PALSAR Polarimetric accuracy and stability evaluated for three years using the Amazon data

Masanobu Shimada
Masato Ohki

Japan Aerospace Exploration Agency
Earth Observation Research Center

PolInSAR2009
Contents

Polarimetric Calibration

1. Calibration model
2. Calibration site and the data
3. Discussions
   1.3.1 Cross-talk
   1.3.2 Channel Imbalance
   1.3.3 Faraday Rotation and TEC values

2. Conclusions
1. Calibration model

\[
\begin{pmatrix}
  Z_{hh} & Z_{hv} \\
  Z_{vh} & Z_{vv}
\end{pmatrix}
= A \frac{1}{r} e^{-\frac{4 \pi r}{\lambda}} \begin{pmatrix}
  1 & \delta_3 \\
  \delta_4 & f_2
\end{pmatrix}
\begin{pmatrix}
  \cos \Omega & \sin \Omega \\
  -\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
  S_{hh} & S_{hv} \\
  S_{vh} & S_{vv}
\end{pmatrix}
\cdot
\begin{pmatrix}
  \cos \Omega & \sin \Omega \\
  -\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
  1 & \delta_1 \\
  \delta_2 & f_1
\end{pmatrix}
\]

where \( Z_{ij} \) is the measurement matrix, \( i \) is the transmission polarization, \( j \) is the reception polarization, \( A \) is the amplitude, \( r \) is the slant range, \( S_{ij} \) is the true scattering matrix of the target, \( f_1 \) is the channel imbalance of the transmission distortion matrix, \( f_2 \) is that for the reception matrix, \( \delta_1 \) (\( \delta_2 \)) are the cross talks of transmission, and \( \delta_3 \) (\( \delta_4 \)) are the those for the reception. Here, noise is ignored.

Solutions are obtained by using Quegan’s method and assuming \( \Omega = 0 \)
Proposed method

Prepare two independent non-symmetric distortion matrices, one for transmission and one for reception.

Assume that L band backscattering from the uniform forest consists of the volume scattering and double bounce and has polarization dependency of double bounce.

Use a corner reflector and the forest region for parameter determination.

Assume that L band antennas have been well calibrated.
Theoretical models

Two step determinations of 10 parameters (7 complex, and 3 real) needs a trihedral corner reflector and forest region.

\[
\begin{align*}
Z_{hh}^* & = f_v + |x|^2 f_d + \epsilon_x f_1 f_2 \\
Z_{hv}^* & = f_i \left( \delta_1 + \frac{1}{3} f_i (\delta_3 + x \delta_3^* + \delta_2^*) \right) + \left( |x|^2 \delta_1 - x \delta_1 \right) f_d \\
Z_{vh}^* & = f_i \left( \delta_4 + \frac{1}{3} f_i (\delta_2 + x \delta_2^* + \delta_3^*) \right) + \left( |x|^2 \delta_4 - x \delta_2 \right) f_d \\
Z_{vv}^* & = f_i f_2 \left( \frac{1}{3} f_v - x f_d \right) \\
Z_{hh} & = f_i \left( \varepsilon_{hv} \varepsilon_{hh}^* \right) \\
Z_{hv} & = \frac{1}{3} f_i |f|^2 \\
Z_{vh} & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right) \\
Z_{vh}^* & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right) \\
Z_{vv} & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right) \\
Z_{vv}^* & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right) \\
Z_{hh}^* & = f_v + |x|^2 f_d + \epsilon_x f_1 f_2 \\
Z_{hv}^* & = f_i \left( \delta_1 + \frac{1}{3} f_i (\delta_3 + x \delta_3^* + \delta_2^*) \right) + \left( |x|^2 \delta_1 - x \delta_1 \right) f_d \\
Z_{vh}^* & = f_i \left( \delta_4 + \frac{1}{3} f_i (\delta_2 + x \delta_2^* + \delta_3^*) \right) + \left( |x|^2 \delta_4 - x \delta_2 \right) f_d \\
Z_{vv}^* & = f_i f_2 \left( \frac{1}{3} f_v - x f_d \right) \\
Z_{hh}^* & = f_i \left( \varepsilon_{hv} \varepsilon_{hh}^* \right) \\
Z_{hv}^* & = \frac{1}{3} f_i |f|^2 \\
Z_{vh}^* & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right) \\
Z_{vh}^* & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right) \\
Z_{vv}^* & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right) \\
Z_{vv}^* & = \frac{1}{3} f_i f_1 f_2 \left( \varepsilon_{vh} \varepsilon_{hh}^* \right)
\end{align*}
\]

noise\( (n_0) \)
\( f_1, f_2, f_d, f_v, x(|x| \text{ and } \theta_x) \)
\( \delta_1, \delta_2, \delta_3, \text{ and } \delta_4 \)
Covariance matrix (4 x 4) and its notation

\[ \langle \mathbf{Z} \cdot \mathbf{Z}^* \rangle = \begin{pmatrix}
Z_{hh}^* Z_{hh} & Z_{hv}^* Z_{hh} & Z_{vh}^* Z_{hh} & Z_{vv}^* Z_{hh} \\
Z_{hh}^* Z_{hv} & Z_{hv}^* Z_{hv} & Z_{vh}^* Z_{hv} & Z_{vv}^* Z_{hv} \\
Z_{hh}^* Z_{vh} & Z_{hv}^* Z_{vh} & Z_{vh}^* Z_{vh} & Z_{vv}^* Z_{vh} \\
Z_{hh}^* Z_{vv} & Z_{hv}^* Z_{vv} & Z_{vh}^* Z_{vv} & Z_{vv}^* Z_{vv}
\end{pmatrix} \]

\[
\begin{pmatrix}
a & 0 & 0 & b \\
0 & c & d & 0 \\
0 & d^* & e & 0 \\
b^* & 0 & 0 & f
\end{pmatrix}
\]

1) Distributed targets

a = 1, f < 1, c ~ e ~ |d| ~ 1/3

d, b: complex

|b| < c & d & e because b contains double bounce.

All components to be subtracted by the noise

Cross talks

2) Corner reflector

Channel imbalances
PALSAR Calibration sites

Amazon: Antenna Pattern, cal factors
CRs: Cal factors and location
CRs in Amazon: POL calibration (ASF, IGBE)

612 match up
Geomagnetic field lines
必ずしも磁力線と筋状雑音の方向が一致している訳ではない
Calibration test sites
In Rio Branco, Amazon

Red circle shows the location of the 2.5 meter corner reflector deployed by IGBP, Brazil.

Sites in Amazon: Minimize the ionospheric effect: almost zero geomagnetic latitude.

> Rio Branco
A list of POLCAL data

Amazon Rio Branco

(1) Ascending : 10 datasets (July 20 2006 ~ Oct. 25 2008)

(2) Descending : 17 datasets (July 21 2006 ~ Dec. 11 2008)

(3) Off nadir : 21.5 degrees
Channel Imbalances

- $f_1 = 1.015 (0.028)$
- $f_2 = 0.725 (0.023)$

- $a_{f1} (\text{deg}) = 20.287, 0.91$
- $a_{f2} (\text{deg}) = -3.174, 1.05$
Comparison of the distortion matrix

Current data
f1=1.030979981/21.80501612(degrees)
f2=0.722099971/-1.878998903
δ1=-25.538(3.09)/79.36933594)
δ2=-25.385(0.077)/-151.5032329
δ3=-25.077(3.107)/131.4864021
δ4=-25.897(2.416)/-1.878998903

Averaging the data (Calculation from the Rio Branco data)
f1=1.015(0.028)/20.287(0.91):degrees
f2=0.725(0.023)/-3.174(1.05)
δ1=-25.538(3.09)/-171.03(4.778)
δ2=-25.385(0.077)/-1.719(1.740)
δ3=-25.077(3.107)/8.378(3.868)
δ4=-25.897(2.416)/170.22(2.525)
Polarimetric calibration
Trend of the CAL factor for Polarimetry

Unexpected Rise of values

> CR of descend displaced?
Results of the calibration

CRs are selected from the world calibration test sites.

**Amp ratio of VV/HH**

- Mean: 1.0134
- Std: 0.0619

**Phase diff VV/HH**

- Mean: 0.0612 (deg)
- Std: 2.6613 (deg)
Evaluation of the cross talk between H and V polarization. Specification $< -30$ dB

Method:
1) Cross correlation of signal from the natural targets
2) Direct evaluation of the point target response.

\[
cross_1 = 10 \cdot \log_{10} \left( \frac{\langle s_{hv}^* s_{hh} \rangle}{\sqrt{\langle s_{hv}^* s_{hv} \rangle} \cdot \sqrt{\langle s_{hh}^* s_{hh} \rangle}} \right)
\]

\[
cross_2 = 10 \cdot \log_{10} \left( \frac{P_{hv} - P_{sur,hv}}{P_{hh} - P_{sur,hh}} \right)
\]
Cross talks

Specification: -30 dB

Crosstalks: crhv/hh = -35.178 (6.927), crvh/vv = -27.964 (4.895), cross = -34.332 (3.793)
Calculating Faraday Rotation

\[
\begin{pmatrix}
    Z_{hh} & Z_{hv} \\
    Z_{vh} & Z_{vv}
\end{pmatrix}
= \begin{pmatrix}
    \cos \Omega & \sin \Omega \\
    -\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
    S_{hh} & S_{hv} \\
    S_{vh} & S_{vv}
\end{pmatrix}
\begin{pmatrix}
    \cos \Omega & \sin \Omega \\
    -\sin \Omega & \cos \Omega
\end{pmatrix}
\]

1) \[s_{hv} = s_{vh}\]
\[\Omega = \frac{1}{2} \tan^{-1}\left(\frac{Z_{hv} - Z_{vh}}{Z_{hh} + Z_{vv}}\right)\]
Freeman (2004)

2) \[\Omega = \frac{1}{4} \text{Arg}(Z_{LR} \cdot Z_{RL}^*)\]
\[
\begin{pmatrix}
    Z_{LL} & Z_{LR} \\
    Z_{RL} & Z_{RR}
\end{pmatrix}
= \begin{pmatrix}
    1 & j \\
    j & 1
\end{pmatrix}
\begin{pmatrix}
    Z_{hh} & Z_{hv} \\
    Z_{vh} & Z_{vv}
\end{pmatrix}
\begin{pmatrix}
    1 & j \\
    j & 1
\end{pmatrix}
\]
(Bickel and Bates, 1965):

3) \[\left\langle s_{hv} \cdot s_{hv}^*\right\rangle = \left\langle s_{vh} \cdot s_{vh}^*\right\rangle\]
\[\alpha = \left\langle (Z_{hv} + Z_{vh}) \cdot (Z_{hh} + Z_{vv})^*\right\rangle + \left\langle (Z_{hv} + Z_{vh})^* \cdot (Z_{hh} + Z_{vv})\right\rangle\]
\[\beta = \left\langle Z_{hv} \cdot Z_{hv}^* - Z_{vh} \cdot Z_{vh}^*\right\rangle\]
\[\Omega_i = \tan^{-1}\left\{\frac{\beta}{\alpha} \left(1 - \tan^4 \Omega_0\right) - \tan^3 \Omega_0\right\}\]
\[\Omega_0 = \tan^{-1}\left(\frac{\beta}{\alpha}\right)\]

Several methods
Correlation between Faraday Rotations

Needs more evaluations
Calibration Factor and location error measured using the CR
Validation of cal results: using the Amazon forest data

0.22 dB (FBS) and 0.4 dB (SCAN)
NESZ:

FBD HH ~ -31 dB, FBD HV ~ -34 dB

- NESZ (FBS343HH): Green land
- NESZ (FBO343HH)
- NESZ (FBO343HV)
L1.0 data (raw data)

Telemetry analysis and correction (ATT)

Doppler/Doppler rate analysis

Range compress (Chirp rate correction)

Range antenna pattern correction

Range migration

Azimuth compression (SRC)

Multilook

16 bits

$DN = \sqrt{z \cdot z^* / B}$

IEEE four byte complex, POLCAL (Distortion correction) multilook + 16 bits

$\begin{pmatrix}
z_{hh} & z_{hv} \\
z_{vh} & z_{vv}
\end{pmatrix} = [full\ pol]$

$\begin{pmatrix}
1 & \delta_3 \\
\delta_4 & f_2
\end{pmatrix}^{-1} \begin{pmatrix}
Z_{hh} & Z_{hv} \\
Z_{vh} & Z_{vv}
\end{pmatrix} \begin{pmatrix}
1 & \delta_1 \\
\delta_2 & f_1
\end{pmatrix}^{-1}$

[others]

$\begin{pmatrix}
Z_{hh} & Z_{hv} / f_1 \\
Z_{vh} / f_2 & Z_{vv} / f_1 \cdot f_2
\end{pmatrix}$

$\sigma^0 = 10 \log_{10} \langle DN^2 \rangle + CF$

① L1.5

② L1.1

③ L1.5

④ DN-σ0 conversion
We have installed the new version of PALSAR processing software that improves radiometric accuracies synthetically using the Corner Reflector responses and Amazon forest data for the following two points

1) Calibration of the HV for FBD343 and FBD415
2) Minor calibration updates for the other incidence angles (PLR215, FBS415, FBS508)

After the processing date (January 7 2009), the normalized radar cross section of any of the polarization component can be obtained by the following formula with single calibration factor, i.e.,

\[
\text{NRCS(dB)} = 10\log_{10}(\langle \text{DN}^2 \rangle + CF) \quad (1.5 \text{ product})
\]
\[
\text{NRCS(dB)} = 10\log_{10}(\langle I^2+Q^2 \rangle) + CF - 32.0 \quad (1.1 \text{ product})
\]

where \( CF = -83.0 \).

Before this processing date, the calibration factor (CF) can be given as follows:

<table>
<thead>
<tr>
<th>Date</th>
<th>Before Jan 6, 2009</th>
<th>After Jan 7 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBS099HH</td>
<td>-83.16</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBS215HH</td>
<td>-83.55</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBS343HH</td>
<td>-83.4</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBD343HH</td>
<td>-83.2</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBD343HV</td>
<td>-80.2</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBS415HH</td>
<td>-83.65</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBD415HH</td>
<td>-83.19</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBD415HV</td>
<td>-80.19</td>
<td>-83.0</td>
</tr>
<tr>
<td>FBS508HH</td>
<td>-83.30</td>
<td>-83.0</td>
</tr>
</tbody>
</table>
PALSAR calibration results (summary)

<table>
<thead>
<tr>
<th>items</th>
<th>values</th>
<th>NOD</th>
<th>spec</th>
<th>remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>geometry</td>
<td>9.3m(RMS:distance)***</td>
<td>615</td>
<td>100m</td>
<td>all modes</td>
</tr>
<tr>
<td>radiometry</td>
<td>0.64dB****/0.17dB*</td>
<td>478/16</td>
<td>1.5dB</td>
<td>all modes</td>
</tr>
<tr>
<td>polarimetry</td>
<td>VV/HH amp ratio(dB) : 0.02dB(0.04)</td>
<td>79</td>
<td>0.2dB</td>
<td>POL</td>
</tr>
<tr>
<td></td>
<td>VV/HH phase diff.(deg) : 0.321(1.01)</td>
<td>79</td>
<td>5deg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cross talk : 31~40dB</td>
<td>79</td>
<td>30dB</td>
<td></td>
</tr>
<tr>
<td>NESZ</td>
<td>-34dB</td>
<td></td>
<td>-23dB</td>
<td>all modes</td>
</tr>
<tr>
<td>resolution(m)</td>
<td>azimuth : 4.49m(0.1m)</td>
<td>478</td>
<td>4.5m</td>
<td>all modes</td>
</tr>
<tr>
<td></td>
<td>range(14MHz) : 9.6m(0.1m)</td>
<td>478</td>
<td>10.7m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>range(28MHz) : 4.7m(0.1m)</td>
<td>478</td>
<td>5.4m</td>
<td></td>
</tr>
<tr>
<td>side lobe(dB)</td>
<td>PSLR(azimuth) : -16dB</td>
<td>478</td>
<td>-10dB</td>
<td>all modes</td>
</tr>
<tr>
<td></td>
<td>PSLR(range) : -12.5dB</td>
<td>478</td>
<td>-10dB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ISLR : -8.6dB</td>
<td>478</td>
<td>-8dB</td>
<td></td>
</tr>
<tr>
<td>ambiguity</td>
<td>azimuth : zero</td>
<td></td>
<td>16dB**</td>
<td>all modes</td>
</tr>
<tr>
<td></td>
<td>range : ~23dB@ image end</td>
<td></td>
<td>16dB</td>
<td></td>
</tr>
</tbody>
</table>

note)all the values are average , value in blanket is a standard deviation, * is at Sweden site,,**70km swath,***1m improved,**** : 0.1dB improved
Polarimetric coverage of Greenland. Browse products (21.5 degrees)

HH( R), HV(G), VV(B)
Conclusions

PALSAR has been being calibrated using the CR and natural forest data. It shows the good stability for all the mode, including the polarimetry.

Stability over the three years has been confirmed.
PALSAR polarimetry mode (specification)

Off nadir angles 7.9~30.1(degrees)
Image swath 30.6 km at off nadir 21.5
number of bits 3/5
data rate 240Mbps
NE$\sigma^0$ (dB) -31.1(hh), -32.3(hv), -31.1(vh), -32.3(vv)
SA(range) 46.8(hh), 24.7(hv), 27.1(vh), 44.7(vv)
SA(azimuth) 20.6(hh), 21.4(hv), 21.5(vh), 21.9(vv)
resolution(R) 31.2m(ground range)
resolution(A) 20m(4look)
X-pol. cross talk -25dB (spec.) : ~-30 dB(measured)

<table>
<thead>
<tr>
<th>V偏波</th>
<th>1255MHz</th>
<th>28.87</th>
<th>28.08</th>
<th>&gt;25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1270MHz</td>
<td>31.32</td>
<td>34.02</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1285MHz</td>
<td>30.93</td>
<td>33.56</td>
<td>&gt;25</td>
</tr>
<tr>
<td>H偏波</td>
<td>1255MHz</td>
<td>35.45</td>
<td>31.51</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1270MHz</td>
<td>28.90</td>
<td>29.48</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1285MHz</td>
<td>27.09</td>
<td>29.96</td>
<td>&gt;25</td>
</tr>
<tr>
<td>V偏波</td>
<td>1255MHz</td>
<td>28.92</td>
<td>27.55</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1270MHz</td>
<td>32.87</td>
<td>34.42</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1285MHz</td>
<td>32.87</td>
<td>33.03</td>
<td>&gt;25</td>
</tr>
<tr>
<td>H偏波</td>
<td>1255MHz</td>
<td>34.99</td>
<td>33.39</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1270MHz</td>
<td>30.75</td>
<td>29.27</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1285MHz</td>
<td>32.34</td>
<td>31.02</td>
<td>&gt;25</td>
</tr>
<tr>
<td>V偏波</td>
<td>1255MHz</td>
<td>27.46</td>
<td>27.61</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1270MHz</td>
<td>33.01</td>
<td>34.18</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1285MHz</td>
<td>32.70</td>
<td>32.73</td>
<td>&gt;25</td>
</tr>
<tr>
<td>H偏波</td>
<td>1255MHz</td>
<td>35.14</td>
<td>31.77</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1270MHz</td>
<td>36.61</td>
<td>29.67</td>
<td>&gt;25</td>
</tr>
<tr>
<td></td>
<td>1285MHz</td>
<td>32.10</td>
<td>31.23</td>
<td>&gt;25</td>
</tr>
</tbody>
</table>

Note: Pulse width = 20μsec

NE$\sigma^0$ (Noise equivalent sigma-zero)
Antenna Pattern - Elevation(left) and Azimuth(right)

Off Nadir angle (degrees)

- Weighting in receive
- No weighting in transmit

9.9 34.3 47.7 50.8

Antenna pattern data
2. Conclusions

PALSAR has been operated around two years. During this, we have collected global datasets. Because of the resource limitation, the polarimetry cannot cover the global earth. But, it may give another cross section of the earth surface. As shown in this short paper, the polarimetry shows a good stability. The channel imbalance shows very good stability. The cross talk is less than -30 dB. This means that the more accurate estimation of the cross talk better than this is very difficult and discrimination of this small term contaminated by the Faraday rotation is very difficult. Thus, it can be possible to assume the cross talk is zero.
Evaluation of L-band signal penetration property
Use of the calibrated data
Evaluation of the HH/VV for phase information.
Dependence of the coherence on scattering

Evaluation target:

Area A: Amazon rain forest (Riobranco: -9.98, -68.10 and 300km neighbors)
Area B: Boreal forest: Tomakomai in Japan (42.744, 141.518)

Pauli basis data were used.

Coherence of the HH-HH and VV-VV are almost the same, and, different from HV-HV.

This means that HH and VV have good similarity on the scattering and penetration.

HH+VV represents the double bounce (even number scattering).

HH-VV for the odd scattering, and HV+VH for volume scattering.
Scattering characteristics

Evaluation of the PolCaled Amazon (Riobranco)

HH and VV, HV and VH are the same.

Power ratio (power decomposition with unknowns)
DB to Volume ratio is 1:9 (\(-\) InSAR coherence 0.1)
DB in HH exceeds that in VV by 30%

Total power in VV is 15% smaller than HH.
Covariance of cross component is 25% of HH
\(x: fdh/fdv\)
Both parameters for channel imbalance are very stable.
Faraday rotation and TEC evaluation

Red: theoretical values from the TEC (GPS)
Blue: Measurement from Polarimetry

26-30 January 2009 | ESA-ESRIN | Frascati (Rome) Italy
Covariance of the forest data is expressed by explicitly double bounce, surface scattering and volume scattering.

Their polarization dependence should be clarified for the cross talk determination.
Features of the proposed method

• Find the two channel imbalances using CR and the distributed targets.

• Subtract the noise component in each term.

• Determine the channel imbalances using the corner reflectors and forest data (First order calculation).

• Estimate the HH and VV double bounces and use them for estimating the cross talks (second order calculation).

• Correct the range dependent parameters (phase of the channel imbalances)
First order terms (channel imbalances and forest parameters)

Equations

1) distributed target
\[ f_v + |x|^2 f_d = a' \]
\[ F_1 F_2 \left( \frac{f_d}{3} \gamma - f_d x \right) \left\{ \cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2) \right\} = b \]
\[ x = \cos \theta_x + j \cdot \sin \theta_x \]
\[ f_v F_1^2 = 3c \]
\[ f_v F_1 F_2 = 3d \]
\[ f_v F_2^2 = 3e \]
\[ (f_v + f_d) F_1^2 F_2^2 = f \]

2) corner reflector
\[ \theta_2 + \theta_1 = \arg(b_{cr}) = \theta_{cr} \]
\[ F_1^2 F_2^2 = f_{cr} \]

Unknowns: \( F_1, F_2, \theta_1, \theta_2, f_v, f_d, x(x, \theta_x) \)

Solutions

\[ \theta_2 = \frac{1}{2} \left( \arg(d) + \theta_{cr} \right) \]
\[ \theta_1 = \frac{1}{2} \left( \theta_{cr} - \arg(d) \right) \]
\[ F_1 F_2 = \sqrt{f_{cr}} \]
\[ 3d \left( \frac{1}{3} - \frac{f_d x \cos \theta_x}{f_v} - \frac{f_d x \sin \theta_x j}{f_v} \right) e^{i\theta_x} = b \]
\[ \theta_x = \tan^{-1} \left\{ \begin{array}{l}
\frac{b}{3d} e^{-j\theta_x} \\
\frac{b}{3d} e^{j\theta_x} + \frac{1}{3}
\end{array} \right. \]
\[ x = \sqrt{(a - f_v)/f_d} \]
\[ f_v = \frac{f}{g^2} a + 1 - c^2 - \sqrt{\left( \frac{f}{g^2} a + 1 - c^2 \right)^2 - 4a \frac{f}{g^2}} \]
\[ f_d = f \sqrt{\frac{f}{9d^2} f_v - 1} \]
\[ g = 3d \]
\[ F_1 = \sqrt{3c/f_v} \]
\[ F_2 = \sqrt{3e/f_v} \]
Second Order Terms (Crosstalk’s of distortion matrices)

\[ z_{hh}^* z_{hh}^* = (f_v + x^2 f_u) \delta_1^* + \frac{f_v}{3} f_v \delta_2^* + f_i (\frac{f_v}{3} - x f_u) \delta_3^* + \frac{f_v}{3} f \delta_3^* \]
\[ z_{hh}^* z_{hh}^* = f_2 \left( \frac{f_v}{3} - x f_u \right) \delta_2^* + \frac{f_v}{3} f \delta_2^* + (f_v + x^2 f_u) \delta_4^* \]
\[ z_{vh}^* z_{hh}^* = \frac{f_v}{3} f \delta_2^* \delta_3^* + f_i f_2 \left( \frac{f_v}{3} - x f_u \right) \delta_3^* + \frac{f_v}{3} |f_i|^2 \delta_4^* \]
\[ z_{vh}^* z_{vh}^* = \frac{f_v}{3} |f_i|^2 \delta_1^* + \frac{f_v}{3} f_i (f_v + f_i) \delta_2^* + \frac{f_v}{3} f_i \delta_4^* + f_i f_2 \left( \frac{f_v}{3} - x f_u \right) \delta_4^* \]

Then, the solution is given by

\[ \mathbf{MY} = \mathbf{N} \]

\[ \mathbf{M} = \begin{pmatrix}
    m_{1r} & -m_{1i} & m_{2r} & m_{2i} & m_{3r} + m_{4r} & m_{3i} + m_{4i} & 0 & 0 \\
    m_{1i} & m_{1r} & m_{2r} & -m_{2i} & m_{3r} + m_{4i} & m_{3i} - m_{4r} & 0 & 0 \\
    0 & 0 & m_{5r} + m_{6r} & m_{5i} + m_{6i} & m_{7r} & m_{7i} & m_{8r} & m_{8i} \\
    0 & 0 & m_{5i} + m_{6i} & m_{5r} - m_{6r} & m_{7i} & -m_{7r} & m_{8i} & m_{8r} \\
    m_{9r} + m_{10r} & m_{9i} + m_{10i} & 0 & 0 & m_{11r} & m_{11i} & m_{12r} & -m_{12i} \\
    m_{9i} + m_{10i} & m_{9r} - m_{10r} & 0 & 0 & m_{11i} & -m_{11r} & m_{12i} & m_{12r} \\
    m_{13r} & -m_{13i} & m_{14r} & m_{14i} & 0 & 0 & m_{15r} + m_{16r} & -m_{15i} + m_{16i} \\
    m_{13i} & m_{13r} & m_{14i} & -m_{14r} & 0 & 0 & m_{15i} + m_{16i} & m_{15r} - m_{16r}
\end{pmatrix} \]

\[ \mathbf{Y} = \begin{pmatrix}
    A \\
    B \\
    C \\
    D \\
    E \\
    F \\
    G \\
    H
\end{pmatrix} \]

\[ \mathbf{N} = \begin{pmatrix}
    \text{Re}(z_{hh}^* z_{hh}^*) \\
    \text{Im}(z_{hh}^* z_{hh}^*) \\
    \text{Re}(z_{vh}^* z_{hh}^*) \\
    \text{Im}(z_{vh}^* z_{hh}^*) \\
    \text{Re}(z_{vh}^* z_{vh}^*) \\
    \text{Im}(z_{vh}^* z_{vh}^*)
\end{pmatrix} \]

\[ \mathbf{Y} = \mathbf{M}^{-1} \mathbf{N} \]
Noise estimation: determined by the following iterative operation

\[
\begin{align*}
\langle (S_{hh} + N_{hh})(S_{hh} + N_{hh})^* \rangle &= \langle S_{hh}S_{hh}^* + N_{hh}N_{hh}^* \rangle = \langle S_{hh}S_{hh}^* \rangle + 2n_0 \\
\langle S_{hv}S_{hv}^* + N_{hv}N_{hv}^* \rangle &= \langle S_{hv}S_{hv}^* \rangle + n_0 \\
\langle S_{vh}S_{vh}^* + N_{vh}N_{vh}^* \rangle &= \langle S_{vh}S_{vh}^* \rangle + n_0 \\
\langle S_{vv}S_{vv}^* + N_{vv}N_{vv}^* \rangle &= \langle S_{vv}S_{vv}^* \rangle + 2n_0 \\
\langle S_{hh}S_{hh}^* + N_{hh}N_{hh}^* \rangle &= \langle S_{hh}S_{hh}^* \rangle \\
\langle S_{hv}S_{hv}^* + N_{hv}N_{hv}^* \rangle &= \langle S_{hv}S_{hv}^* \rangle \\
\end{align*}
\]

\[
\begin{align*}
\left( \frac{f_v}{3} + n_0 \right) F_1^2 &= c \\
\left( \frac{f_v}{3} + n_0 \right) F_2^2 &= e \\
\frac{f_v}{3} F_1 F_2 &= d
\end{align*}
\]

\[
n_0 = \frac{eF_1^2 + cF_2^2 - \sqrt{(eF_1^2 + cF_2^2)^2 - 4F_1^2F_2^2(ce - d^2)}}{2F_1^2F_2^2}
\]
\[ \sigma_{\text{sigma-sar, } Q16}^0 = 10 \cdot \log_{10} \left( D N^2 \right) + CF_1 \]

\[ \sigma_{\text{sigma-sar, slc}}^0 = 10 \cdot \log_{10} \left( I^2 + Q^2 \right) + CF_1 - A \]

<table>
<thead>
<tr>
<th>CF</th>
<th>mean (dB)</th>
<th>std (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF_1</td>
<td>-83.0</td>
<td>0.67</td>
</tr>
<tr>
<td>A</td>
<td>32.0</td>
<td>-</td>
</tr>
</tbody>
</table>

Parameter summary

<table>
<thead>
<tr>
<th>factors</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range time offset</td>
<td>-0.31539μs</td>
</tr>
<tr>
<td>Azimuth time offset</td>
<td>-2.239ms (Strip)</td>
</tr>
<tr>
<td></td>
<td>-67ms (SCAN)</td>
</tr>
<tr>
<td>Polarimetric distortion mats</td>
<td></td>
</tr>
<tr>
<td>PLR215</td>
<td>(1.000000e+00, 0.000000e+00)</td>
</tr>
<tr>
<td></td>
<td>(-2.804701e-02, -2.933507e-03)</td>
</tr>
<tr>
<td></td>
<td>(3.164040e-02, -1.038148e-02)</td>
</tr>
<tr>
<td></td>
<td>(9.352351e-01, 4.073565e-01)</td>
</tr>
<tr>
<td></td>
<td>(1.000000e+00, 0.000000e+00)</td>
</tr>
<tr>
<td></td>
<td>(-3.699034e-02, 8.453709e-04)</td>
</tr>
<tr>
<td></td>
<td>(2.115907e-02, 5.648345e-03)</td>
</tr>
<tr>
<td></td>
<td>(7.249998e-01, 5.535966e-04)</td>
</tr>
</tbody>
</table>
Faraday rotation angle ($\Omega$)

\[
\Omega = \frac{K}{f^2} B \cos \psi \times \sec \theta_0 \times TEC
\]

(Bickel and Bates, 1965):

\[
\Omega = \frac{1}{4} \arg \left( Z_{LR} \cdot Z_{RL}^* \right)
\]

\[
\begin{pmatrix}
Z_{LL} & Z_{LR} \\
Z_{RL} & Z_{RR}
\end{pmatrix} = \begin{pmatrix}
1 & j \\
j & 1
\end{pmatrix}
\begin{pmatrix}
Z_{hh} & Z_{hv} \\
Z_{vh} & Z_{vv}
\end{pmatrix} \begin{pmatrix}
1 & j \\
j & 1
\end{pmatrix}
\]

Freeman (2004)

\[
\Omega = \frac{1}{2} \tan^{-1} \left( \frac{Z_{hv} - Z_{vh}}{Z_{hh} + Z_{vv}} \right)
\]

where $K = 2.365 \times 10^4$ in SI units, $f$ is the transmission frequency (Hz), TEC is the total electron contents (electrons/m$^2$), $B$ is the geomagnetic flux density (Tesla), $\psi$ is the angle between the geomagnetic field vector and the radar line-of-sight (radian), $\theta_0$ the incidence angle, and the over-bar indicates averaging.