Impact of Polarimetric Dimensionality of Forest Parameter Estimation by Means of Polarimetric SAR interferometry

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Microwaves and Radar Institute
Polarimetry and Interferometry

- Polarimetry is sensitive to dielectric structure / shape.
- Interferometry is sensitive to height / density.
- Interferometric coherence depends on polarisation.
Coherence Region
-In Complex unit circle

Small decorrelation = high coherence

High decorrelation = small coherence

Coherence Region

Tree height

Quad-pol

Comp-pol

δ
Structure of coherence region
- $\alpha$-angle dependency

- P-Band
- L-Band
**Structure of Coherence region**

- **Important $\gamma$ values and compact-pol**

- Complex eigenvalues are placed in the coherence region and they are foci of boundary.

- Depending on the transmission basis, coherence region changes their shape and position.
**Study Area and Datasets**

**Mawas**
- Flat terrain
- Hilly terrain,
- Homogeneous, and Sparse Peat Swamp forest with understory

**Sungai Wain**
- Hilly terrain,
- Heterogeneous, and Dense Dipterocarp forest
Study Area and Datasets - Interferometric parameters

P-Band  Sungai Wain  L-Band
The Dependency of $\delta/\kappa_\zeta$ as a function of…

1. **Wavelength ($\lambda$)**
2. **(Spatial) Baseline ($B$)**
3. **Polarization Configuration ($A$)**
   1. Quad Polarisation (Quad)
   2. Compact Pol
      - Tx: Linear (45 deg) Rx: H,V (L1)
      - Tx: Linear (-45 deg) Rx: H,V (L2)
      - Tx: Circular Right Rx: H,V (CR)
      - Tx: Circular Left Rx: H,V (CL)
   3. Dual Pol
      - HH and VH (DH)
      - VV and HV (DV)
      - HH and VV (HV)
4. **Incidence angle ($\theta$)**
5. **Terrain slope ($\alpha$)**
Dependency on (Spatial) Baseline

Mawas P-Band

Quad-pol

Right Circular

Dual-pol Vertical

B=16m

B=32m
Dependency on Wavelength

Sungai Wain $\kappa_\xi \sim 0.1$

Quad-pol | Right Circular | Dual-pol Vertical

P-Band

L-Band
## Dependency on Spatial Baseline

*Mawas*

<table>
<thead>
<tr>
<th></th>
<th>P-Band</th>
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<tbody>
<tr>
<td></td>
<td>16m</td>
</tr>
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<td>08/11</td>
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<tr>
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<tr>
<td>05/11</td>
<td>7.00</td>
</tr>
<tr>
<td>02/11</td>
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</table>

Δ/κ_ζ decreases with wavelength

Δ/κ_ζ independent of baseline
### Sungai Wain: Dependency on Spatial Baseline

<table>
<thead>
<tr>
<th>P-Band</th>
<th>14m</th>
<th>28m</th>
<th>42m</th>
<th>56m</th>
</tr>
</thead>
<tbody>
<tr>
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<td>01/02</td>
<td>02/03</td>
<td>02/04</td>
<td>01/03</td>
</tr>
<tr>
<td>L1</td>
<td>6.00</td>
<td>6.40</td>
<td>6.45</td>
<td>4.80</td>
</tr>
<tr>
<td>L2</td>
<td>6.15</td>
<td>6.20</td>
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<td>4.80</td>
</tr>
<tr>
<td>CR</td>
<td>6.10</td>
<td>5.95</td>
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</tr>
<tr>
<td>CL</td>
<td>5.95</td>
<td>6.35</td>
<td>6.45</td>
<td>4.90</td>
</tr>
<tr>
<td>HH,VV</td>
<td>6.40</td>
<td>6.20</td>
<td>6.65</td>
<td>5.00</td>
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<tr>
<td>HH,VH</td>
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<td>5.80</td>
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<tr>
<td>HV,VV</td>
<td>5.90</td>
<td>5.80</td>
<td>6.20</td>
<td>4.80</td>
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<tr>
<td>Avr.</td>
<td>49.7</td>
<td>47.7</td>
<td>48.0</td>
<td>44.2</td>
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Dependency on Baseline

**Sungai Wain**

<table>
<thead>
<tr>
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<th>L-Band</th>
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<tbody>
<tr>
<td></td>
<td>5m</td>
<td>10m</td>
<td>15m</td>
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<td>02/04</td>
<td>01/04</td>
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</tr>
<tr>
<td>Quad</td>
<td>12.20</td>
<td>9.65</td>
<td>9.70</td>
<td>9.80</td>
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<tr>
<td></td>
<td>9.75</td>
<td>9.40</td>
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</tbody>
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\[ \delta/\kappa_\zeta \text{ is about half with compact-pol} \]
Dependency on Incidence Angle

- L0103
  - B=10m
  - B=28m

- P0103
  - B=10m
  - B=28m

- L0204
  - B=10m
  - B=28m

- P0305
  - B=10m
  - B=28m
Dependency on Incidence Angle

- **L0103**
  - **B=10m**
  - **B=28m**

- **L0204**
  - **B=10m**

- **P0103**
  - **B=28m**

- **P0305**
  - **B=10m**
  - **B=28m**
## Dependency on Incidence Angle

### Mawas

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<tr>
<td>16m</td>
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</tr>
<tr>
<td>02/05</td>
<td>5.90(52.5)</td>
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<tr>
<td>05/08</td>
<td>6.10(50.8)</td>
</tr>
<tr>
<td>08/11</td>
<td>6.10(52.5)</td>
</tr>
<tr>
<td>32m</td>
<td></td>
</tr>
<tr>
<td>02/08</td>
<td>5.20(44.2)</td>
</tr>
<tr>
<td>05/11</td>
<td>4.90(49.0)</td>
</tr>
<tr>
<td>48m</td>
<td></td>
</tr>
<tr>
<td>02/11</td>
<td>3.00(50.0)</td>
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</table>

<table>
<thead>
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<tbody>
<tr>
<td>6m</td>
<td></td>
</tr>
<tr>
<td>02/05</td>
<td>5.10(54.9)</td>
</tr>
<tr>
<td>12m</td>
<td></td>
</tr>
<tr>
<td>05/08</td>
<td>9.30(39.8)</td>
</tr>
<tr>
<td>18m</td>
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</tr>
<tr>
<td>02/08</td>
<td>6.60(57.6)</td>
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## Dependency on Incidence Angle

### Sungai Wain

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<tr>
<td>56m</td>
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</tr>
<tr>
<td></td>
<td>04/05</td>
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$\delta/\kappa$ is about half in co-polarization.
## Dependency on Incidence Angle

**Sungai Wain**

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<tr>
<td>5m</td>
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</tr>
<tr>
<td></td>
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</tr>
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<td>15m</td>
<td>01/04 10.30 (44.7)</td>
</tr>
</tbody>
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\[ \delta/\kappa_\zeta \text{ increases with incidence angle} \]
Dependency on Terrain Slope

\[ \frac{\delta}{\kappa} \text{ is more sensitive in P-Band} \]
Dependency on Terrain Slope

Compact-pol perfoms better near 0 slope
Dependency on Terrain Slope

Also along azimuth slope
Conclusions ... from the analysis of the two tropical data sets

- $\delta/\kappa_\zeta$ is widely independent on spatial baseline.
  The baseline dependency observed in Sungai Wain is probably due to terrain induced
  decorrelation effects.

- $\delta/\kappa_\zeta$ increases with wavelength: at P-band about 20-30% longer than at L-band

- $\delta/\kappa_\zeta$ decreases significantly (40-50%) in any partial polarimetric scenario.
  
  Quad-pol L-band performs better than Compact-Pol P-band !!!

Surprising:
There is no big difference between Compact/Hybrid configurations and Dual-Pol
configurations. This is probably due to the very weak dihedral scattering component in
both test sites.

- $\delta/\kappa_\zeta$ increases with incidence angle -> wider ground scattering dynamic in far range.

- The performance of partial-polarimetry degrades in the presence of even weak slopes
  ($\alpha > 2-3^\circ$)

- Compact-polarimetry works better when ground is flat. (Sungai Wain shows smaller
  CP/QP ratio than Mawas)
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Compact Polarimetry
-on Poincaré Sphere

Conventional Polarizations

Compact Polarimetry

Horizontal

Circular Right

Vertical

$\pi/4$ - mode
Compact Polarimetry (Analogy)

H pol obs.

V pol obs.

Compact-pol obs.
**Projection Matrix**

- **Quad-pol to Comp-pol**

- Definition of scattering vector:

\[
\begin{pmatrix}
S_{hh} + S_{vv} \\
S_{hh} - S_{vv} \\
2S_{hv}
\end{pmatrix}
\]

\[
\vec{k} = \frac{1}{\sqrt{2}} \begin{pmatrix}
S_{hh} + S_{vv} \\
S_{hh} - S_{vv} \\
2S_{hv}
\end{pmatrix}
\]

- Scattering vector of Compact Polarimetry:

\[
\vec{j} = \frac{1}{\sqrt{2}} \begin{pmatrix}
S_{hh} & S_{hv} \\
S_{vh} & S_{vv}
\end{pmatrix}
\begin{pmatrix}
1 \\
e^{i\delta}
\end{pmatrix}
= \frac{1}{\sqrt{2}} \begin{pmatrix}
S_{hh} + e^{i\delta}S_{hv} \\
S_{vh} + e^{i\delta}S_{vv}
\end{pmatrix}
\]

\[
\vec{r} = \frac{1}{2} \begin{pmatrix}
1 & 1 \\
e^{i\delta} & -e^{i\delta}
\end{pmatrix}\vec{k} = \mathbf{A}^T \vec{k}
\]

- Covariance matrix of Compact Polarimetry:

\[
\mathbf{J} = \langle \vec{j} \cdot \vec{j}^* \rangle = \langle \left( \mathbf{A}^T \vec{k} \right) \cdot \left( \mathbf{A}^T \vec{k} \right)^* \rangle
\]

\[
= \mathbf{A}^T \langle \vec{k} \cdot \vec{k}^* \rangle \mathbf{A} = \mathbf{A}^T \mathbf{T} \mathbf{A}
\]
Various projection matrices

- **Linear Transmissions** ($\delta = 0, \pi$)
  
  $$A^{*T}_{\pi/4} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} : L_1$$  
  $$A^{*T}_{-\pi/4} = \frac{1}{2} \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix} : L_2$$

- **Circular Transmissions** ($\delta = \pi/2, -\pi/2$)
  
  $$A^{*T}_{\pi/2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & i \\ i & -i & 1 \end{pmatrix} : CR$$  
  $$A^{*T}_{-\pi/2} = \frac{1}{2} \begin{pmatrix} 1 & 1 & -i \\ -i & i & 1 \end{pmatrix} : CL$$

- **Dual polarizations**
  
  $$A^{*T}_H = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : DV$$  
  $$A^{*T}_V = \frac{\sqrt{2}}{2} \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} : DV$$  
  $$A^{*T}_{HV} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} : HV$$
Coherence region (Field of Values)

Complex $\gamma$ in PolInSAR (How $\gamma$ varies as $w$ changes?)

$$\gamma = \frac{\bar{w}^* T \Omega \bar{w}}{\bar{w}^* T \bar{w}}$$

where

$$T = \frac{1}{2} \sum_i \left< \vec{k}_i \cdot \vec{k}_i^T \right>$$

and

$$\Omega = \left< \vec{k}_1 \cdot \vec{k}_2^* \right>$$
Interferometric Coherence:

\[
\tilde{\mathcal{Y}}(\tilde{w}_1, \tilde{w}_2) = \frac{< \tilde{w}_1 [\Omega] \tilde{w}_2^+ >}{\sqrt{< \tilde{w}_1 [T_{11}] \tilde{w}_1^+ > < \tilde{w}_2 [T_{22}] \tilde{w}_2^+ >}}
\]

Optimisation Problem:

\[
\tilde{w}_1 = \tilde{w}_2
\]

\[
[T]^{-I}[\Omega_\phi] \tilde{w} = \lambda \tilde{w}
\]

\[
[T] = \frac{1}{2} ([T_{11}] + [T_{22}]), \quad \lambda = -(\lambda_1 + \lambda_2^*)
\]

\[
[\Omega_\phi] = \frac{1}{2} (\exp(i\phi)[\Omega] + \exp(-i\phi)[\Omega]^*)
\]

Coherence Region: \( \forall \phi \rightarrow \lambda_{\text{max}}, \lambda_{\text{min}} \) that have to be connected to provide the boundary of the cr.

Shape and size are characterised by the acquisition and scattering parameters.