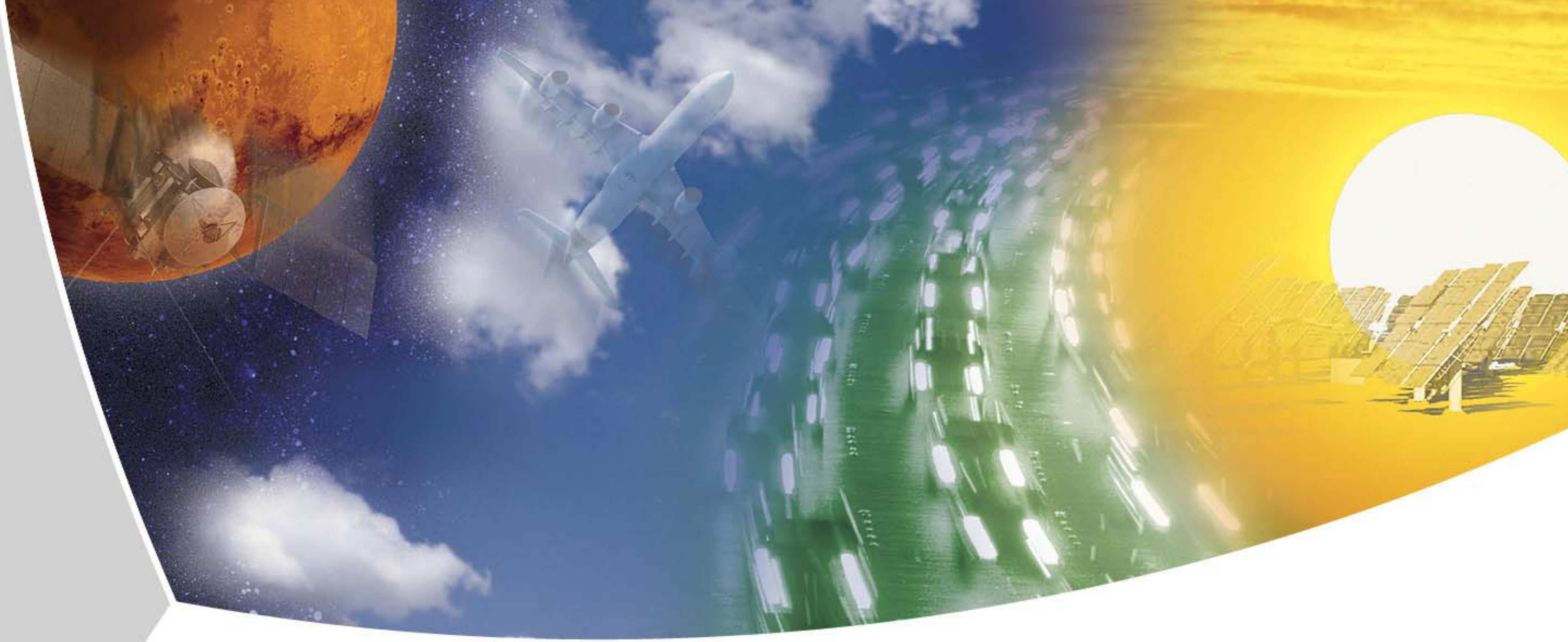

Tandem-L Forest Parameter Performance Analysis

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Overview

- Tandem-L interferometric polarimetric L-band satellite mission
  - Biosphere (forest height, structure, biomass)
  - Lithosphere (fault monitoring, volcanoes, earthquakes)
  - Cryosphere/Hydrosphere (ice flows, ocean currents)
- Mission design helped by performance models
- Forest parameters estimation from limited single-pass interferograms
  - Model/Estimators (forest structure)
  - Theoretical limits and baseline distribution study
  - Performance measures: classification approach, number of looks
- Conclusions & Open Issues

Mission design

Lithosphere (d-InSAR)

Biosphere (Pol-InSAR, MB-InSAR)

Cryosphere (d/Pol/MB -InSAR)

- Different applications with different necessities/requirements
  - Resolution/coverage (space & time)
  - Polarimetry, Single-pass baseline
  - …
- Feasibility constraints
- Goal: to develop a mission design that satisfies as many applications as possible with a systematic acquisition plan

Data rate & storage

Fuel

Instrument

Scheduling

Performance analyses for different applications

- To facilitate the task of allocating system resources to each application we want performance models and analyses for at least the most important applications
- These analyses can be used to make preliminary studies (How many looks? Which baselines?) or in conjunction with a mission simulator to estimate the final performance and optimize the mission schedule
- Forest height is assumed to be obtained by means of Pol-InSAR inversion[(1)]
- With forest structure we mean the reconstruction of a vertical scattering profile (low wavenumber components): this can be considered a final product in its own right or an input to biomass estimation (another Tandem-L goal)

1. **Papathanassiou and Cloude, *Single-baseline polarimetric SAR Interferometry.* IEEE Trans. Geosci. Remote Sensing, 2001**

Profile estimates from single-pass interferograms

- Which kind of estimator do we consider?
  - Combinations of single-pass interferograms to avoid temporal decorrelation
  - Limited baseline span (fuel) and number of acquisitions
  - Each interferogram is a sample of the baseline autocorrelation function and is related to the vertical scattering profile

Single-pass interferograms

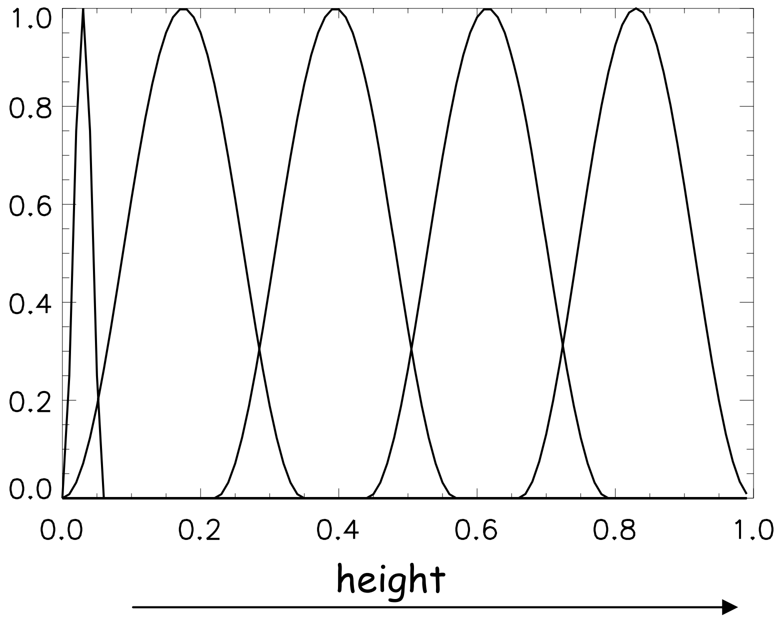
Forest height

⇨ **ML/COMET/PCT CAPON/FOURIER** ⇨ Profile estimates

- Since we are dealing with few baselines we are somehow forced to move from proper tomography (i.e. Fourier analysis) to model inversion (parametric)

Four-layer + ground model

- We want a simple model to study the baseline distribution
- We employ a four-layer + ground model for the forest profile
- Each layer is a cosine on a pedestal with arbitrary power



Cramér-Rao lower bound with a forest model

If we collect in a vector all the observations for a given point we can model the covariance matrix ($\mathbf{R}$).
$\mathbf{C}_k$ is the covariance matrix corresponding to layer k, σ^2 is the corresponding power. The total covariance is then:

$$\mathbf{R} = \sum_k \sigma_k^2 \mathbf{C}_k$$

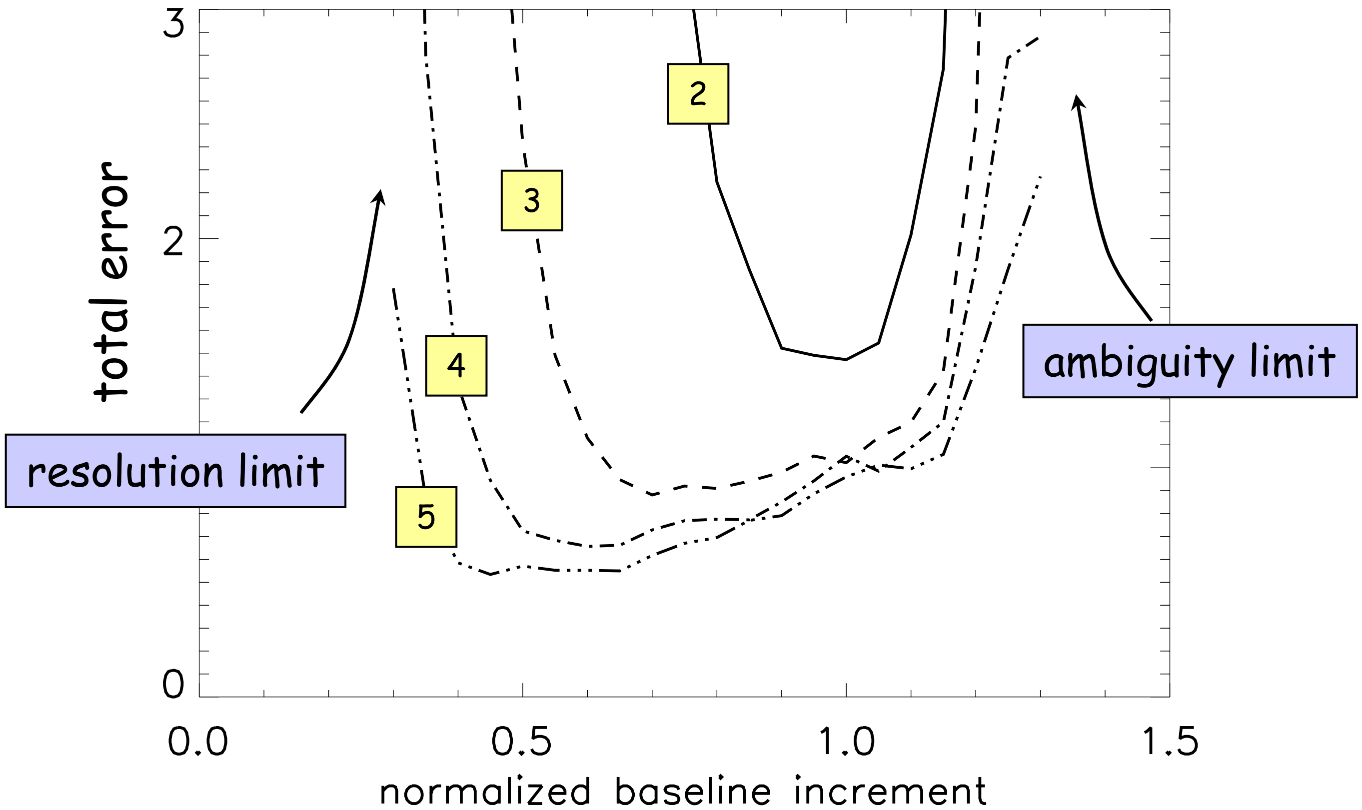
This model can also easily account for noise and a delta for the trunk-ground double bounce. The Fisher-information matrix is (for Complex Gaussian samples)

$$i(h,k) = trace\left[\mathbf{R}^{-1}\left(\frac{\partial}{\partial \sigma_h^2}\mathbf{R}\right)\mathbf{R}^{-1}\left(\frac{\partial}{\partial \sigma_k^2}\mathbf{R}\right)\right]$$

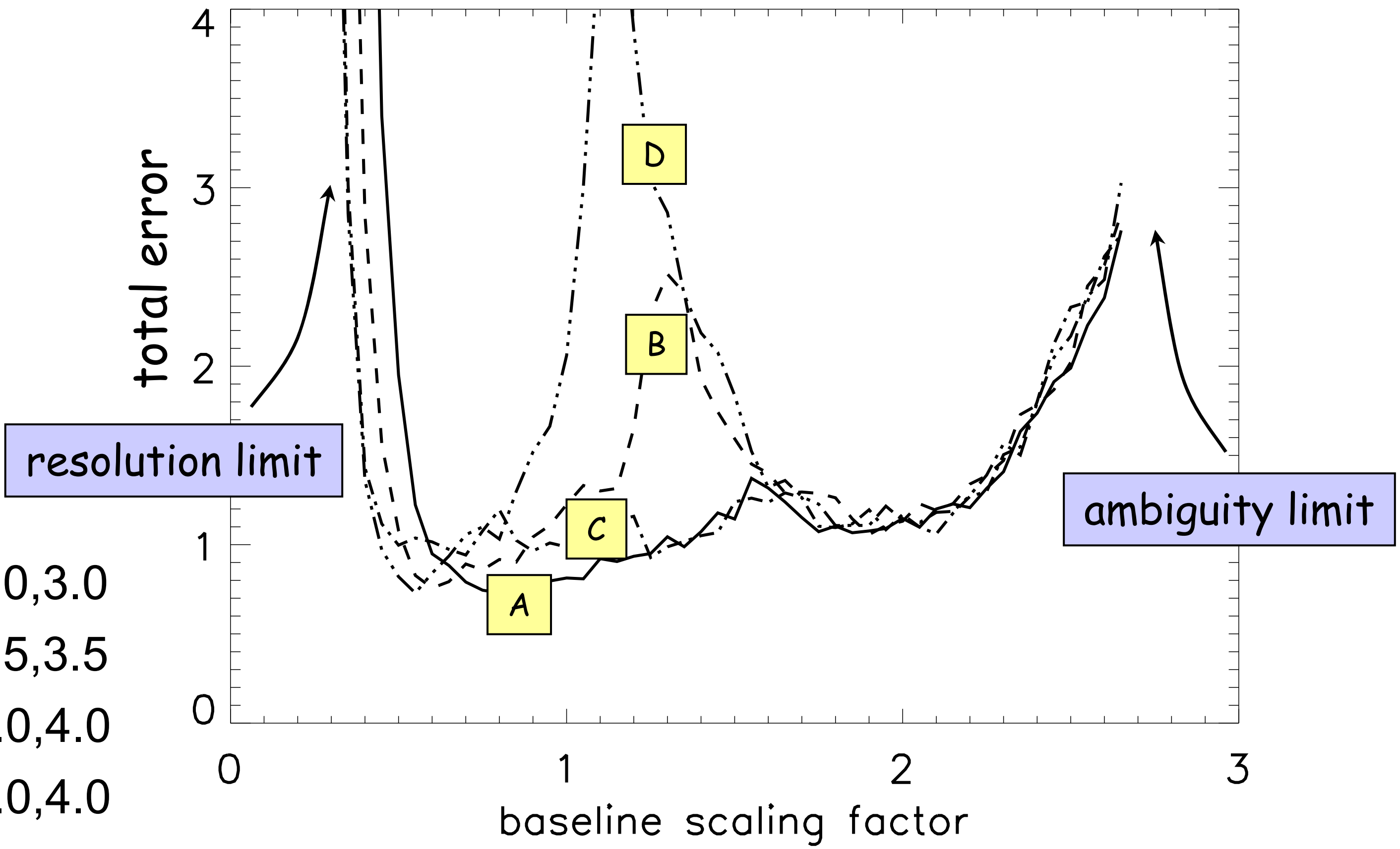
$$= trace\left[\mathbf{R}^{-1}\mathbf{C}_h\mathbf{R}^{-1}\mathbf{C}_k\right]$$

Regularly spaced baselines: total error

- From 2 to 5 baselines (single pass pairs), regularly spaced
- The baselines are normalized (it is possible to read height dependence)
- More baselines give more flexibility with respect to the forest height



Irregular baselines (Pol-InSAR campaign combination)



- A: 0.5,1.0,2.0,3.0
- B: 0.5,1.0,2.5,3.5
- C: 0.5,1.0,2.0,4.0
- D: 0.5,1.0,3.0,4.0

The ambiguity limit is extended thanks to the small baselines !

COMET (Covariance Matching Estimation Techniques)

- COMET[1][2] is a kind of model inversion based on comparing the measured covariance matrix and a modeled one (which depends on parameters)
- The metric to be minimized is :

$$trace(\hat{\mathbf{R}}^{-1}(\hat{\mathbf{R}}-\mathbf{R})\hat{\mathbf{R}}^{-1}(\hat{\mathbf{R}}-\mathbf{R}))$$

- Advantages
  - It is asymptotically equivalent to ML and thus asymptotically efficient[1]
  - Reduced computational costs for “linear parameters”, i.e. for intensities and noise power : explicit formulas!
- Can the performance of this estimator be compared with the CRLB? This would justify the use of CRLB to predict the performance… (the theory says that it is for many looks!)

1. **Stoica et al., *Covariance matching estimation techniques for array signal processing applications*. Digital Signal Processing, 1998**
2. **Tebaldini, *Forest SAR Tomography: A Covariance Matching Approach*, Proc. of IEEE Radar ‘08**

COMET expression for our case

$$
\begin{aligned}
J &= \operatorname{trace}\left(\hat{R}^{-1}(\hat{R} - \sum_k \sigma_k^2 C_k)\hat{R}^{-1}(\hat{R} - \sum_k \sigma_k^2 C_k)\right) \\
&= \operatorname{trace}\left((I - \sum_k \hat{R}^{-1}\sigma_k^2 C_k)(I - \sum_k \hat{R}^{-1}\sigma_k^2 C_k)\right) \\
&= \operatorname{trace}\left(I - 2\sum_k \hat{R}^{-1}\sigma_k^2 C_k + \sum_h \sum_k \hat{R}^{-1}\sigma_h^2 C_h \hat{R}^{-1}\sigma_k^2 C_k\right)
\end{aligned}
$$

$$
\frac{\partial J}{\partial \sigma_k^2} = -2\,\operatorname{trace}(\hat{R}^{-1}C_k) + 2\sum_h \sigma_h^2 \operatorname{trace}(\hat{R}^{-1}C_h\hat{R}^{-1}C_k)
$$

$$
\begin{aligned}
M &= \{\operatorname{trace}(\hat{R}^{-1}C_h\hat{R}^{-1}C_k)\} \\
n &= \{\operatorname{trace}(\hat{R}^{-1}C_k)\}
\end{aligned}
$$

The estimates of the linear parameters are then simply given by

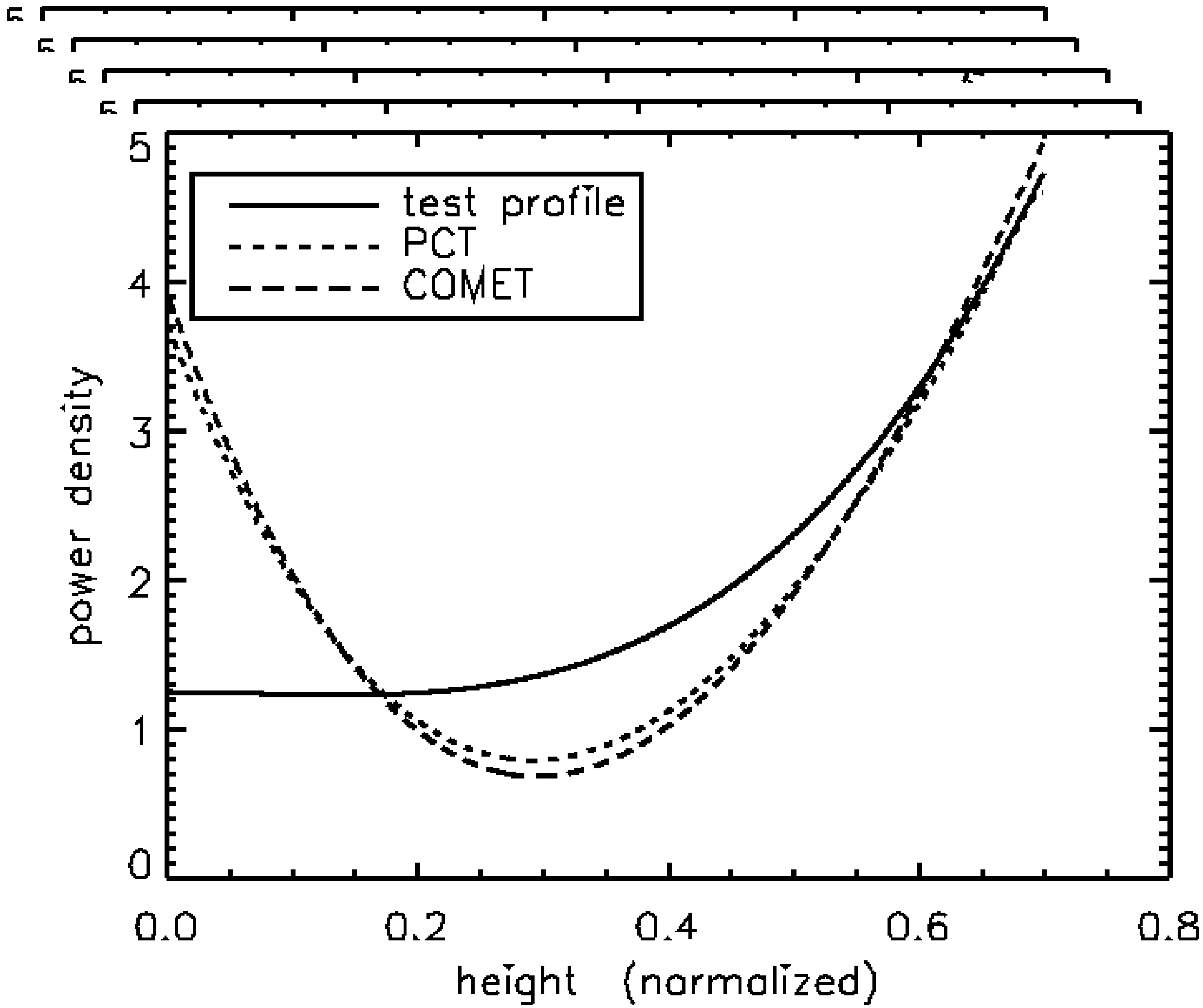
$$
\hat{\sigma}^2 = M^{-1}n
$$

COMET estimator is close to the CRLB

- Simulations with 5 baselines, 3 layers (powers: 0.5,0,1) at positions 0.2,0.4,0.6
- For 50 looks the bias is not large (underestimation) and the performance is very close to the Cramér-Rao lower bound
  - Fast performance computation!

LOOKS	E[]	true	Var[]	CRLB
50	0.47	0.5	0.0042	0.0045
	0.005	0.0	0.0023	0.0026
	0.92	1.0	0.0078	0.0077
36	0.45	0.5	0.0055	0.0063
	0.007	0.0	0.0031	0.0037
	0.90	1.0	0.0098	0.0107
25	0.44	0.5	0.0083	0.0091
	0.02	0.0	0.0047	0.0053
	0.85	1.0	0.0142	0.0154

COMET and PCT (1/2)



1. Cloude, *Polarization Coherence Tomography*, Radio Science, 2006

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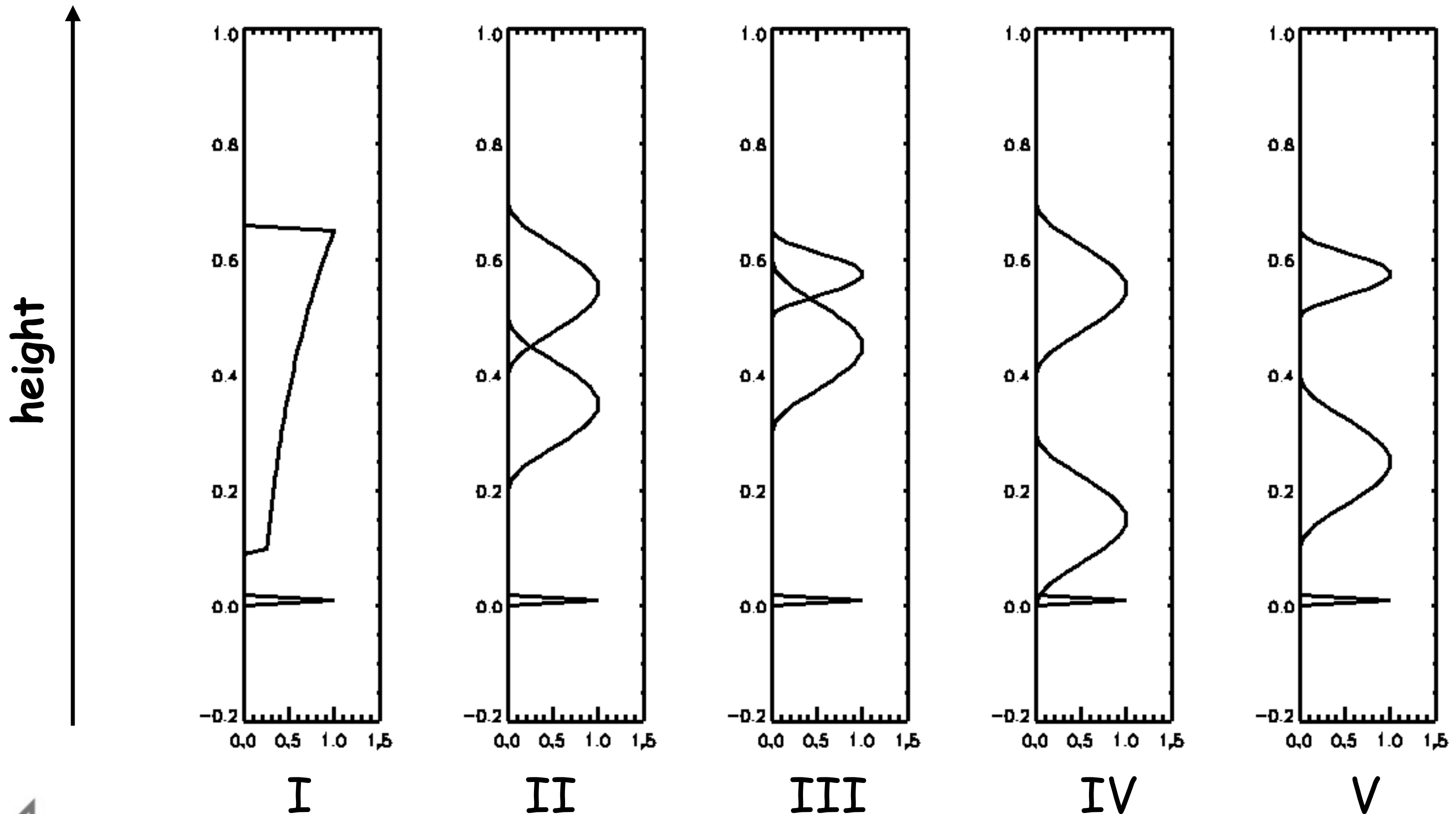
COMET and PCT (2/2)

- But
  1. COMET can model any profile, if you have a better model than Legendre polynomials you can use it (e.g. a delta for double-bounce!)
  2. COMET can model additive noise, PCT can't
  3. COMET claims to have a better metric than L^2

Confusion matrix

- Fixed number of models (layer intensities are not fixed). How well are they separable with a given configuration? (for images this would be called resolution)



15 looks, baselines 1,2,3,4,5

Model #

Recognized Model #

	I	II	III	IV	V
I	488	153	30	40	36
II	271	796	191	9	24
III	39	20	724	1	0
IV	87	5	47	934	18
V	115	26	8	16	922
Total	1000	1000	1000	1000	1000

77%

25 looks, baselines 1,2,3,4,5

Model #

Recognized Model #

	I	II	III	IV	V
I	674	81	4	14	9
II	211	910	163	1	7
III	13	5	804	0	0
IV	47	1	28	982	6
V	55	3	1	3	978
Total	1000	1000	1000	1000	1000

87%

50 looks, baselines 1,2,3,4,5

Model #

Recognized Model #

	I	II	III	IV	V
I	83	3	0	1	0
II	14	97	7	0	0
III	0	0	93	0	0
IV	3	0	0	99	0
V	0	0	0	0	100
Total	100	100	100	100	100

94%

Some conclusions from this study

- The baseline distribution need not to be regular but cannot be totally arbitrary either
- Three to four single-pass baselines should suffice to cover forest heights over a span of 1:4
- A reasonable baseline number of looks is in the range of 30-50

Further questions

- Forest model mismatch / forest profile modeling (how to address this problem?)
- Forest structure stability, this puts constraints on the campaign duration (2-3 months?)
- Phase calibration errors and requirements?