A COMPARATIVE ANALYSIS OF SIMPLE RADIATIVE TRANSFER APPROACHES FOR AQUATIC ENVIRONMENTS

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ABSTRACT

Upwelling water-leaving radiance transmitted through the atmosphere and received by satellite sensors carries information on color-imparting water components, either naturally occurring or man-induced. Therefore, our understanding of optical processes operating on the air-water surface and within the water column has an utmost importance. However, rigorous theory of underwater light field based on solution of radiative transfer equation (RTE) is very complicated and cannot be used in routine remote sensing practice. On the other hand, it appears that simple radiative transfer models for semi-infinite plane-parallel layers, such as by Hulst (1974, 1980), Madgett-Richards (1971), Pierce-Marcus (1997), Gordon-Brown-Jacobs (1975), Morel-Prieur (1977) Chandrasekhar (1960)-Klier (1972), Hitachi (2004), and some others may be the key to a practical solution of various optical and bio-optical problems. We made a comparison of some selected approaches and evaluated their usefulness for bio-optical modeling of aquatic environments.

1. INTRODUCTION

Establishment of the proper relationships between irradiance ratio (reflectance) $R = E_u/E_d$ (E_u and E_d are upward and downward irradiances, respectively) and inherent optical properties, IOPs, such as volume absorption (a), scattering (b), attenuation (c) and backscattering (b_b) coefficients is one of the most intrigue tasks of aquatic optics. Such relationships may be used further for solution of various bio-optical problems, arising during the field, laboratory and remotely-sensed (RS) observations. In last decades two main factors contributed to extensive RS bio-optical investigations, namely, a conspicuous progress in the development of air- or satellite-born sensors and an establishment of a simple relationships between reflectance just below water-air interface $R(0-)$ and b_b/a ratio ([1], [2]). However, numerous theoretical and experimental investigations carried out in the last decades show significant impact of other factors, besides b_b/a, on R. Examples of these factors include, but are not limited to: size, form and concentration of various water components, scattering phase function for particulate matter, sun position, sky conditions, wavelength of the incoming irradiation, and geographical coordinates of the site under consideration. A large number of publications devoted to radiative transfer theory (RTT) problems is published in monographs and journals that are not focused on aquatic optics problems, and, hence, poorly known to limnologists and oceanographers. Thus, the aim of the present study to acquaint limnologists and oceanographers with existing and new radiative transfer approximations suitable for application in aquatic environments.

2. IRRADIANCE REFLECTANCE MODELING

Irradiance reflectance R within any semi-infinite plane-parallel layer can be represented as superposition of reflectance coefficient due to direct illumination R_{dir} and reflectance coefficient due to diffuse illumination R_{dif} as follows:

$$R = \frac{E_u^{dir} + E_u^{dif}}{E_d^{dir} + E_d^{dif}} = \frac{R_{dir} E_d^{dir} + R_{dif} E_d^{dif}}{E_d^{dir} + E_d^{dif}} = \frac{sR_{dir} + R_{dif}}{1+s}, \quad s = \frac{E_d^{dir}}{E_d^{dif}}, \tag{1}$$

where E_d^{dir} and E_d^{dif} are direct and diffuse components of downward irradiance, respectively; E_u^{dir} and E_u^{dif} are components of diffuse upward irradiance, forcing by direct and diffuse illumination, respectively.

Such optical properties as R_{dir} and s can be estimated from the knowledge of solar position, incoming radiation wavelength, sky conditions and geographical coordinates (e.g., [3]-[9]), however, these issues will not be considered here. Below we shall only concentrate on the question of R_{dif} prediction.

Calculation of R_{dif} in the frame of RTT is very complicated, that is doing it ill-suited for practical application. There is, however, a simple and accurate approximation by van de Hulst ([10], 11]):

$$R_{dif} = \frac{(1-0.139t)(1-t)}{1+1.170t}, \tag{2}$$

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where

$$t = \sqrt{\frac{1-\omega_0}{1-g\omega_0}} = \sqrt{1-\omega_0^{'}} \qquad (3)$$

based on fitting to exact solution of Chandrasekhar's radiative transfer equation (RTE) [12] and on using similarity relations, which significantly simplifies calculations in situations with isotropic and anisotropic scattering. Here $\omega_0 = b/c$ is the single scattering albedo and $\omega_0^{'}$ is the "reduced" scattering albedo

$$\omega_0^{'} = \frac{b(1-g)}{a+b(1-g)} = \frac{1-g}{1/\omega_0 - g}; \qquad (4)$$

g is the average cosine of scattering (asymmetry factor), closely connected with the b_b/b ratio. The form of such relationship does depend on the phase function for scattering. To start with, we shall use an analytic Henyey-Greenstein (HG) phase function, popular in radiative transfer calculations in astrophysics and atmospheric and oceanic optics (e.g., [11], [13], [14]):

$$p_{HG}(\mu, g) = \frac{1-g^2}{(1-2g\mu+g^2)^{1.5}}, \quad \mu = \cos\vartheta, \qquad (5)$$

where ϑ is a scattering angle, and b_b/b is defined by the following equation [13]:

$$\begin{aligned}(b_b/b)_{HG} &= \frac{1}{2}\int_{-1}^{0} p_{HG}(\mu, g)d\mu \\ &= \frac{1-g}{2g}\left[\frac{1+g}{\sqrt{1+g^2}} - 1\right].\end{aligned} \qquad (6)$$

Calculations of R_{dif} via ω_0 (and hence via b_b/a) for selected values g (0.0, 0.25, 0.50, 0.75, and 0.875) have been carried out by both exact (numerical values were taken from the tables published in [10], [11], [15]) and approximated (Eqs. 2 and 3) methods. The results show almost perfect coincidence between them (Fig. 1). Average error, expressed by the normalized (to the mean value) root-mean-squared errors (NRMSE, in %), is less than 0.5% for all range of parameters.

Unfortunately, despite its widespread usage and clear theoretical merits, HG phase function ill-suited to model scattering in real natural waters ([13], [16]). To overcome this shortcoming, we shall further use the relationship between b_b/b and g derived by me from the analysis of experimental data, collected by

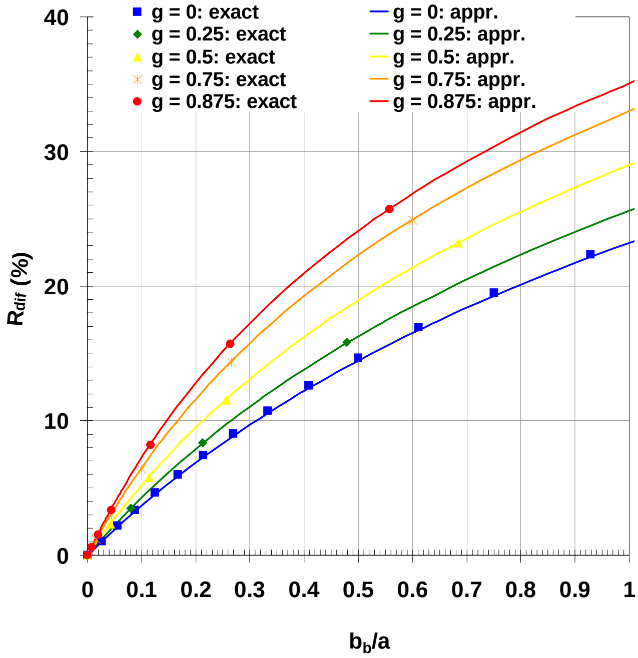

Fig. 1. Irradiance reflectance R_{dif} vs. b_b/a and g computed by exact (curves) and approximated (symbols) radiative transfer theory. Henyey-Greenstein phase function used in calculations.

Kirk [5] as follows:

$$(b_b/b)_{Kirk} = \frac{0.5(1-g)^2}{1-0.928g} \qquad (7)$$

A set of 19 different models for R_{dif} as a function of b_b/a, b_b/b (optionally) and g (optionally) is presented in Table 1. They can be classified into the following five main groups:

(1) Monte Carlo simulations [1, 2]
(2) Approximate solutions of Chandrasekhar's RTE ([10]-[12], [19], [20], [23], [25], [32], [33], [36])
(3) Two-flux approximations ([17], [18], [26], [28], [31], [34])
(4) Photon diffusion theory ([21], [22], [24], [27])
(5) Bio-optical modeling ([29], [30]).

Hulst's model (Eqs. 2 and 3) was chosen as a basic model, and all other models were compared with it at three sets of parameters g and R_{dif} : {$g = 0$, $R_{dif} \in [0.1, 10\%]$ } (isotropic scattering), {$g = 0.50$, $R_{dif} \in [0.1, 10\%]$ (clear oceanic waters, see, e.g., [5], [14]) and {$g = 0.95$, $R_{dif} \in [1, 30\%]$ } (turbid inland waters - [5], [14]). In the models presented, K and S denote diffuse absorption and diffuse scattering coefficients, respectively, in the two-flux Kubelka-Munk theory. Several models (*Hu*, *FPWW*) use Hulst's similarity conversion (Eqs. 3 and 4); we also will use

Table 1. Selected models for irradiance reflectance by semi-infinite plane-parallel surface illuminated by diffuse illumination R_{dif}. Normalized root-mean-squared errors (NRMSE, in %) are computed for the media with isotropic scattering, clear oceanic and turbid inland water types, respectively (see text for definitions)

Model	Source	NRMSE (%)	Abbreviation for the model
$\frac{b_b}{a}=\frac{S}{K}=\frac{2R_{dif}}{(1-R_{dif})^2}$; $R_{dif}=1+\frac{K}{S}-\sqrt{\frac{K}{S}\left(\frac{K}{S}+2\right)}$	Kubelka and Munk (1931) [17]; Kortüm (1969) [18]	25.3 17.6 37.7	KM
$\frac{b_b}{a}=\frac{8}{3}\frac{b_b/b}{1-g}\frac{S}{K}$; $\frac{K}{S}=\frac{8b_b/b}{3(1-g)(b_b/a)}$	Richards (1970) [19]; Madgett-Richards (1971) [20]	5.3, 5.4, 3.2	MR
$\frac{b_b}{a}=\frac{8}{3}\frac{b_b/b}{1-g}\left(\frac{S}{K}+\frac{1}{2}\right)$; $\frac{K}{S}=\frac{8b_b/b}{3(1-g)(b_b/a)-4(b_b/b)}$	Gate (1971) [21]; Brinkworth (1971) [22]	—, —, 51.8	GB
$R_{dif}=\frac{ln(1+\xi)-\xi}{ln(1-\xi)+\xi}$, $\omega_0'=\frac{2\xi}{ln[(1+\xi)/(1-\xi)]}$	Chandrasekhar (1960) [12]; Klier (1972) [23]; this publication	17.8, 18.1, 7.3	CK
$R_{dif}=\frac{(1-0.139t)(1-t)}{1+1.170t}$, $t=\sqrt{\frac{1-\omega_0}{1-g\omega_0}}\equiv\sqrt{1-\omega_0'}$	Van de Hulst (1974) [10]	0.0, 0.0, 0.0	Hu
$R_{dif}=0.0003+0.3687G+0.1802G^2+0.0740G^3$, $G=\frac{b_b/a}{1+b_b/a}$	Gordon, Brown, and Jacobs (1975) [1]	2.5, 9.3, 49.1	GBJ
$R_{dif}=0.33\frac{b_b}{a}$	Morel and Prieur (1977) [2]	5.2, 11.0, 42.6	MP
$\frac{b_b}{a}=\frac{8}{3}\frac{b_b/b}{1-g}\left(\frac{S}{K}+\frac{1}{8}\right)$; $\frac{K}{S}=\frac{8b_b/b}{3(1-g)(b_b/a)-(b_b/b)}$	Meador and Weaver (1979) [24]	70.3, 70.1, 18.6	MW-2
$\kappa=\frac{224}{132-55\omega_0'+35\sqrt{1+2(1-\omega_0')/35+121(1-\omega_0')^2/49}}$, $\sigma=\frac{15-(1-\omega_0')\kappa(16-3\kappa)}{\omega_0'(16-3\kappa)}$, $\kappa=\frac{K}{a}$, $\sigma=\frac{S}{b_b}$	Meador and Weaver (1979) [24]; this publication	10.7, 12.3, 52.7	MW-4
$\frac{b_b}{a}=\frac{4R_{dif}}{(1-\sqrt{R_{dif}})^2(1+4\sqrt{R_{dif}}+R_{dif})}$	Khalturin (1985) [25]	17.2, 23.8, 55.5	Kh

$R_{dif} = \frac{r_{du} b_b / a}{(1 + r_{du} b_b / a) + m_{du}(1 + r_{ud} b_b / a)}$	Aas (1987) [26]	18.6, 16.2, 30.4	A
$R_{dif} = \frac{\omega_0'}{1 + 2(1-\omega_0') + (5/3)\sqrt{3(1-\omega_0')}}$	Flock, Patterson, Wilson, and Wyman (1989) [27]	16.9, 17.2, 15.1	FPWW
$R_{dif} = \frac{r_{du} b_b / a}{(1 + r_{du} b_b / a)(1 + m_{du})}$	Sathyendranath and Platt (1991) [28]	18.6, 12.3, 24.6	SP
$R_{diff} = [0.6279 - 0.2227\eta_b - 0.0513\eta_b^2 - 0.676 * (0.3119 - 0.2465\eta_b)](b_b / a), \eta_b = 0.000965 / b_b,$ $b_b = (0.00193 + 0.3C^{0.62})(b_b / b), a = 0.064 + 0.0223C^{0.65}$	Morel and Gentili (1991, 1993) [29, 30]	26.3, 17.3, 55.2	MG
$R_{dif} = (1-t)\left[\frac{1}{t} - \frac{0.5\ln(1+2t)}{t^2}\right],$ $t = \sqrt{1-\omega_0'}$	Pierce and Marcus (1997) [31]; this publication	11.9 12.1 6.4	PM
$R_{dif} = \frac{(1-H)\left(H - \sqrt{1+H^2}\right)^2}{1+H},$ $H = \frac{1}{\sqrt{1 + (4 + 2\sqrt{2})(b_b / a)}}$	Haltrin (1998) [32]	10.2, 17.1, 51.3	H
$R_{dif} = \exp\left[-\frac{6.3744 + 0.35688 J^{0.28786}}{\sqrt{3(1+J)}}\right],$ $J = \omega_0' / (1-\omega_0')$	Jacques (1999) [33]	54.2, 55.9, 60.0	J
$R_{dif} = \frac{r_{du} b_b / a}{(1 + b_b / a)(1 + m_{du})}$	Hirata (2003) [34]	18.6, 9.9, 13.4	Hi
$R_{dif} = \exp\left[-4\sqrt{\frac{1-\omega_0}{3(1-g)}}\right]$	Zege, Ivanov, and Katsev (1991) [35]; Kokhanovsky (2003) [36]	152.8, 49.1, 18.2	ZIKK

the same conversion in *CK*, *MW-4* and *PM* models ("extended models"). The last model (extended *PM* model) was derived by me from the two-flux approximation of Pierce and Marcus (1997) developed by them for the isotropically-scattered medium illuminated by diffuse light and viewed from the angle $\theta = \arccos \mu$ ([11], [12], [14], [31]):

$$R_{dif}(\mu) = 1 - H(\mu)\sqrt{1-\omega_0}\,, \qquad (8)$$

where $H(\mu)$ is Chandrasekhar's function, expressed approximately as [31]:

$$H(\mu) = \frac{1 + 2\mu}{1 + 2\mu\sqrt{1-\omega_0}}\,. \qquad (9)$$

Eqs. (8) and (9) were generalized in order to obtain solution for the case of diffuse viewing and for any anisotropic media, using the conversion formula ([11], [12], [14])

$$R_{dif} = 1 - 2\int_0^1 \mu H(\mu)\sqrt{1-\omega_0}\,d\mu\,, \qquad (10)$$

performing integration in Eq. (10) and replacing ω_0 by ω_0' (Eq. 4). The shape factors r_{du} and r_{ud} in *A*, *SP* and *Hi* models are explain the ratios of the upward scattering and downward scattering, respectively, to b_b ([26], [32], [34]), and have been computed by equations derived by Walker ([14], p. 58):

$$r_{du} = \frac{1+g-1.5g\bar{\mu}_d}{1-0.5g} \qquad (11)$$

and

$$r_{ud} = \frac{1+g-1.5g\bar{\mu}_u}{1-0.5g}\,. \qquad (12)$$

where $\bar{\mu}_d$ and $\bar{\mu}_u$ are the average directions (cosines) of the light field in the lower and upper hemispheres, respectively; $m_{du} = \bar{\mu}_d / \bar{\mu}_u$. Average cosines were computed by formulae of Walker ([14], p. 93) as follows:

$$\bar{\mu}_d = \frac{1}{ln(1-\xi)} + \frac{1}{\xi} \ (13), \quad \bar{\mu}_u = \frac{1}{ln(1+\xi)} - \frac{1}{\xi}\,, \ (14)$$

where $\xi = K_\infty/c$ (K_∞ is the diffuse attenuation coefficient in the asymptotic light regime; $c = a+b$ is the beam attenuation coefficient) is the positive real root of the characteristic equation for the reduced single scattering albedo ω_0'. Bio-optical model by Morel and Gentili ([29], [30]) was expoited at wavelength λ = 550 nm, isotropic incoming sky radiation, and for any values of chlorophyllous pigment concentration *C*.

The accuracy of the different models is varied widely (Table 1). Figs. 2-4 present R_{dif} vs. *g* and b_b/a computed only for several models which can be regarded as more or less acceptable. Two analytical models that yielded the good results (accuracy better than 12%) at any values of parameters are Madgett-Richards and extended Pierce-Marcus models. The other three models [Gordon-Brown-Jacobs, Morel-Prieur and extended δ-Eddington fourth-order approximation (MW-4)] demonstrate good accuracy (better than 12%) for weakly anisotropic ($g < 0.5$) underwater light field. Two-flux model by Hirata show acceptable results (NRMSE < 13%) for both clear and turbid waters, and extended Chandrasekhar-Klier model yields good results (NRMSE < 7%) for very turbid waters.

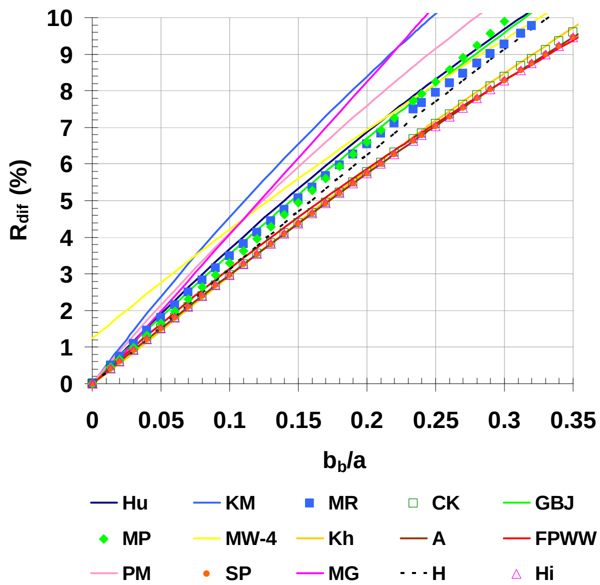

Fig. 2. Different models used for calculation of irradiance reflectance of a layer, illuminated by diffuse light: isotropic scattering case ($g = 0$).

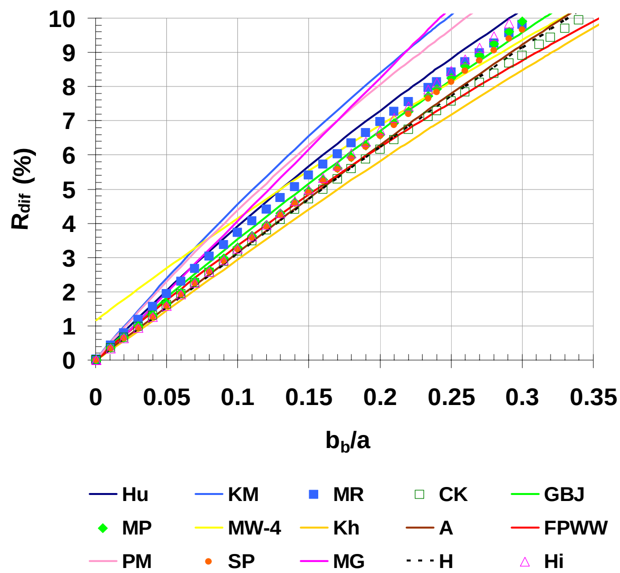

Fig. 3. The same as Fig. 2, but for clear oceanic waters ($g = 0.50$). Kirk's (1991) data-based relationship between b_b/b and *g* (Eq. 7) used in modeling.

2. CONCLUSION

19 different radiative transfer reflectance models intensively used in various scientific fields are collected and compared with almost exact radiative transfer solution (by van de Hulst). Three of them (Hulst's, Madgett-Richards and extended Pierce-Marcus models) demonstrate the best results and can be proposed for further use in aquatic optics investigations. Some others models also proposed for different water types.

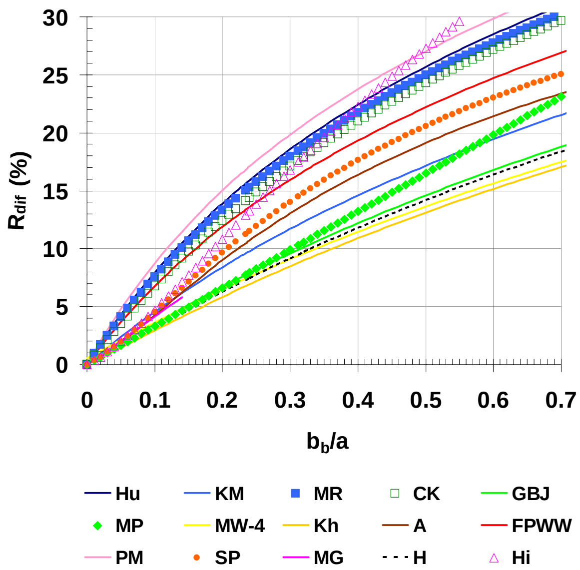

Fig. 4. The same as Fig. 3, but for turbid inland waters (*g = 0.95*).

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