Building height estimation using multibaseline L-band SAR data and polarimetric weighted subspace fitting methods

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Objectives of urban area analysis

- Estimate height locations of scatterers
- Extract physical features of scatterers

MB-PolinSAR approach

- MB-InSAR: heights, layover sources
- PolinSAR: physical features
MB-PolinSAR methods and limitations

- Classical methods (Capon, Beamforming): low resolution
- High Resolution methods (MUSIC, Det-ML, Stoch-ML)
  - Adapted to responses with specific statistical properties: coherence, correlation, ...
  - Irregularly sampled baseline: spurious sidelobes.

Proposed method

PolWSF: polarimetric weighted subspace fitting
General array signal model (m sensors)

\[ y = A(\theta)x + n \in \mathbb{C}^m \]

- \( y \in \mathbb{C}^m \): observed noisy data
- \( x \in \mathbb{C}^d \): source (reflected) signals (\( d \) elements)
- \( n \in \mathbb{C}^m \): additive noise
- \( m \times d \) steering matrix

\[ A(\theta) = [a(\theta_1), \ldots, a(\theta_d)] \]

- \( a(\theta_i) \) \( m \)-element steering vector
- \( a(\theta_i) = a(z_i) \) with height \( z_i \)

\[ a(z_i) = [1, \exp\{jk_2z_i\}, \exp\{jk_mz_i\}]^T \]
Unconditional MB-PolinSAR signal model

\[
y_u = \sum_{i=1}^{d} \sqrt{\sigma_i} x_{ui} \odot a(z_i) + n
\]

- Valid for **Distributed Scatterers with speckle affected** responses
- **Stochastic** source signal
  \[
x_i = \sqrt{\sigma_i} x_{ui} \quad \text{with} \quad x_{ui} \sim \mathcal{N}(0, I)
\]
- \(y_u \sim \mathcal{N}(0, R_y)\)
- \(\sigma_i, z_i\) estimated from \(R_y\)

![Diagram showing rough surface scattering, roof scattering, and volume scattering with urban areas and buildings.]
Conditional MB-PolinSAR signal model

\[ y_c = \sum_{i=1}^{d} \sqrt{\sigma_i} x_{ci} a(z_i) + n \]

- Valid for **coherent scatterers**: frequently encountered over urban areas
- \( x = \sqrt{\sigma_i} x_{ci} \) is deterministic (frozen) over \( N \) observations (looks)
- \( y_c \sim \mathcal{N}(Ax, \sigma_n^2 I) \)
- \( \sigma_i, z_i \) estimated from \( Ax \)
Hybrid SAR signal model

- Mixture of coherent and distributed scattering contributions * (Sauer et al: 2007)

\[
y = y_c + y_u = \sum_{i=1}^{d_1} \sqrt{\sigma_i} x_{ci} a(z_i) + \sum_{i=1}^{d_2} \sqrt{\sigma_i} x_{ui} \circ a(z_i) + n
\]

- MUSIC: degraded performance for coherent scatterers
- Det-ML and Stoch-ML: partially sub-optimal techniques

Model adaptive method
\[ \Rightarrow \text{Weighted Subspace Fitting (WSF)} \]
Characteristics of WSF

- Applicable to arbitrary array structure
- Multi-dimensional optimization (MUSIC: 1-D optimization)
- Statistically efficient for correlated signals compared to MUSIC
- Optimal for coherent and distributed scatterers
  - Coherent scatterers
    - $C_{WSF}(\text{error}) \leq C_{Det-ML}(\text{error})$
    - Asymptotically $C_{WSF}(\text{error}) \rightarrow \text{CRLB}$
  - Distributed scatterers
    - $C_{WSF}(\text{error}) = C_{Stoch-ML}(\text{error})$
    - $C_{WSF}(\text{error}) \rightarrow \text{CRLB}$
WSF principle * (M. Viberg et al: 1991)

Subspace fitting problem

\[ \hat{z}, \hat{T} = \arg \min \| E_s W^{1/2} - AT \|_F^2 \]

- **T**: fitting matrix; **W**: weighting matrix; **A**: steering matrix;
  **E_s**: signal subspace; **E_n**: noise subspace
- **Objective**: determine \( T \) that fits the steering matrix space to signal subspace
- **Subspace fitting cost function** \( \| E_s W^{1/2} - AT \|_F^2 \)

Total Least-Square Solution

\[ \hat{T} = (A^HA)^{-1}A^HE_sW^{1/2} \]
\[ \hat{z} = \arg \min \text{tr}\{A^H\hat{E}_n\hat{E}_n^HAW\} \]
WSF principle * (M. Viberg et al: 1991)

Various weighting matrix choices

- \( W = I \) → MUSIC
- \( W = \hat{\Lambda}_s - \hat{\sigma}^2 I = \tilde{\Lambda} \) → Det-ML for coherent scatterers

Optimal weighting matrix: WSF

- \( W_{opt} = (A^H \hat{E}_s \Lambda_s W_s^{-1} \hat{E}_s^H A)^{-1} \)
- Estimation error covariance \( C(W_{opt}) \leq C(W) \)
- \( C(W_{opt}) = C(W_{stoch-ML}) \) for distributed scatterers

\[ \Rightarrow \text{WSF is optimally model adaptive} \]
Extension to the fully polarimetric case

\[(\hat{z}, \hat{\Phi}) = \arg\min tr\{A^H(z, \Phi)\hat{E}_n\hat{E}_n^H A(z, \Phi)W\}\]

\[W_{opt} = (A^H(z, \Phi)\hat{E}_s(\Lambda_s - \sigma^2 I)^{-2}\Lambda_s\hat{E}_s^H A(z, \Phi))^{-1}\]

Polarimetric steering vector and matrix

- \(a(z, \phi) = a_1(z)\phi_1 + a_2(z)\phi_2 + a_3(z)\phi_3\)
- \(\phi = [\phi_1, \phi_2, \phi_3]^T\): unitary target vector (4 parameters)
- For \(d\) sources, \(A(z, \Phi) = [a(z_1, \phi_1), \ldots, a(z_d, \phi_d)]\)

Optimization of POL-WSF estimation

- Originally search dimension = \(5 \times d\)
- Analytic optimization solution over polarization space (see paper)
  Optimization algorithm maintained to \(1 \times d\) search dimension
### Simulations of distributed and coherent models

<table>
<thead>
<tr>
<th>SNR= 20dB</th>
<th>Distributed model</th>
<th>Coherent model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source height (m)</td>
<td>0.0</td>
<td>15.0</td>
</tr>
<tr>
<td>MUSIC</td>
<td>0.0</td>
<td>14.97</td>
</tr>
<tr>
<td>WSF</td>
<td>-0.1</td>
<td>14.96</td>
</tr>
</tbody>
</table>

**MUSIC** cannot localize coherent scatterers or too closely spaced contributions.
Urban zone in SAR image

- Acquired by E-SAR (DLR)
- L-Band (1.3 GHz)
- Resolution: 0.5 m × 2.5 m
- Fully polarimetric
- Dual-baseline InSAR
  - Small baseline of 10 m
    height of ambiguity: 55 m to 73 m
  - Large baseline of 40 m
    height of ambiguity: 14 m to 18 m

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Coherent and Distributed scatterers in SAR image
*(L. Ferro-Famil et al: 2007)
SP-MUSIC vs SP-WSF tomograms (Model order=1)

MUSIC pseudo-tomogram (order=1)

MUSIC discrete pseudo-tomogram (order=1)
SP-MUSIC vs SP-WSF tomograms (Model order=1)

MUSIC pseudo-tomogram (order=1)

MUSIC: sidelobes at about 15m ~ height ambiguity of large baseline

WSF: no sidelobes

MUSIC discrete pseudo-tomogram (order=1)

WSF pseudo-tomogram (order=1)
SP-WSF tomograms (Model order=1)
SP-WSF tomograms (Model order=1)

Tomogram in slant range

Tomogram in ground range
SP-MUSIC vs SP-WSF tomograms (Model order=2)

MUSIC discrete pseudo-tomogram (order=2)

WSF pseudo-tomogram (order=2)
SP-MUSIC vs SP-WSF tomograms (Model order=2)

MUSIC pseudo-tomogram (order=2)

MUSIC: cannot separate 2 coherent scatterers
WSF: good resolution for coherent scatterers

MUSIC discrete pseudo-tomogram (order=2)

WSF pseudo-tomogram (order=2)
SP-WSF tomograms (Model order=2)

Tomogram in slant range

Tomogram in ground range
SP-WSF tomograms (Model order=2)

Tomogram in slant range

Tomogram in ground range

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FP-MUSIC vs FP-WSF tomograms (Model order=1)
FP-MUSIC vs FP-WSF tomograms (Model order=1)

FP-MUSIC pseudo-tomogram (order=1)

FP-WSF pseudo-tomogram (order=1)
FP-WSF tomogram (Model order=1)
Scattering mechanism $\alpha$-FP-WSF (order=1)

$\alpha$ in slant range

$\alpha$ in ground range
Scattering mechanism $\alpha$ FP-WSF (order=2)

$\alpha$ in slant range

$\alpha$ in ground range

Surface reflection $0$ $\frac{\pi}{2}$ Double bounce scattering

Surface reflection $0$ $\frac{\pi}{2}$ Double bounce scattering
Conclusion

➤ Building characterization using Polarimetric MB-inSAR methods:
   - Source discrimination, physical characterization
   - Common HR estimation methods may reach serious limitations

➤ Proposed approach: Weighted Subspace Fitting
   - Model adaptive: coherent and distributed scatterers
   - Compared to MUSIC
     - WSF has no or very low sidelobes
     - WSF has a better resolution
     - WSF performs well on coherent signals

➤ Extension to the fully polarimetric case
   - Analytical polarimetric optimization
   - Computation cost equal to the single polarization case
   - Refined characterization of building height and scattering mechanisms