Dependence of Polarimetric Surface Scattering on Spatial Resolution

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Nonlinear Forward Mapping

$F(\Theta) \rightarrow \Omega$

- Dielectric constant
- Height distribution
- Autocorrelation

$\Omega = \{\text{Independent combination of polarization measurements}\}$

$\Theta = \{\text{Dielectric constant, Height distribution, Autocorrelation}\}$

$\Theta = F^{-1}(\Omega)$

Finding the set of unknown surface parameters from polarization measurement
**Aims of the study**

Understanding the effects of the spatial resolution of the radar sensor on 
1) the statistical description of surface roughness properties 
2) the surface backscattering characteristics 
   particularly for the high resolution polarimetric radar systems.
### Random Surface Parameters

<table>
<thead>
<tr>
<th>Height Probability Distribution</th>
<th>Autocovariance (Autocorrelation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero mean Gaussian characterized by its RMS height ( \sigma = \sqrt{\langle z(x)^2 \rangle - \langle z(x) \rangle^2} )</td>
<td>( R(l) = \langle z(x)z(x+l) \rangle - \langle z(x) \rangle )</td>
</tr>
<tr>
<td>( R_N(l) = R(l) / R(0) = R(l) / \sigma^2 )</td>
<td>( R_N(l_C) = 1 / e )</td>
</tr>
</tbody>
</table>

- **Essential to** define an appropriate **mathematical function for the autocovariance**

- The commonly used **Autocovariance Functions (ACVF)** are Gaussian and Exponential
  \[
  \begin{align*}
  R(l)_G &= \sigma^2 \exp[-l^2 / l_C^2] \\
  R(l)_E &= \sigma^2 \exp[-|l| / l_C]
  \end{align*}
  \]
  Correlation length does not define the types of ACF

- Roughness **Power Spectral Density (PSD)**
  \[
  \begin{align*}
  W(K)_G &= FT\{R(\tau)_G\} = \sigma^2 \sqrt{\pi} l_C \exp[-l_C^2 K^2 / 4] \\
  W(K)_E &= FT\{R(\tau)_E\} = \sigma^2 2l_C[1 + l_C^2 K^2]^{-1}
  \end{align*}
  \]

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![ACF Plot]

- **Measured**
- **Gaussian**
- **Exponential**
Statistical properties of random rough surfaces are affected by the spatial resolution of the radar sensor.

**Autocovariance of the infinite surface**

\[
R(l) = \left< [z(x) - \mu][z(x+l) - \mu] \right> = \lim_{L_x \to \infty} \frac{1}{L_x} \int [z(x) - \mu][z(x+l) - \mu] dx
\]

**Autocovariance defined in a range \( L_x \)**

\[
\rho(l) = \left< \frac{1}{L_x} \int [z(x) - \bar{z}][z(x+l) - \bar{z}] dx \right>
\]

\[
= (1 - |l|/L_x)R(l) - (1 - |l|/L_x)\text{var}(\bar{z})
\]

\[
\text{var}(\bar{z}) = \left< (\mu - \bar{z})^2 \right> = \frac{2}{L_x} \int_0^{L_x} (1 - |t|/L_x) R(t) dt
\]
### Autocovariance Function of Truncated Surface (ACVFT)

\[
\rho(l) = \left(1 - \frac{|l|}{L_x}\right)\{R(l) - \text{var}(z)\}
\]

#### Gaussian ACVFT

\[
\rho(l)_G = \left(1 - \frac{|l|}{L_x}\right)\left\{\sigma^2 \exp\left[-\frac{l^2}{l_C^2}\right] - G\right\}
\]

\[G = \sigma^2\left\{\alpha^2\left(\exp[\alpha^{-2}] - 1\right) + \alpha \sqrt{\pi} \text{Erf}[\alpha^{-1}]\right\}\]

\[\alpha = l_C / L_x\]

#### Exponential ACVFT

\[
\rho(l)_E = \left(1 - \frac{|l|}{L_x}\right)\left\{\sigma^2 \exp\left[-\frac{|l|}{l_C}\right] - H\right\}
\]

\[H = 2\sigma^2\left\{\alpha^2\left(\exp[\alpha^{-1}] - 1\right) + \alpha \right\}\]

\[\alpha = l_C / L_x\]

#### Gaussian PSDT

\[
S(K)_G = W(K)_G \otimes L_x \text{sinc}^2(L_x K) - L_x G \text{sinc}^2(L_x K)
\]

#### Exponential PSDT

\[
S(K)_E = W(K)_E \otimes L_x \text{sinc}^2(L_x K) - L_x H \text{sinc}^2(L_x K)
\]
Gaussian and Exponential ACVFT

- Numerical Generation
  - Gaussian and Exponentially Correlated Rough Surface

\[ \sigma = 0.4 \text{cm} \quad l_c = 6 \text{cm} \]

Profile length: 60m (=1000 \( l_c \))
# ACFT of Simulated Surface

<table>
<thead>
<tr>
<th>Distance</th>
<th>Gaussian</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m</td>
<td><img src="image1" alt="Gaussian 6m" /></td>
<td><img src="image2" alt="Exponential 6m" /></td>
</tr>
<tr>
<td>1.8m</td>
<td><img src="image3" alt="Gaussian 1.8m" /></td>
<td><img src="image4" alt="Exponential 1.8m" /></td>
</tr>
<tr>
<td>60cm</td>
<td><img src="image5" alt="Gaussian 60cm" /></td>
<td><img src="image6" alt="Exponential 60cm" /></td>
</tr>
</tbody>
</table>

**Gaussian**

- Infinite
- 10m segments
- ACFT ($L_x=100l_0$)

**Exponential**

- Infinite
- 2m segments
- ACFT ($L_x=30l_0$)

- Infinite
- 0.8m segments
- ACFT ($L_x=10l_0$)
The roughness power spectrum is band limited by a size of the resolution cell

\[ f_c = \frac{1}{L_x} \]
Integral Equation Method

\[ S(K) = W(K) \otimes L_x \text{sinc}^2(L_x K) - L_x G \text{sinc}^2(L_x K) \]

\[ \sigma^0_{pq} = \frac{k^2}{2} \exp\left[ -2(kh \cos \theta)^2 \right] \sum_{n=1}^{\infty} |I_{pq}^n|^2 \frac{S^{(n)}(-2k \sin \theta)}{n!} \]

\[ h = \sqrt{\sigma^2 - G(\alpha)} \]

\[ f \text{ (Fresnel reflectivity)} \]

[Fung, 1992; 1994; Wu et al., 2001]
Dependence of backscattering coefficients on the spatial resolution for the generated rough surface $\sigma = 0.4\,cm$, $l = 6\,cm$
Spatial resolution on the Earth’s surface

\[ L_x = \frac{\delta}{\sin \theta} = \frac{c}{2B \sin \theta} \]
# Backscattering Simulations

<table>
<thead>
<tr>
<th></th>
<th>X-band</th>
<th>C-band</th>
<th>L-band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
<tr>
<td>Exponential</td>
<td><img src="image4.png" alt="Graph" /></td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
ACVFT or PSDT depends on

- spatial resolution, and
- correlation length

The correlation length may affect an analysis of radar backscattering coefficient
Experimental Measurements

Gaussian isotropic surfaces

\[ \sigma = 0.4\text{cm} \]
\[ l = 6\text{cm} \]

- Incidence angle = 40°
- Frequency = 14 GHz
- Resolution = 4.2 cm - 24 cm
Experimental Measurements

$L_x = l$

$L_x = 1.4l$

$L_x = 2l$

$L_x = 4l$

$\sigma_{HH}$

$\sigma_{VV}$

Spatial resolution (cm)
CONCLUSIONS

- An appropriate description of the effect of spatial resolution on statistical characteristics of rough surface has been presented by introducing the ACVF of truncated surface.

- Traditional computation of the surface backscattering based on the autocovariance function of infinite surface leads to an underestimation of the backscattering signature of high resolution radar.

- Forward and Inverse modeling of backscattering response based on the sample roughness statistics has not been completely resolved yet particularly due to the spatial variability of local Fresnel coefficients in high resolution radar.