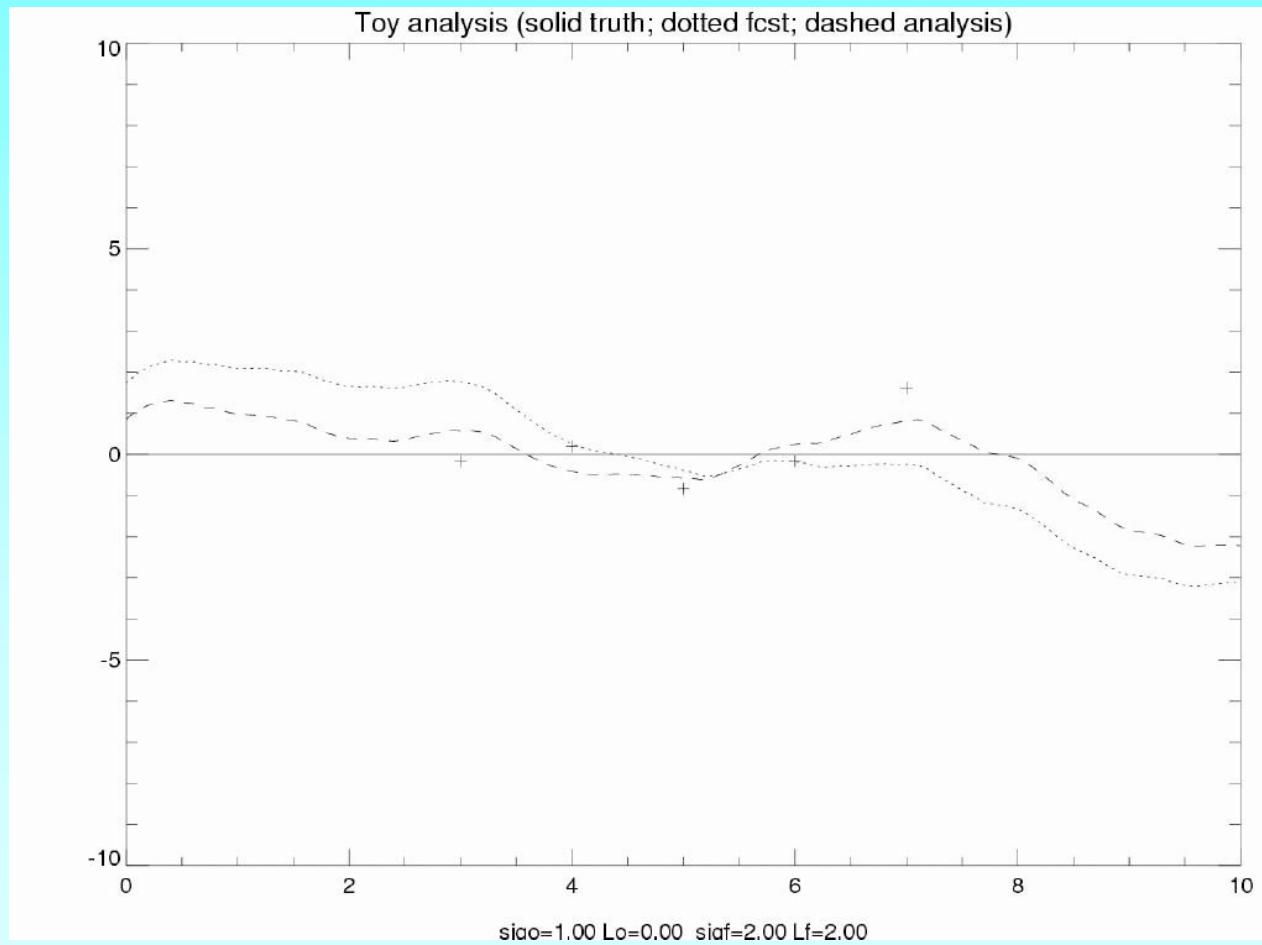


# Effect of statistical analysis parameters: Example

- OBSERVATIONS
- Number of obs, no: 5
- New no (up to 11):
- Current ob locations 5.00000 6.00000 4.00000  
7.00000 3.00000
- Do you want to change them (y or n)? n
- Current observation error,  $s_y$ : 1.00000
- New  $s_y$ :
- Observation error correlation length  $L_y$ : 0.00000
- New  $L_y$  (0.0 for uncorrelated errors):



# Effect of statistical analysis parameters: Example



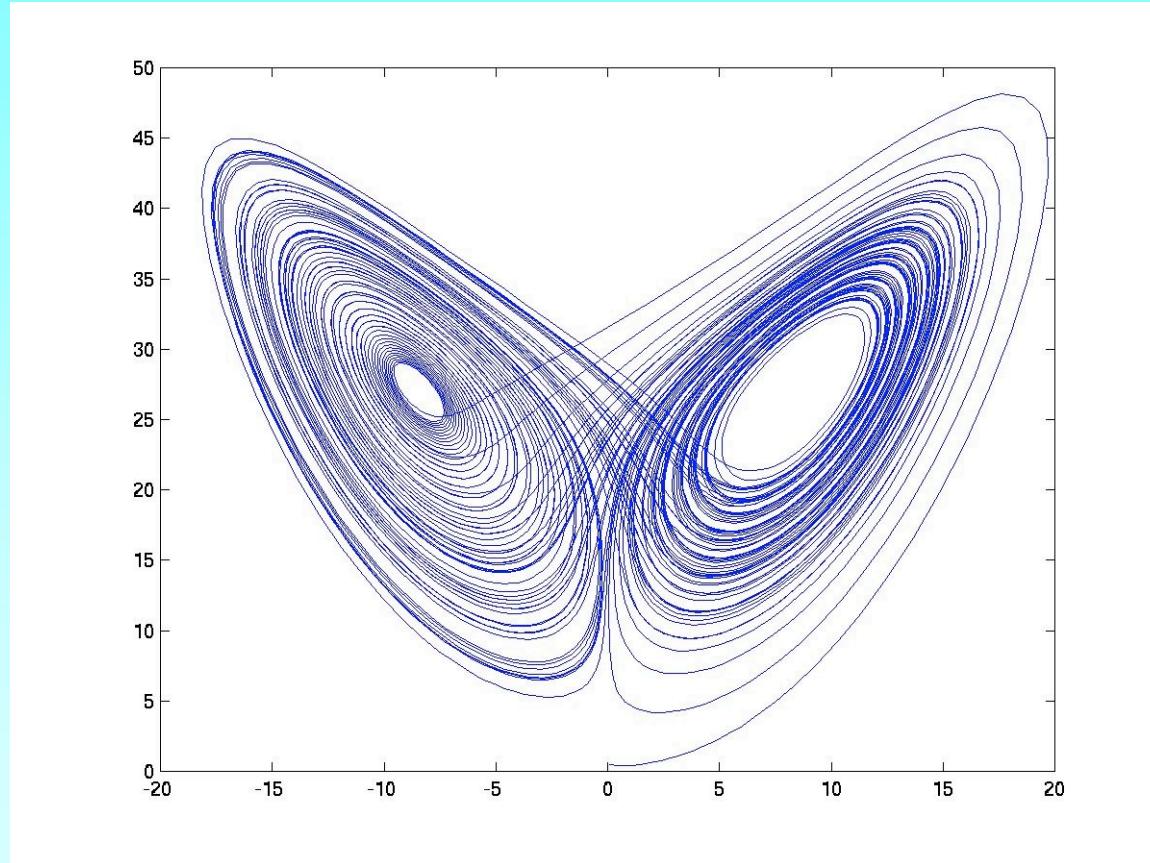


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# Lorenz equations: Sequential DA



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# Lorenz equations: Sequential DA

- You will use the program *lorenz\_menu* in the directory *lorenz/sequential*.
- You can experiment with four different sequential DA schemes
  - Successive correction
  - Analysis correction
  - Optimal interpolation
  - Kalman filter

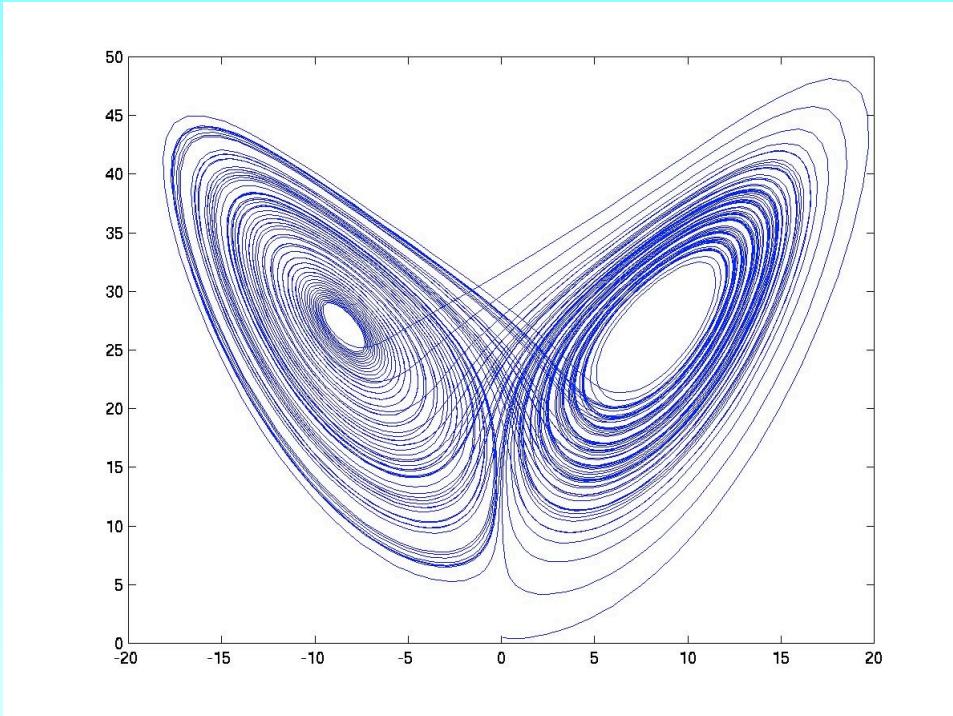


# You can choose various parameters:

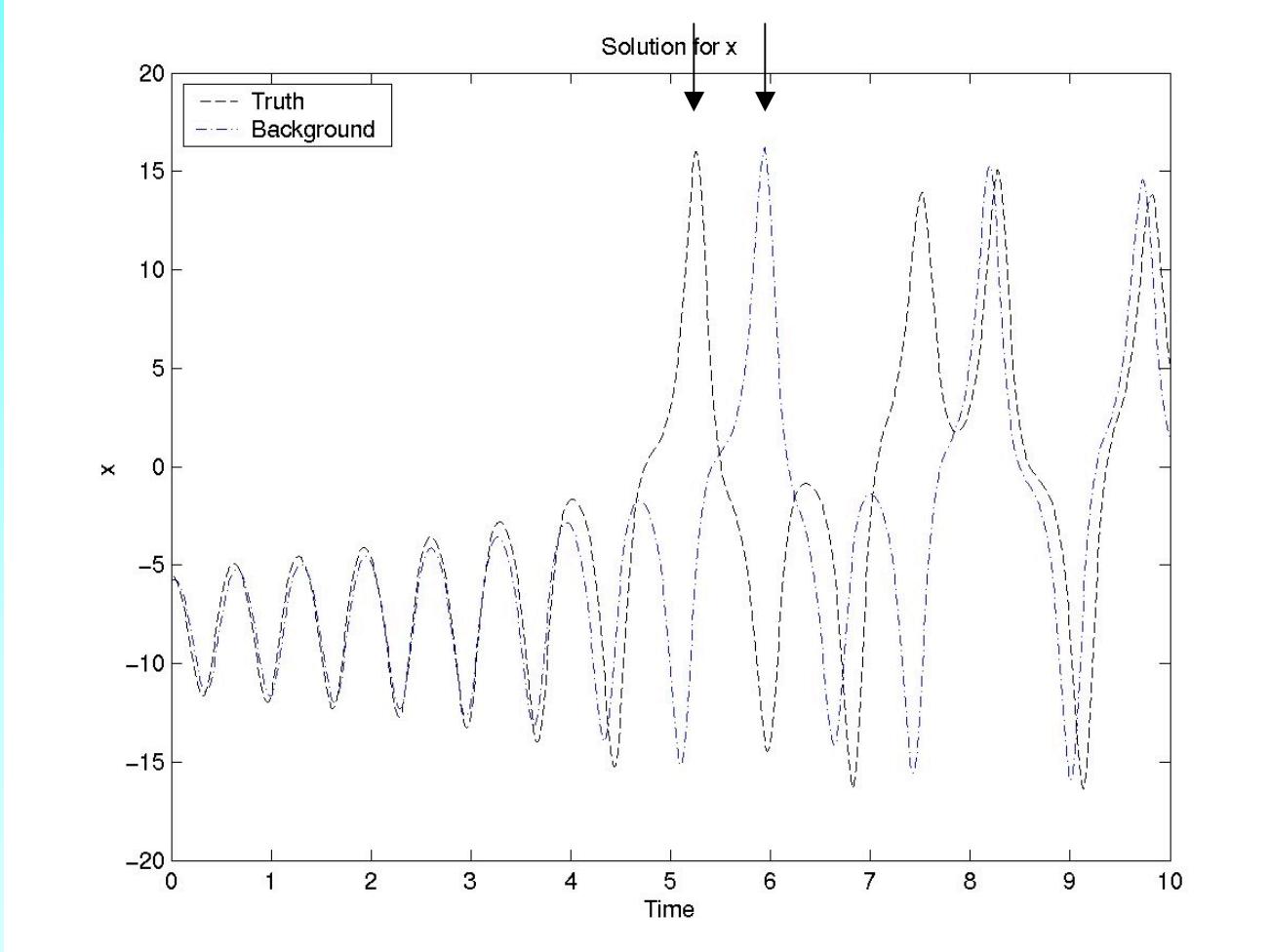
- Iterations
- Correct/ incorrect covariance matrices
- Frequency of observations
- Noise on observations

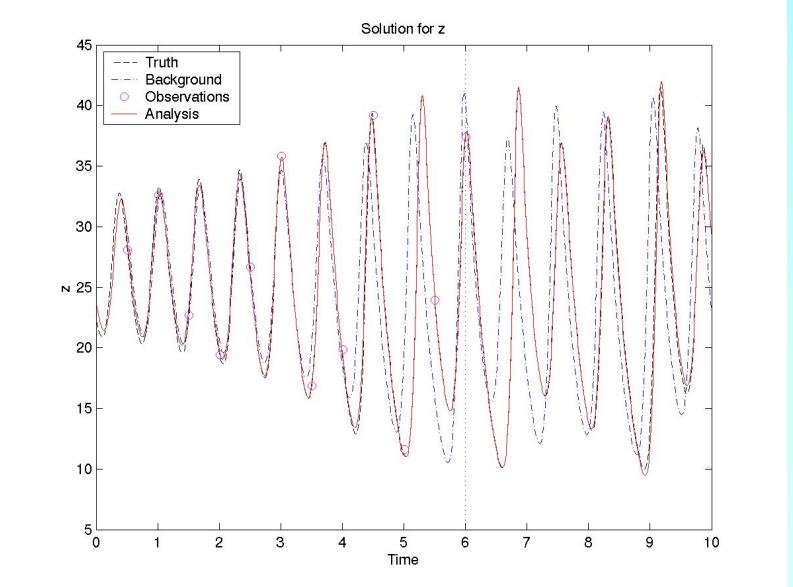
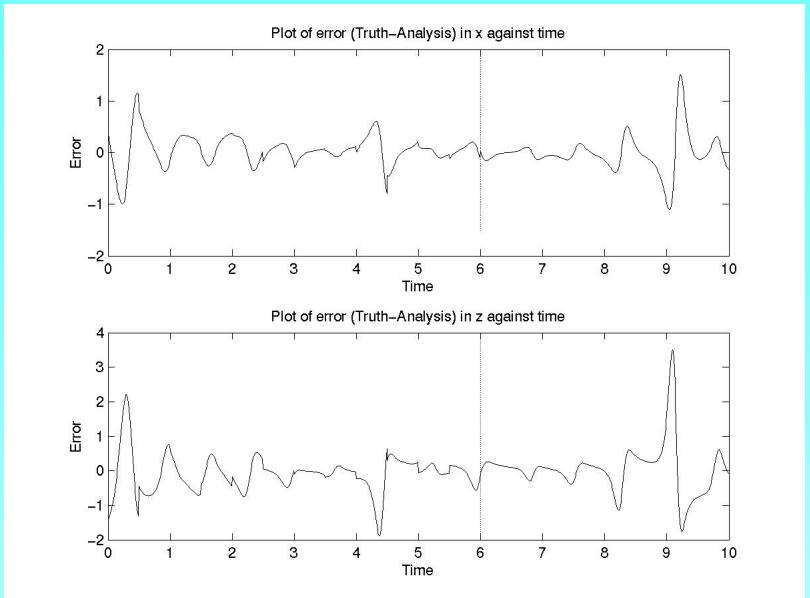
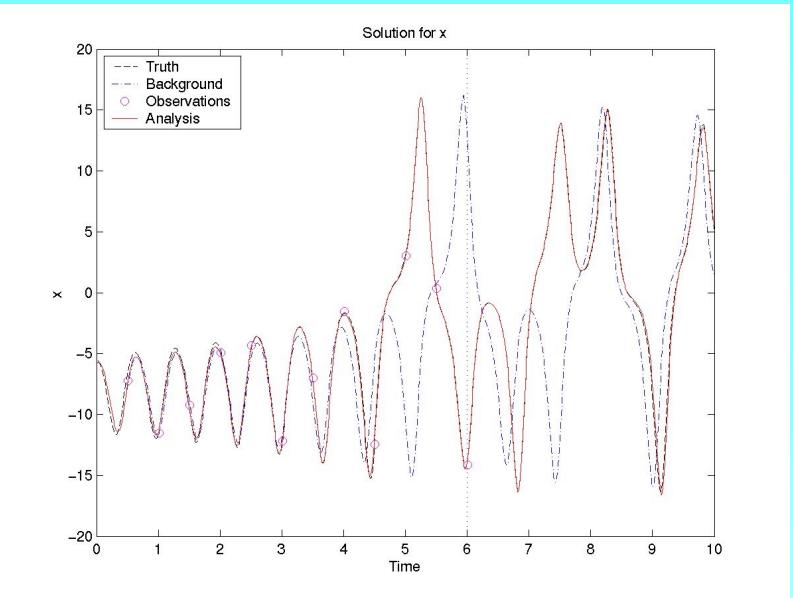


You are provided with a case in which the change in regime in the background occurs later than in the truth



# Truth and background conditions





**List of options chosen**

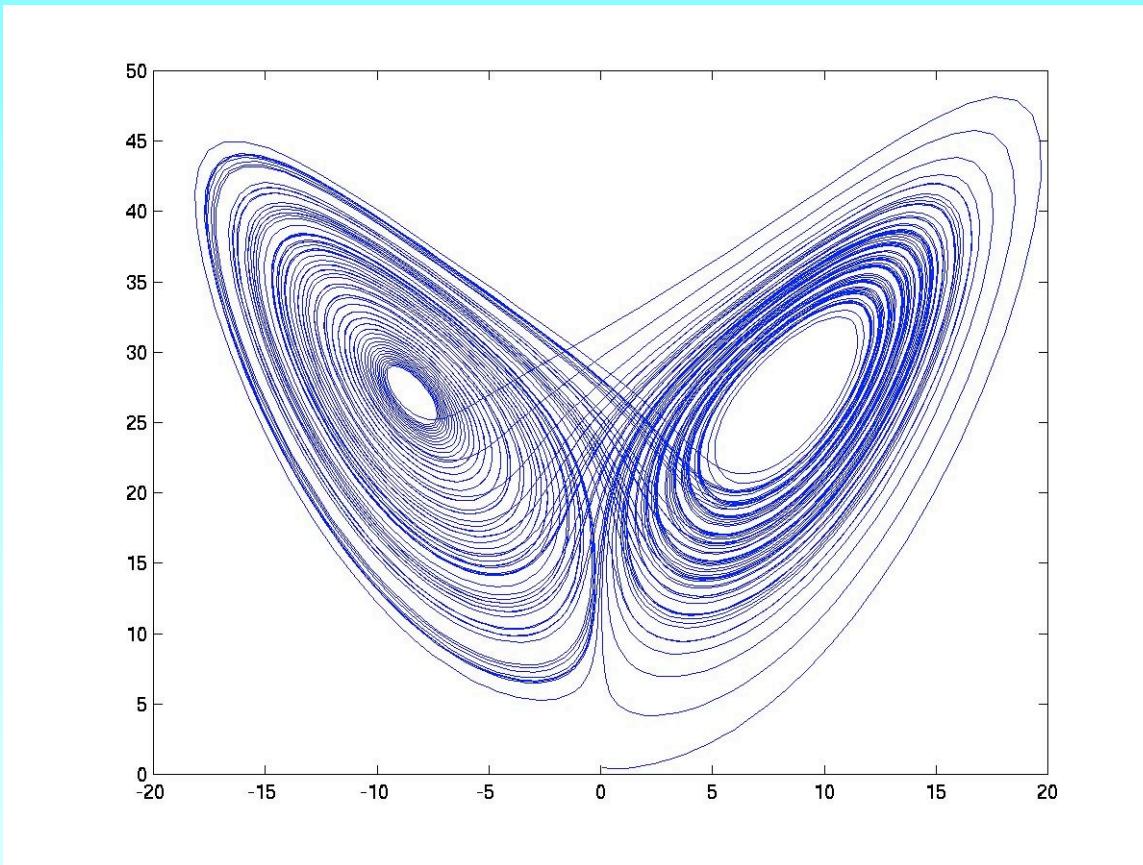
Analysis scheme: Successive correction  
 Number of iterations: 1  
 Time steps between observations: 50  
 Observations have random noise with variance 0.1  
 Noise generated in program and saved to file



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# Lorenz equations: 4D-Var



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# Lorenz equations: 4D-Var

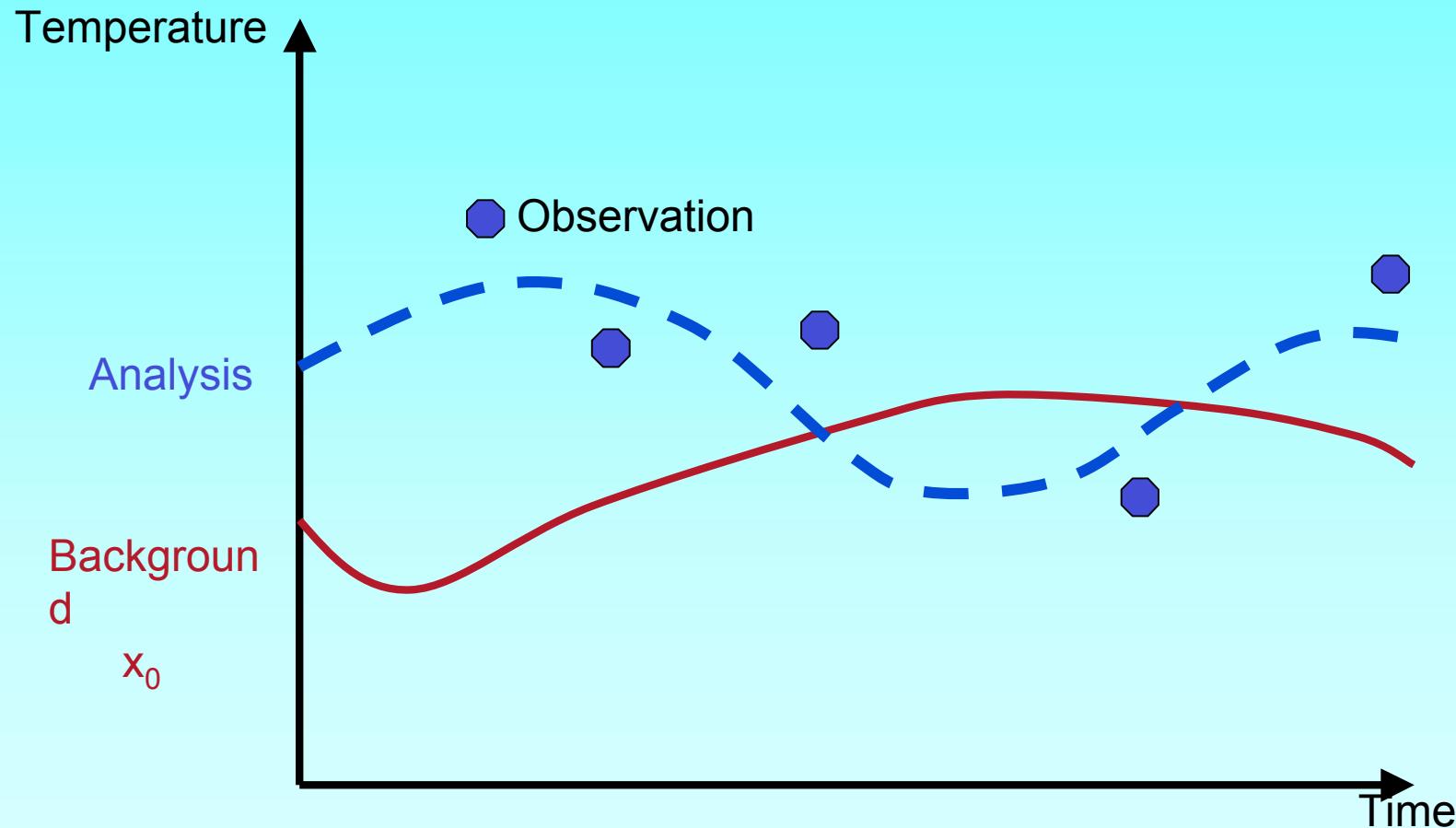
- In the directory *lorenz/var* you have various programs related to 4D-Var.
- The first set of exercises allow you to understand how a 4D-Var system is tested, by running tests of
  - Tangent linear model
  - Adjoint model
  - Gradient of cost function



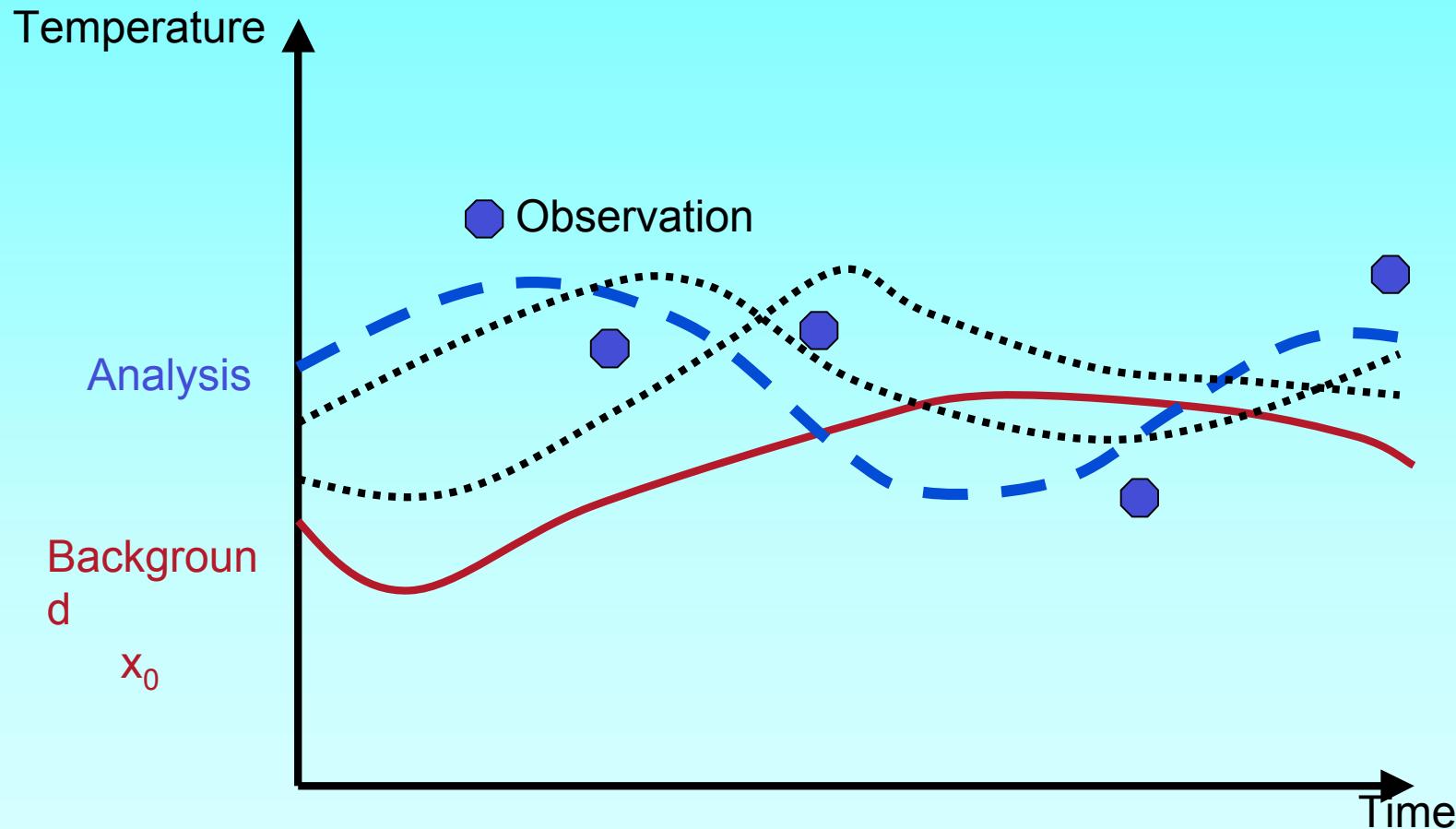
- You can then run two types of 4D-Var
  - Full 4D-Var
  - Incremental 4D-Var
- 
- Start by using the parameters in the documentation and then try changing them to see the effect on the analysis.



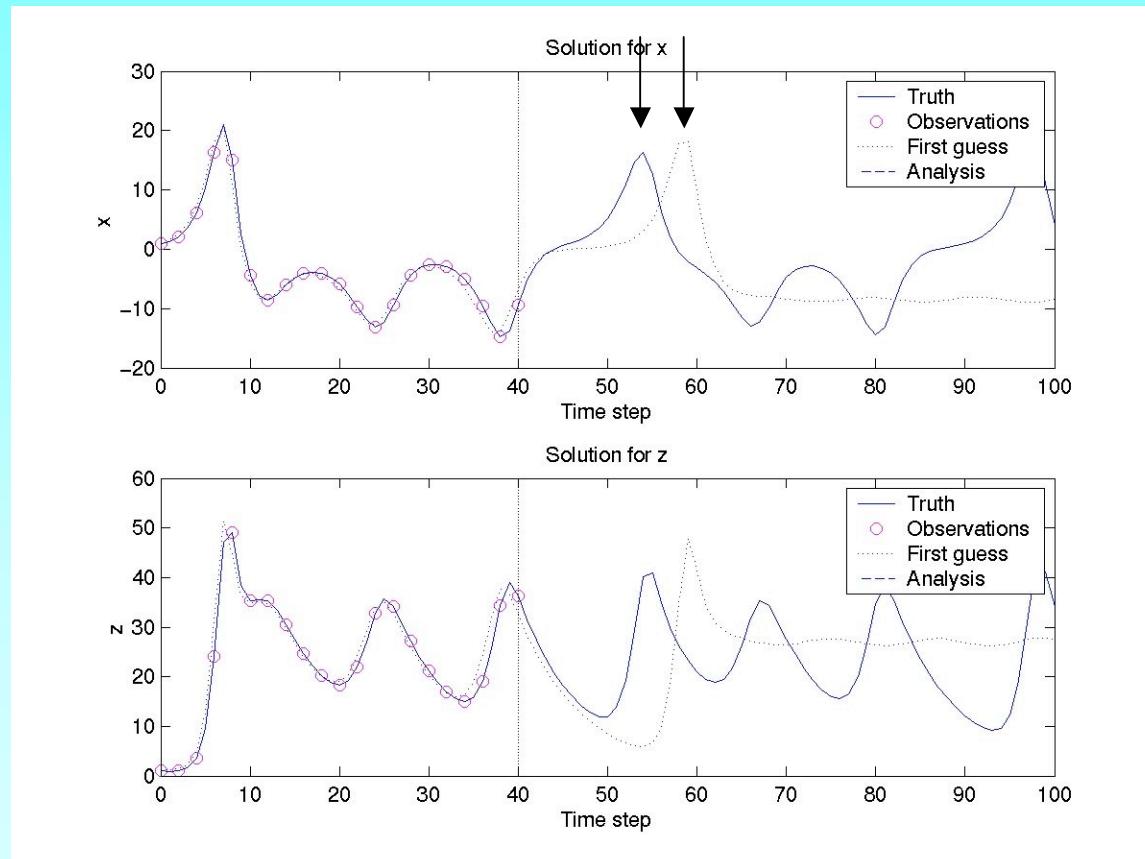
# Full 4D-Var

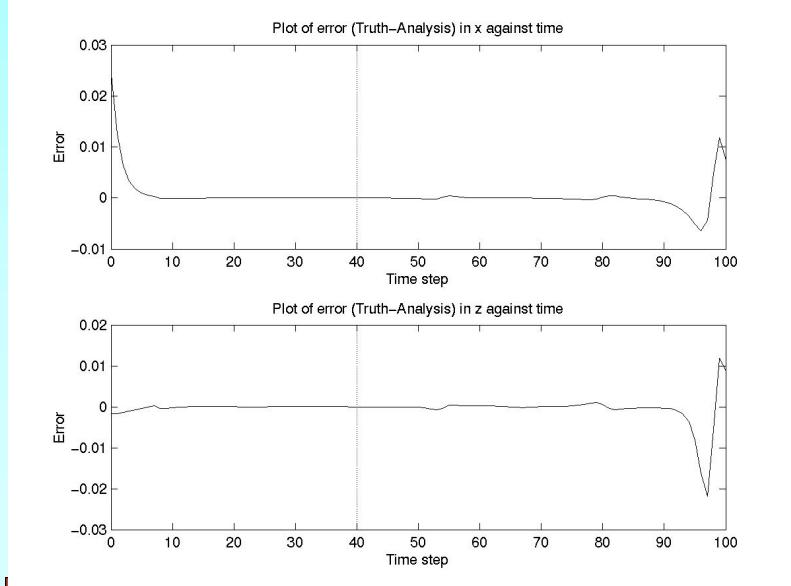
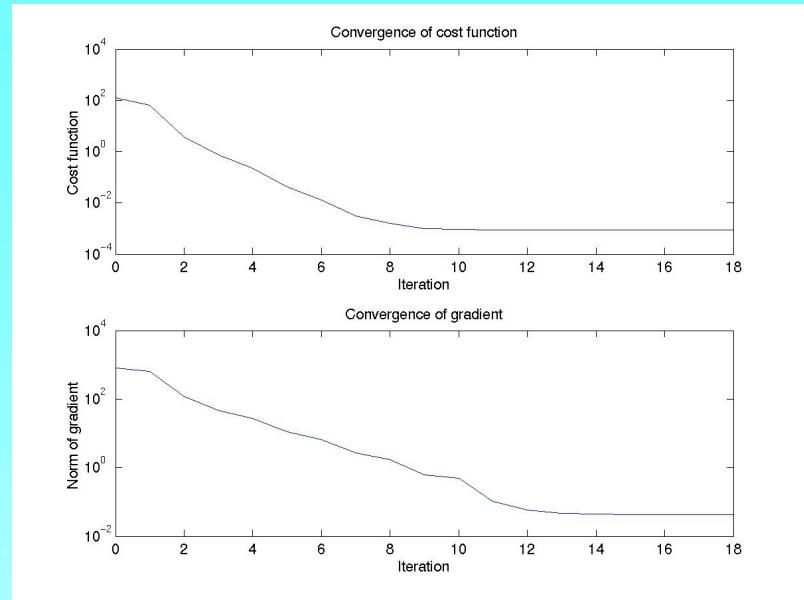
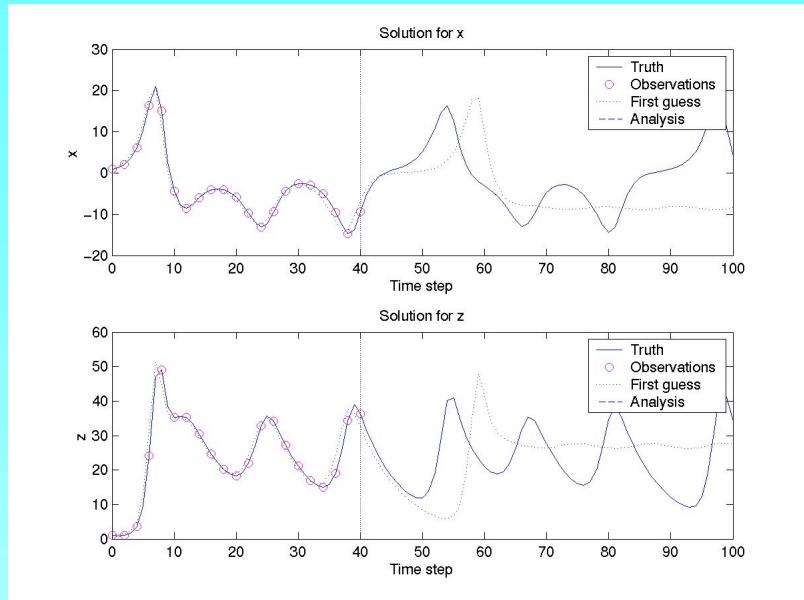


# Incremental 4D-Var



For this case a change of regime occurs towards the start of the forecast and is too late in the background





**List of options chosen**

True  $(x,y,z)$  at  $t=0$ : (1,1,1)  
 First guess  $(x,y,z)$  at  $t=0$ : (1.2,1.2,1.2)  
 Length of assimilation window: 2  
 Length of subsequent forecast: 3  
 Time step: .05  
 Frequency of observations = 2  
 Maximum iterations: 30  
 Tolerance: 1d-5  
 No noise on observations



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