Polarimetric Target Detector by the use of the Polarisation Fork

Armando Marino[1]

Shane R Cloude[2]

Iain H Woodhouse[1]

[1]The University of Edinburgh,
Edinburgh Earth Observatory (EEO), UK
[2]AEL Consultants, Edinburgh, UK

Mathematical formulation

Single (coherent) target

Scattering matrix:

$$[S]=\begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix}$$

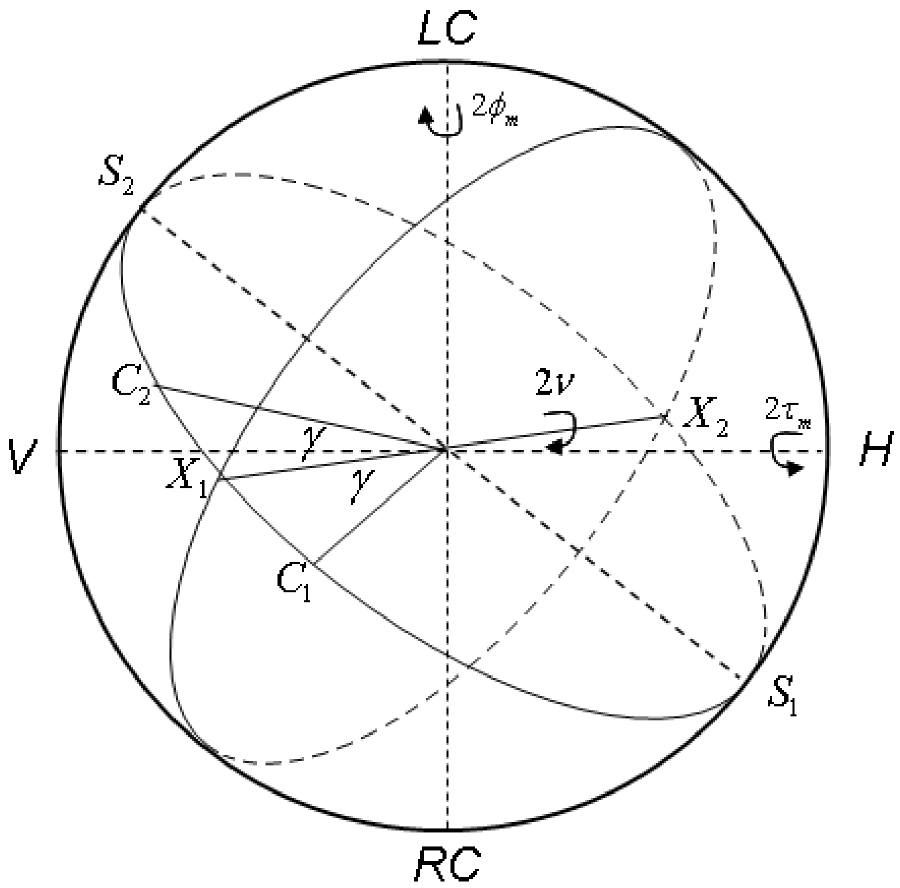
Scattering vector:

$$\underline{k}=\frac{1}{2}Trace\left([S]\Psi\right)=[k_1,k_2,k_3,k_4]^T$$

Backscattering & reciprocity

$$\underline{k}=\frac{1}{2}Trace\left([S]\Psi\right)=[k_1,k_2,k_3]^T$$

Scattering mechanism: $\underline{\omega}=\underline{k}/|\underline{k}|$



6 Huynen parameters:

$$[S]=\left[U^*(\phi_m,\tau_m,\nu)\right]m\begin{bmatrix}1 & 0\\ 0 & \tan^2\gamma\end{bmatrix}\left[U^{*T}(\phi_m,\tau_m,\nu)\right]\exp(j\xi)$$

Polarisation Fork:

$X_1, X_2 = \text{X-pol Nulls}$

$C_1, C_2 = \text{Co-pol Nulls}$

$S_1, S_2 = \text{X-pol Max}$

Partial target: Target Coherency Matrix

The second order statistics are necessary.

$$[C_3] = \langle \underline{k} \cdot \underline{k}^{+} \rangle$$

$$[C_3] = \begin{bmatrix} \langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\ \langle k_2 k_1^* \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\ \langle k_3 k_1^* \rangle & \langle k_3 k_2^* \rangle & \langle |k_3|^2 \rangle \end{bmatrix}$$

Classical formulations:

Lexicographic basis

$$[C_L] = \begin{bmatrix} \langle |S_{HH}|^2 \rangle & \sqrt{2}\langle S_{HH} S_{HV}^* \rangle & \langle S_{HH} S_{VV}^* \rangle \\ \sqrt{2}\langle S_{HV} S_{HH}^* \rangle & 2\langle |S_{HV}|^2 \rangle & \sqrt{2}\langle S_{HV} S_{VV}^* \rangle \\ \langle S_{VV} S_{HH}^* \rangle & \sqrt{2}\langle S_{VV} S_{HV}^* \rangle & \langle |S_{VV}|^2 \rangle \end{bmatrix}$$

Pauli basis

$$[C_P] = \begin{bmatrix} \langle |S_{HH} + S_{VV}|^2 \rangle & \langle (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \rangle & 2\langle (S_{HH} + S_{VV}) S_{HV}^* \rangle \\ \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle & \langle |S_{HH} - S_{VV}|^2 \rangle & 2\langle (S_{HH} - S_{VV}) S_{HV}^* \rangle \\ 2\langle S_{HV} (S_{HH} + S_{VV})^* \rangle & 2\langle S_{HV} (S_{HH} - S_{VV})^* \rangle & 2\langle |S_{HV}|^2 \rangle \end{bmatrix}$$

Polarimetric Detector

Polarimetric coherence:

$$\gamma = \frac{\langle i(\underline{\omega}_1)\cdot i^*(\underline{\omega}_2)\rangle}{\sqrt{\langle i(\underline{\omega}_1)\cdot i^*(\underline{\omega}_1)\rangle\langle i(\underline{\omega}_2)\cdot i^*(\underline{\omega}_2)\rangle}}$$

Where:

$$i(\omega_j) = \underline{\omega}_j^+ \cdot \underline{k} \qquad j = 1,2$$

$$\underline{k} = [k_1, k_2, k_3]$$

In the new basis

Demonstration:

1) A change of **basis** where the target to detect is one axes

Target: $\underline{\omega}_T = [1,0,0]$ ⟹ $i(\omega_T) = \underline{\omega}_T^+ \cdot \underline{k} = k_1$

2) The **Polarisation Fork** (or Huynen parameters) is slightly **changed** to obtain:

Pseudo target:

$$\underline{\omega}_P = [a,b,c]$$

$$a,b,c \in C$$

⟹

$$i(\omega_P) = \underline{\omega}_P^+ \cdot \underline{k} = ak_1 + bk_2 + ck_3$$

$$|a| \approx 1 \quad |b| \approx 0 \quad |c| \approx 0$$

edinburgh earth observatory

Polarimetric Detector

3) Evaluation of the polarimetric coherence

Detector (first attempt): $\left|\gamma\left(\underline{\omega}_T, \underline{\omega}_p\right)\right| > T$

$$\underline{k} = \left[k_1, k_2, k_3\right]$$

$$\underline{\omega}_T = \left[1,0,0\right]$$

$$\underline{\omega}_p = \left[a, b, c\right]$$

$$a, b, c \in C$$

$$\left|\gamma\left(\underline{\omega}_T, \underline{\omega}_p\right)\right| = \frac{\left|\underline{\omega}_T^{+}\langle[C_3]\rangle\underline{\omega}_p\right|}{\sqrt{\left(\underline{\omega}_T^{+}\langle[C_3]\rangle\underline{\omega}_T\right)\left(\underline{\omega}_p^{+}\langle[C_3]\rangle\underline{\omega}_p\right)}}$$

$$i_1 \cdot i_2^* = \underline{\omega}_T^{+}\langle[C_3]\rangle\underline{\omega}_p = a\left\langle\left|k_1\right|^2\right\rangle + b\left\langle k_1 k_2^*\right\rangle + c\left\langle k_1 k_3^*\right\rangle$$

$$i_1 \cdot i_1^* = \underline{\omega}_T^{+}\langle[C_3]\rangle\underline{\omega}_T = \left\langle\left|k_1\right|^2\right\rangle$$

$$i_2 \cdot i_2^* = \underline{\omega}_p^{+}\langle[C_3]\rangle\underline{\omega}_p = \left|a\right|^2\left\langle\left|k_1\right|^2\right\rangle + \left|b\right|^2\left\langle\left|k_2\right|^2\right\rangle + \left|c\right|^2\left\langle\left|k_3\right|^2\right\rangle +$$

$$+ 2\Re\left(a^* b\right)\Re\left(\left\langle k_1 k_2^*\right\rangle\right) + 2\Re\left(a^* c\right)\Re\left(\left\langle k_1 k_3^*\right\rangle\right) + 2\Re\left(b^* c\right)\Re\left(\left\langle k_3 k_2^*\right\rangle\right)$$

Polarimetric detector

After normalisation for:

$$\left|a\right|\left\langle\left|k_1\right|^2\right\rangle$$

$$\left|\gamma\left(\underline{\omega}_T,\underline{\omega}_p\right)\right| = \frac{\left|1+\dfrac{b}{a}\dfrac{\left\langle k_1 k_2^*\right\rangle}{\left\langle\left|k_1\right|^2\right\rangle}+\dfrac{c}{a}\dfrac{\left\langle k_1 k_3^*\right\rangle}{\left\langle\left|k_1\right|^2\right\rangle}\right|}{\sqrt{\dfrac{i_2\cdot i_2^*}{\left|a\right|^2\left\langle\left|k_1\right|^2\right\rangle}}}$$

•If the components of the scattering vector are **uncorrelated**, the cross products correspond to a "noise" residual terms (biasing low coherence).

•If the components are **correlated** the cross product is not 0 and the coherence is biased up/down depending on how they sum with phase.

Where:

$$\frac{i_2}{\left|a\right|^2\left\langle\left|k_1\right|^2\right\rangle} = 1+\frac{\left|b\right|^2}{\left|a\right|^2}\frac{\left\langle\left|k_2\right|^2\right\rangle}{\left\langle\left|k_1\right|^2\right\rangle}+\frac{\left|c\right|^2}{\left|a\right|^2}\frac{\left\langle\left|k_3\right|^2\right\rangle}{\left\langle\left|k_1\right|^2\right\rangle}+2\frac{\Re\left(a^*b\right)}{\left|a\right|^2}\frac{\Re\left(\left\langle k_1k_2^*\right\rangle\right)}{\left\langle\left|k_1\right|^2\right\rangle}+2\frac{\Re\left(a^*c\right)}{\left|a\right|^2}\frac{\Re\left(\left\langle k_1k_3^*\right\rangle\right)}{\left\langle\left|k_1\right|^2\right\rangle}+2\sqrt{2}\frac{\Re\left(b^*c\right)}{\left|a\right|^2}\frac{\Re\left(\left\langle k_3k_2^*\right\rangle\right)}{\left\langle\left|k_1\right|^2\right\rangle}$$

Bias removal

4) Definition of a **new** operator that works on target **powers**

$$\gamma_d = \frac{\underline{\omega}_T^{+} \langle [P] \rangle \underline{\omega}_P}{\sqrt{\underline{\omega}_T^{+} \langle [P] \rangle \underline{\omega}_T \cdot \underline{\omega}_P^{+} \langle [P] \rangle \underline{\omega}_P}}$$

Where: $$[P] = \begin{bmatrix} \langle |k_1|^2 \rangle & 0 & 0 \\ 0 & \langle |k_2|^2 \rangle & 0 \\ 0 & 0 & \langle |k_3|^2 \rangle \end{bmatrix}$$

$$\left|\gamma_d\left(\underline{\omega}_T, \underline{\omega}_p\right)\right| = \frac{|a| \langle |k_1|^2 \rangle}{\sqrt{\langle |k_1|^2 \rangle \left(|a|^2 \langle |k_1|^2 \rangle + |b|^2 \langle |k_2|^2 \rangle + |c|^2 \langle |k_3|^2 \rangle \right)}}$$

Detector:

$$\left|\gamma_d\left(\underline{\omega}_T, \underline{\omega}_p\right)\right| > T$$

$$\left|\gamma_d\left(\underline{\omega}_T, \underline{\omega}_p\right)\right| = \frac{1}{\sqrt{\left(1 + \frac{|b|^2}{|a|^2} \frac{\langle |k_2|^2 \rangle}{\langle |k_1|^2 \rangle} + \frac{|c|^2}{|a|^2} \frac{\langle |k_3|^2 \rangle}{\langle |k_1|^2 \rangle}\right)}}$$

Threshold selection

Sample detector

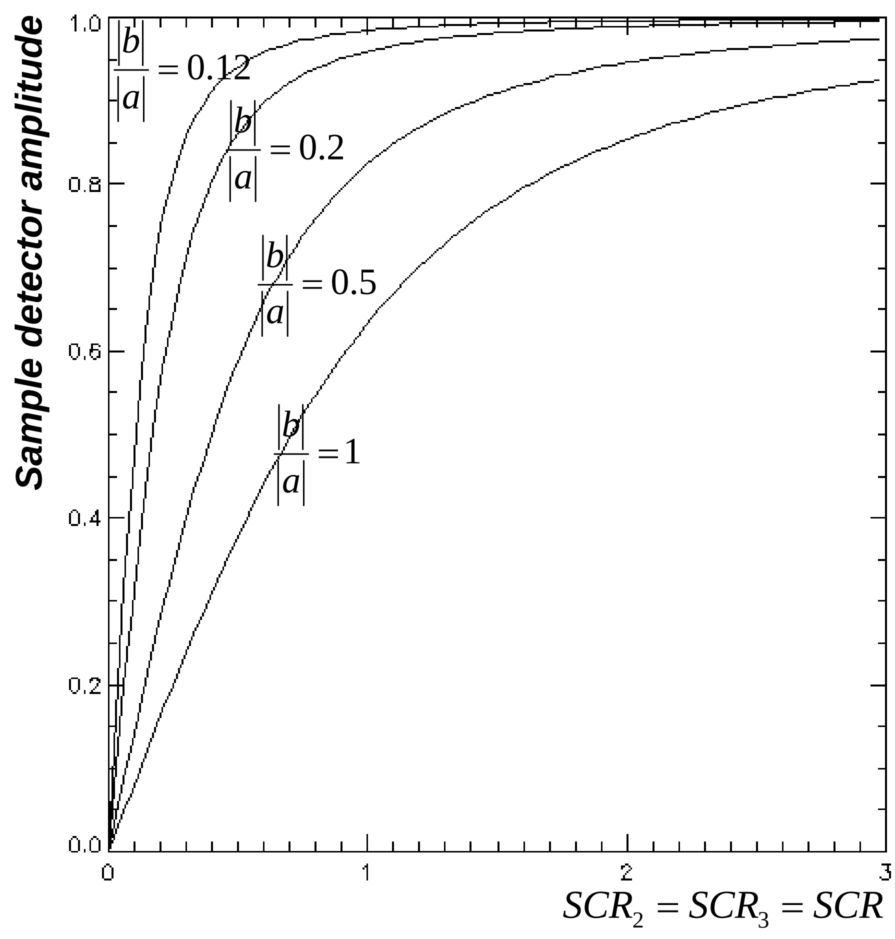
Approximation:

$$\langle x \rangle \approx E[x]$$

$$\left|\gamma_d\left(\underline{\omega}_T, \underline{\omega}_p\right)\right| = \frac{1}{\sqrt{\left(1+\frac{|b|^2}{|a|^2}\frac{E\left[|k_2|^2\right]}{E\left[|k_1|^2\right]}+\frac{|c|^2}{|a|^2}\frac{E\left[|k_3|^2\right]}{E\left[|k_1|^2\right]}\right)}}$$

$$SCR_2 = \frac{E\left[|k_1|^2\right]}{E\left[|k_2|^2\right]} \qquad SCR_3 = \frac{E\left[|k_1|^2\right]}{E\left[|k_3|^2\right]}$$

$$|\gamma_d| \approx \frac{1}{\sqrt{1+\left|\frac{b}{a}\right|^2\frac{1}{SCR_2}+\left|\frac{c}{a}\right|^2\frac{1}{SCR_3}}}$$

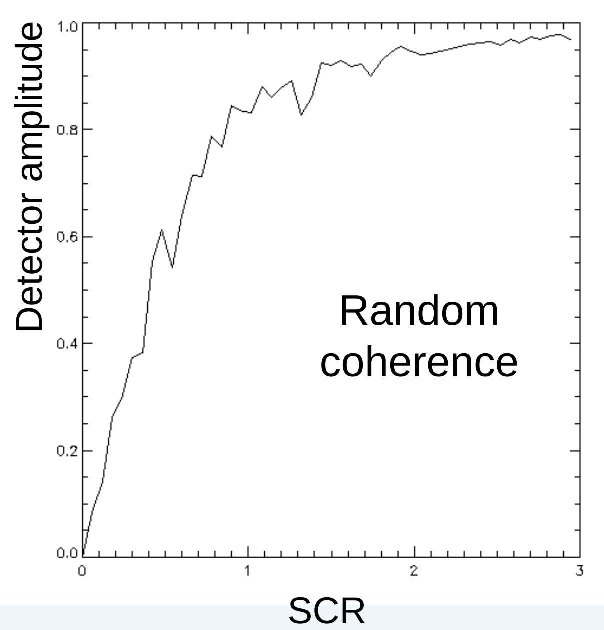


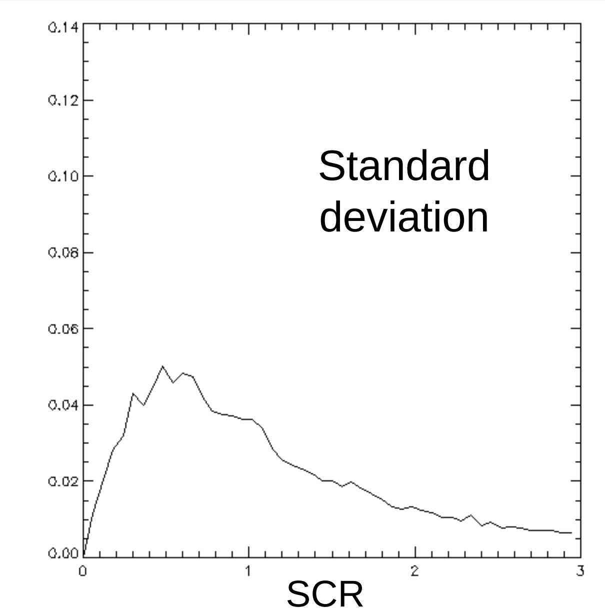
edinburgh earth observatory

Detector: random variable

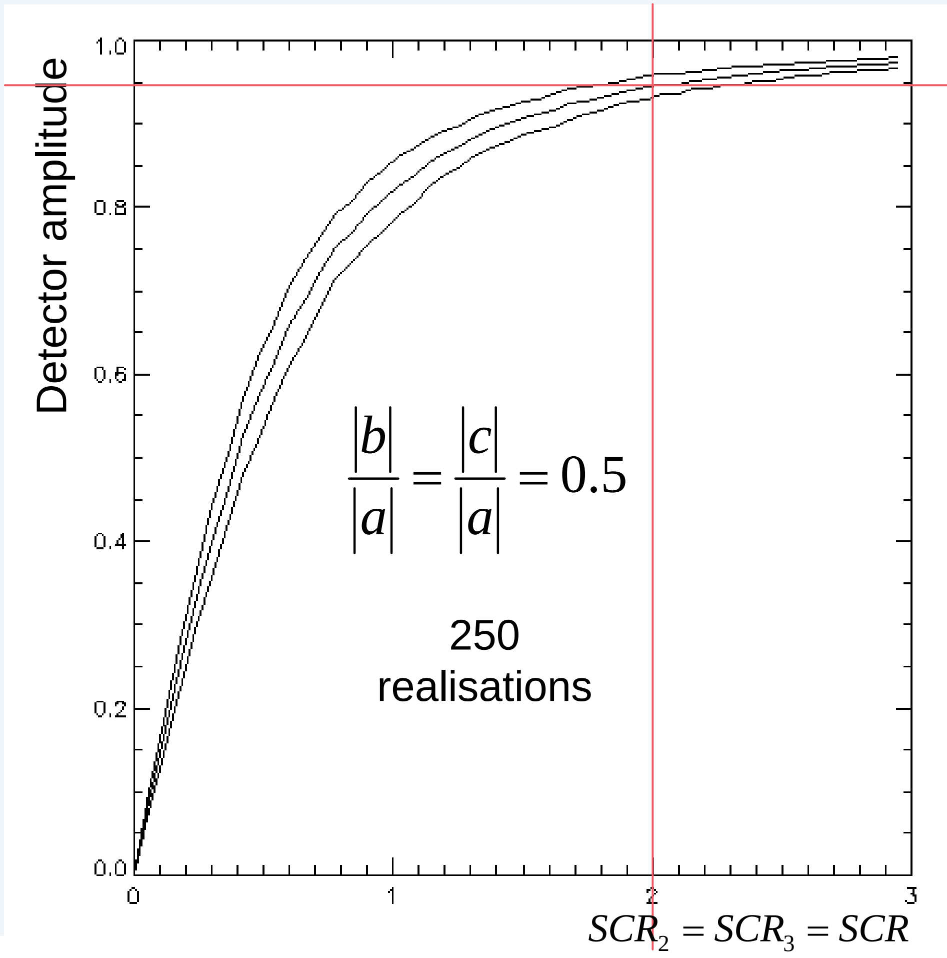
$$c_{2i} \sim N(0, \hat{\sigma}_2)$$
$$c_{2r} \sim N(0, \hat{\sigma}_2)$$

$$k_2 = c_{2r} + jc_{2i}$$





Average window 5x5



Validation

Full-polarimetric Dataset

DLR: E-SAR
L-band

Landsberg
SARTOM project

edinburgh
earth
observatory

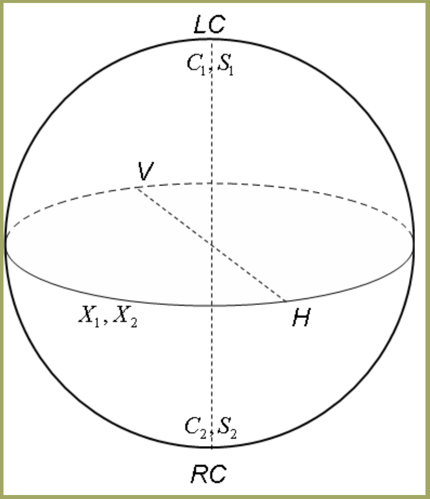
Detection

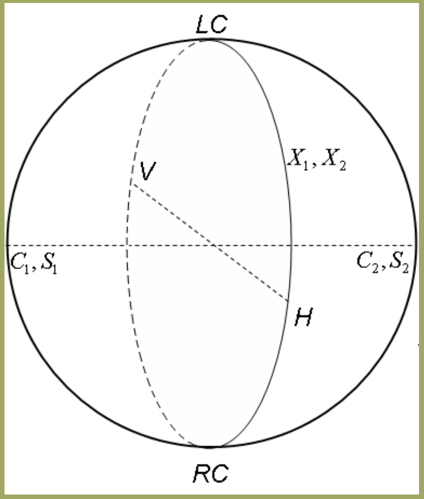
Multiple reflection

$$\underline{k}_P = 1/\sqrt{2}\left[S_{HH}+S_{VV}, S_{HH}-S_{VV}, 2S_{HV}\right]^T$$

Odd-bounce

Even-bounce



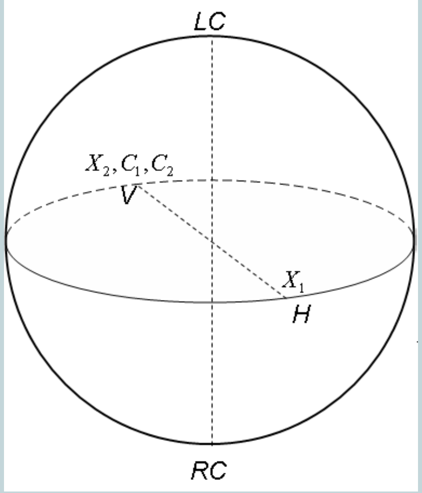


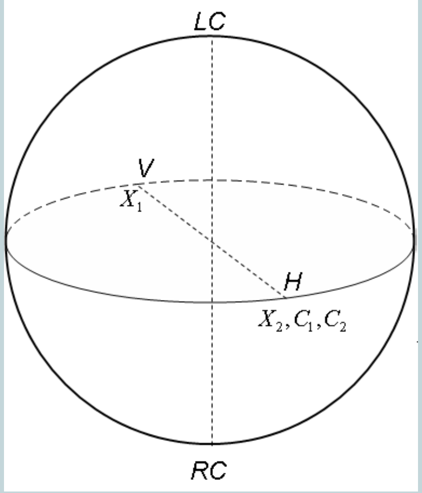
Oriented dipole

$$\underline{k}_L = \left[S_{HH}, \sqrt{2}S_{HV}, S_{VV}\right]^T$$

Horizontal dipole

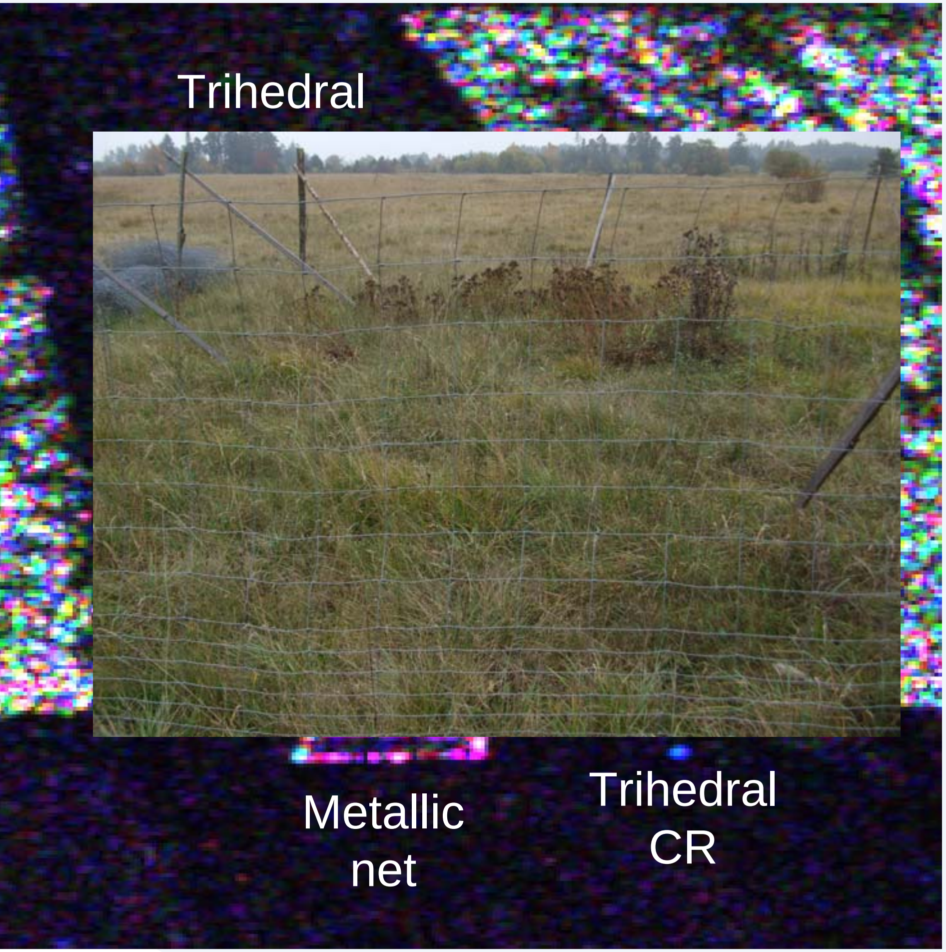
Vertical dipole



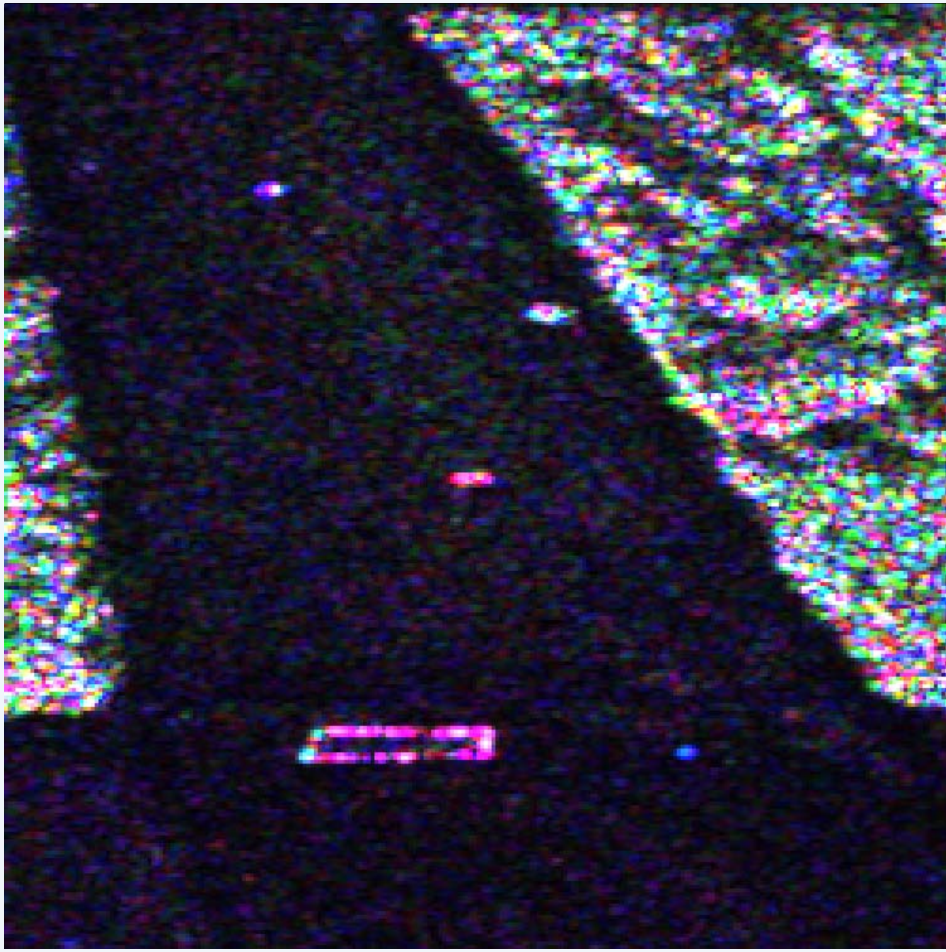


Open field: multiple reflection

L-band



5x5



Red : $S_{HH} - S_{VV}$; Green : $2S_{HV}$; Blue : $S_{HH} + S_{VV}$

Red: Even-bounce

Green: 0

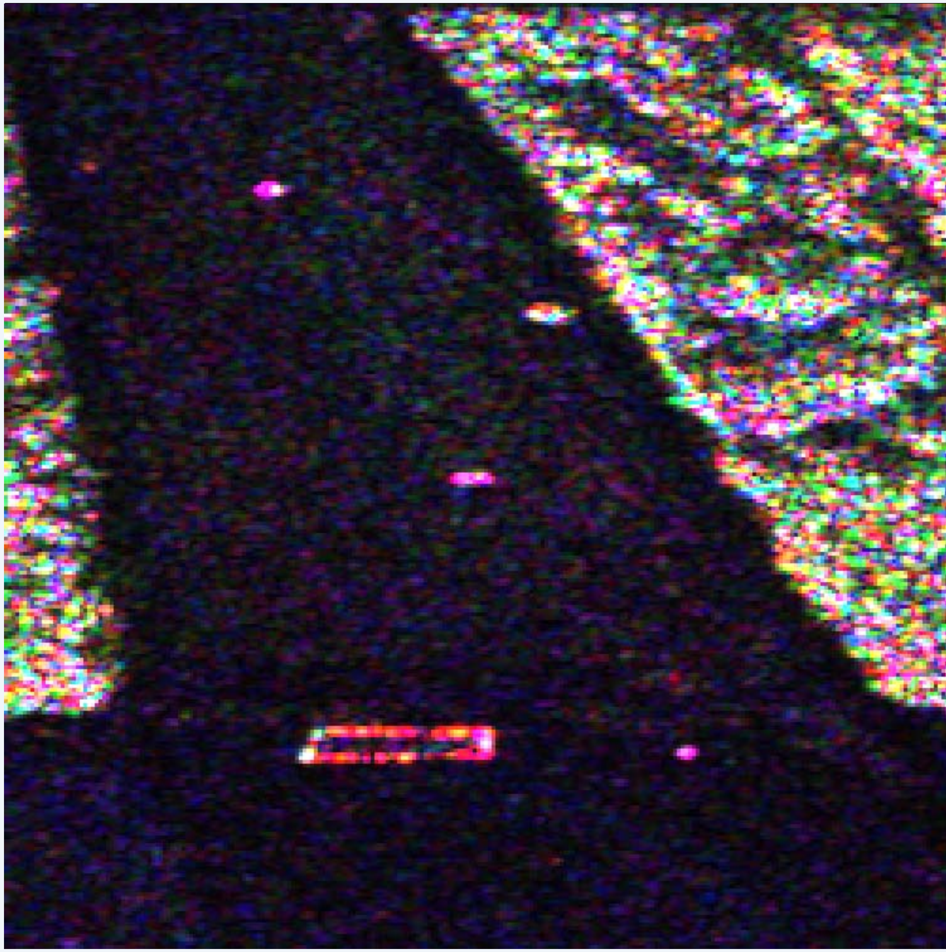
Red: Odd-bounce

edinburgh earth observatory

Open field: oriented dipoles

L-band

5x5



$\text{Red}: S_{HH};\ \text{Green}: \sqrt{2}S_{HV};\ \text{Blue}: S_{VV}$

Red: Horizontal dipole

Green: 0

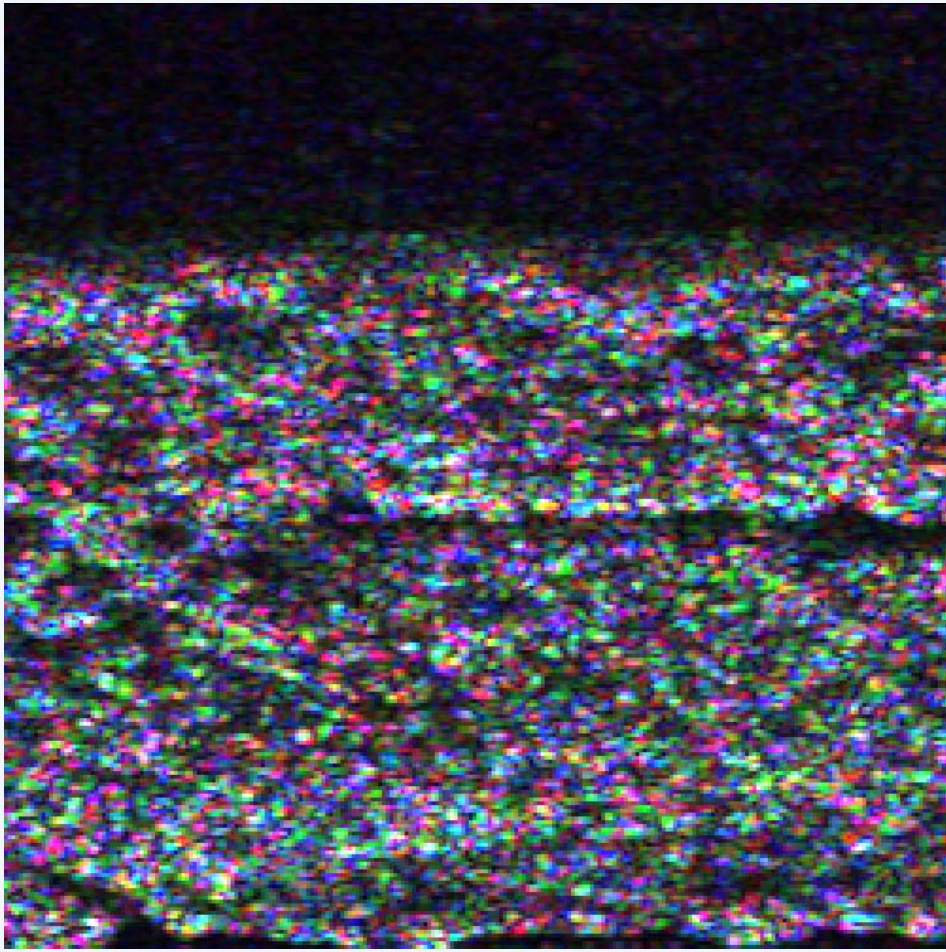
Red: Vertical dipole

edinburgh earth observatory

Forested area: multiple reflection

L-band

5x5



$\mathrm{Red: S_{HH} - S_{VV};\ Green: 2S_{HV};\ Blue: S_{HH} + S_{VV}}$

Red: Even-bounce

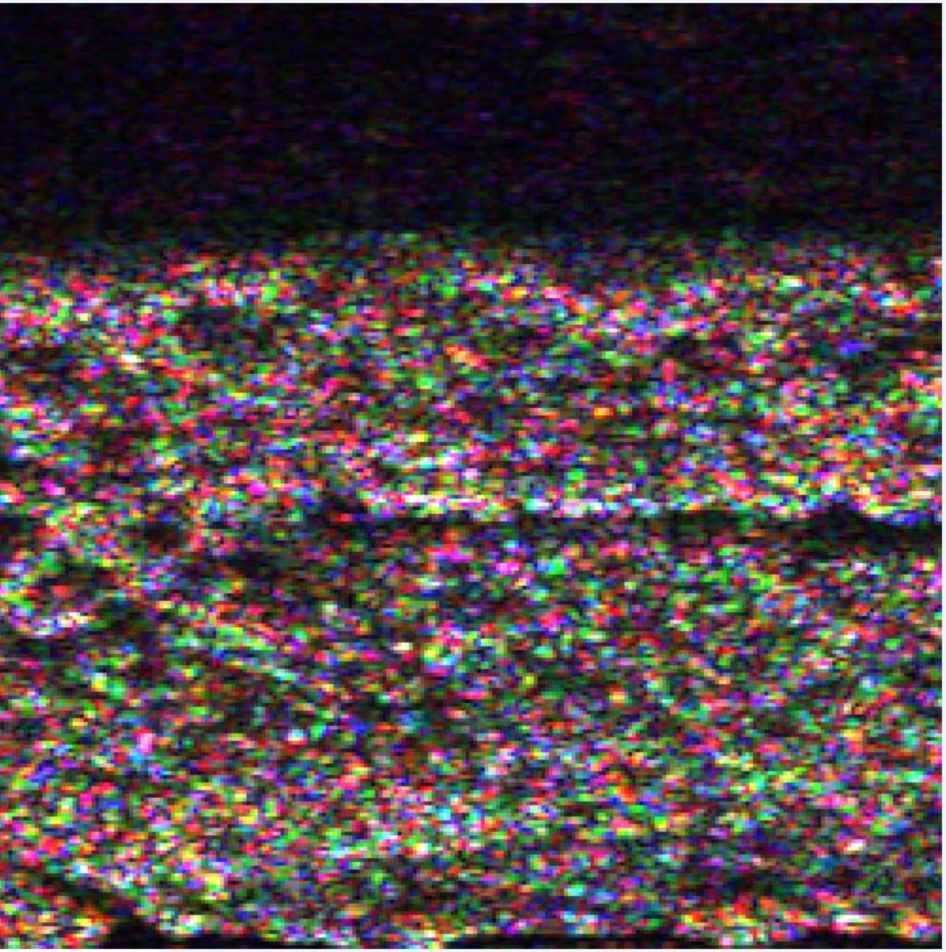
Green: 0

Red: Odd-bounce

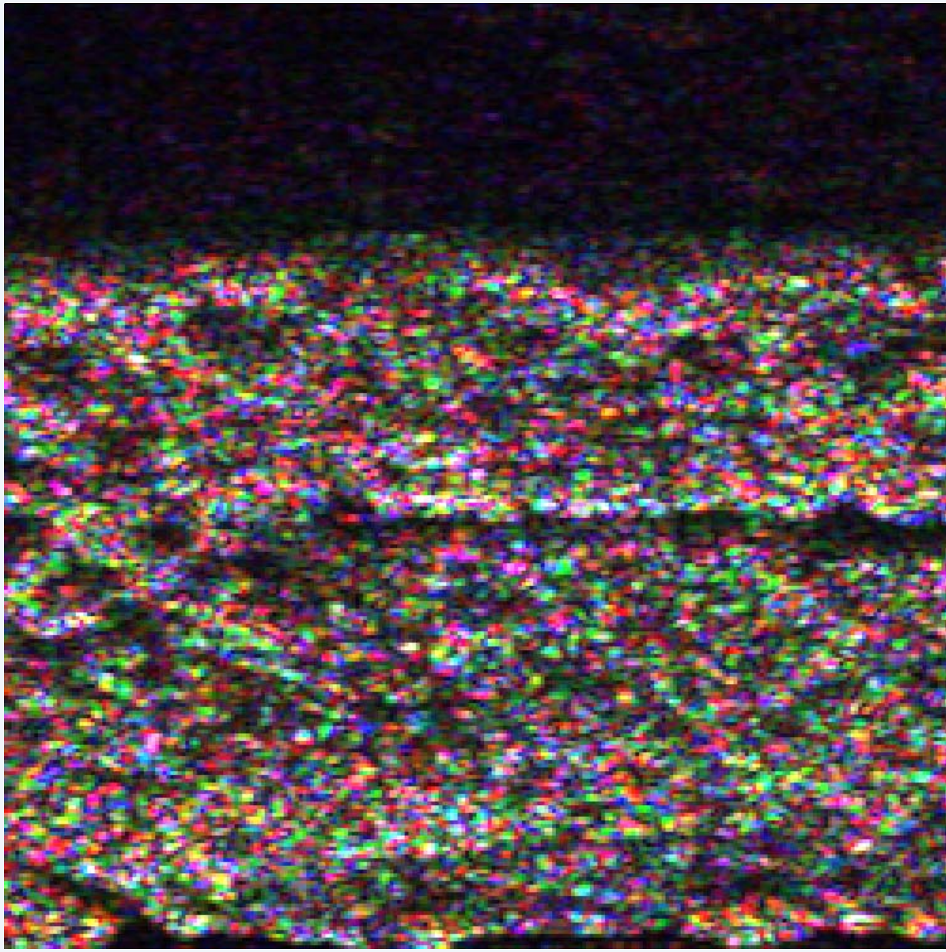
edinburgh earth observatory

Forested area: oriented dipole

L-band



5x5



$\text{Red}: S_{HH};\ \text{Green}: \sqrt{2}S_{HV};\ \text{Blue}: S_{VV}$

Red: Horizontal dipole

Green: 0

Red: Vertical dipole

edinburgh earth observatory

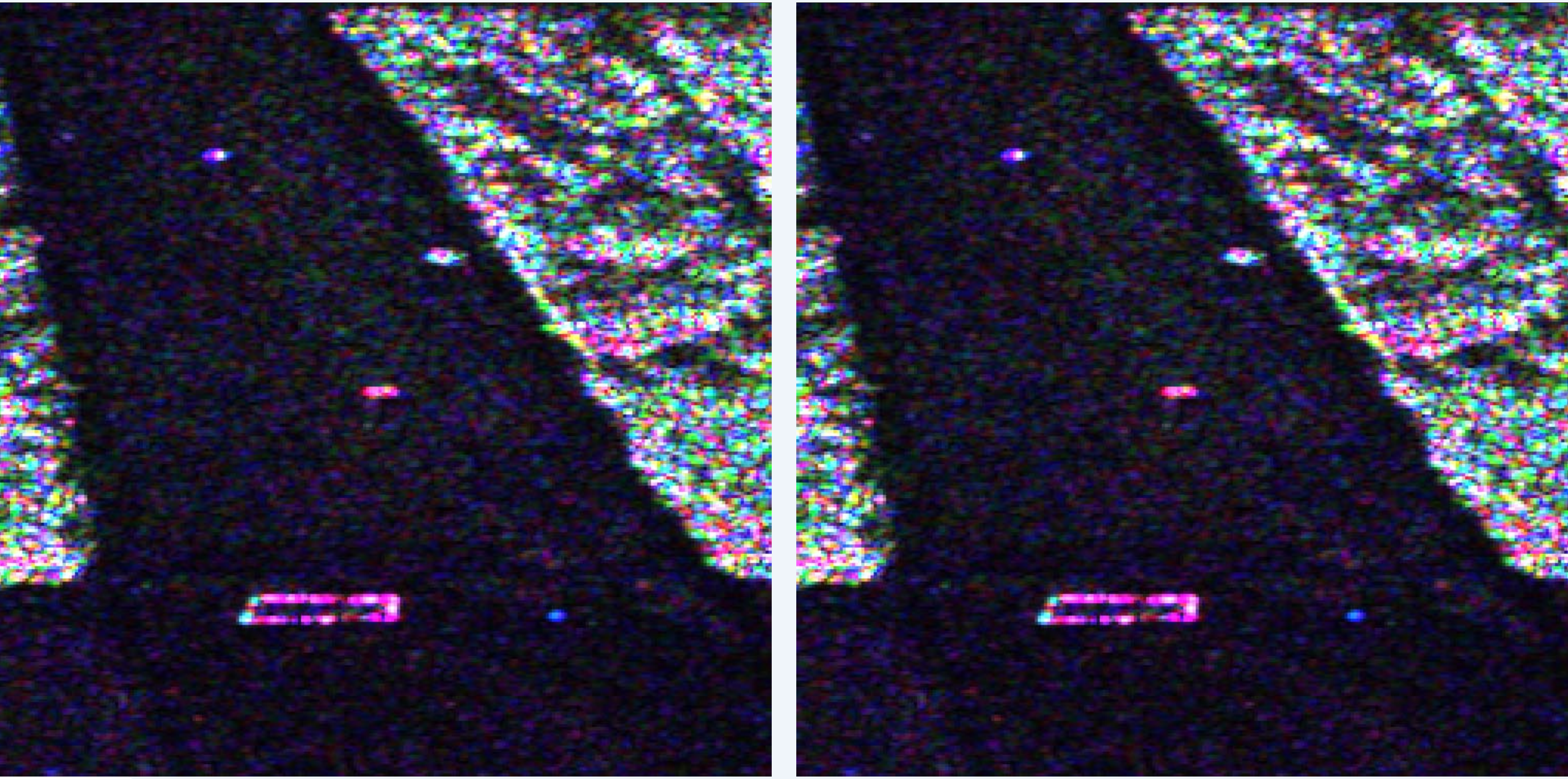
Comparison with Polarimetric Whitening Filter (PWF)

L. M. Novak, M. C. Burl, and M. W. Irving, "Optimal Polarimetric Processing for Enhanced Target Detection," IEEE Trans. Aerospace and Electronic Systems, vol. 20, pp. 234-244,1993.

Open field

L-band

5x5



PWF

Red: Even-bounce

Green: 0

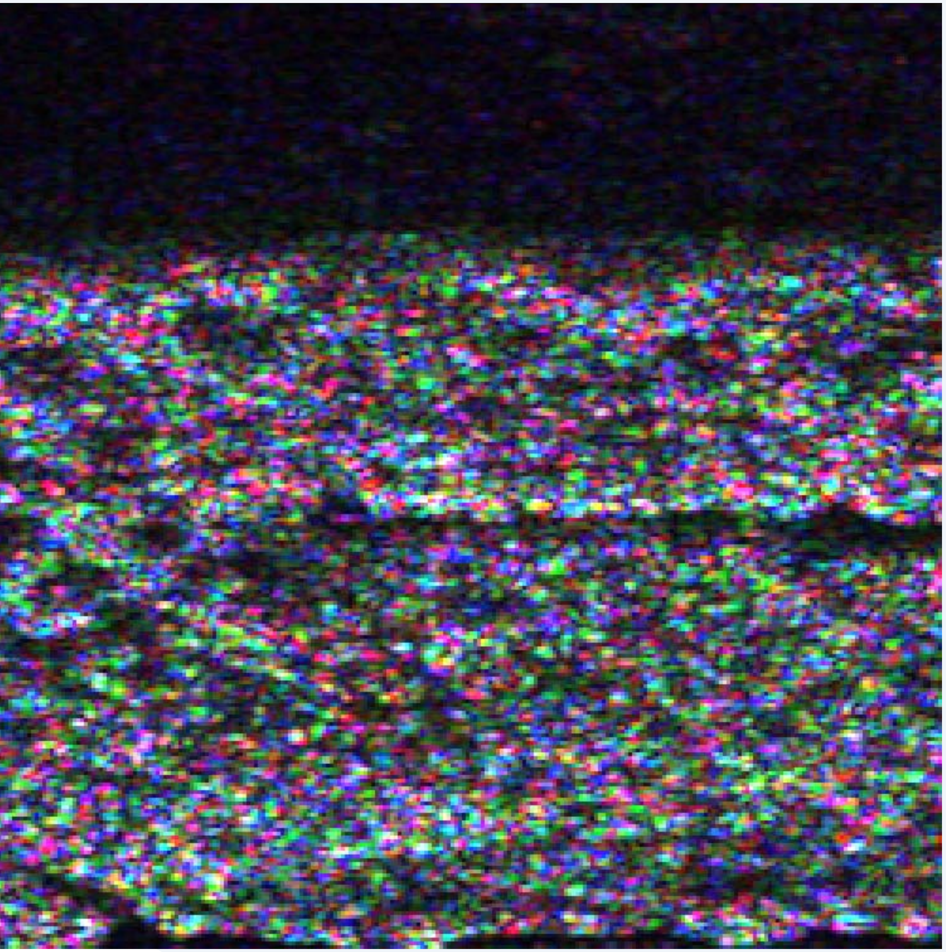
Red: Odd-bounce

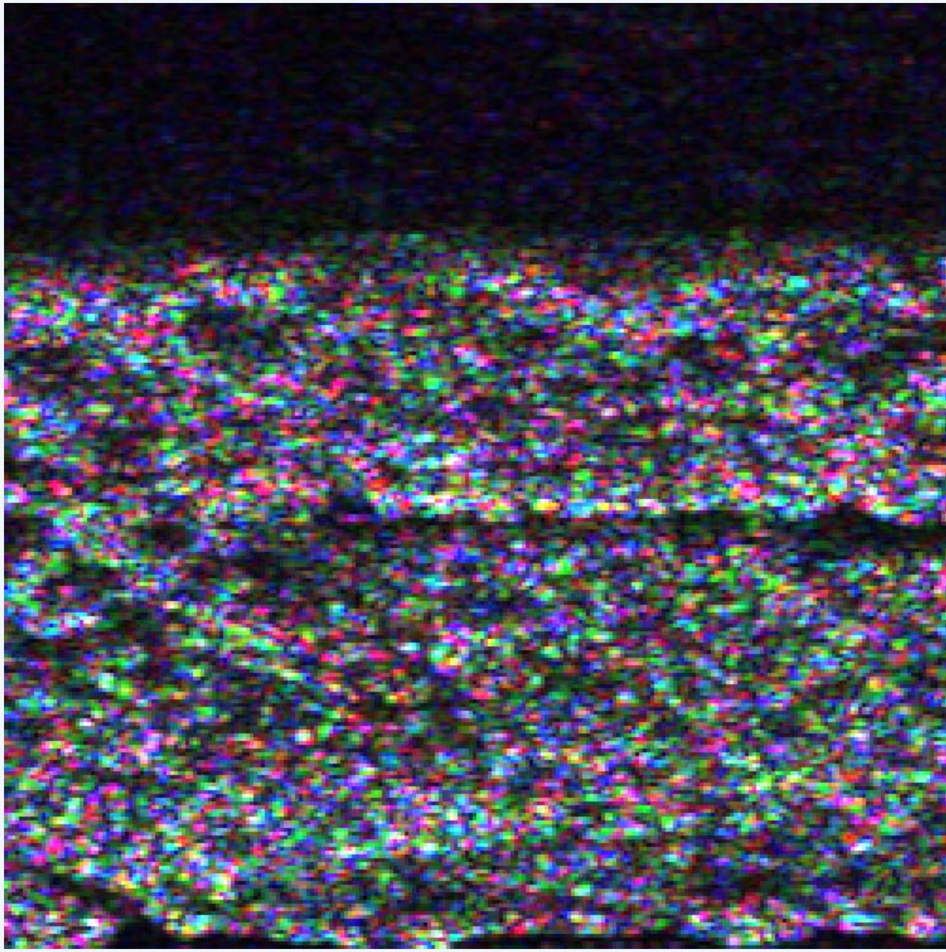
edinburgh
earth
observatory

Forest: POLSAR

L-band

5x5





PWF

Red: Even-bounce
Green: 0
Red: Odd-bounce

edinburgh earth observatory

Conclusions

- A target detector was developed based on the unique polarimetric fork (PF) of the single target (similarly the Huynen parameters can be used).
- The mathematical formulation carried out is general, and so can be applied for any single target of interest (as long as the PF is known).
- The validation was achieved over two categories of targets: multiple reflection and oriented dipoles, with results in line with the expected physical behaviour of the targets.
- A supplementary theoretical validation is carried out, where the algorithm is compared with the Polarimetric Whitening Filter (PWF).

Thank you very much for your attention!

edinburgh earth observatory

Uniqueness of detection

We pass with a projection to the space of Power: $C^3 \rightarrow R^{3+}$

Defined a basis $\hat{e}_i$ the scattering vector in C^3 is represented by:

$$\underline{k} = k_1\hat{e}_1 + k_2\hat{e}_2 + k_3\hat{e}_3 \qquad \underline{k} = [k_1, k_2, k_3] \qquad \underline{k} \in C^3$$

The projective space is obtained with the operator:

$$P_{ii} = \left|\underline{k}^{*T} \cdot \hat{e}_i\right|^2 \qquad C^3 \rightarrow R^{3+}$$

This is a sujective operation, hence the vector in P is uniquelly defined once we select the vector in the 3-D complex space (and we set a basis).

The detection is unique since the detection rule is defined on the power (SCR or peak) and the Power space is uniquelly related with the target space (we need only 3 real numbers).

edinburgh earth observatory

Uniqueness of detection

$$P = \left\langle \underline{k}_1 \cdot \underline{k}_1^{*T} \right\rangle + \left\langle \underline{k}_2 \cdot \underline{k}_2^{*T} \right\rangle + \left\langle \underline{k}_3 \cdot \underline{k}_3^{*T} \right\rangle$$

$$\underline{k}_1 = [k_1, 0, 0]^T$$

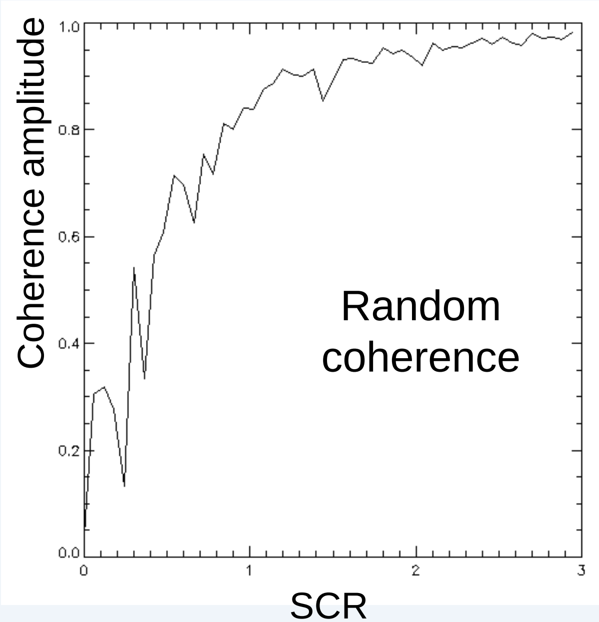
$$\underline{k}_2 = [0, k_2, 0]^T$$

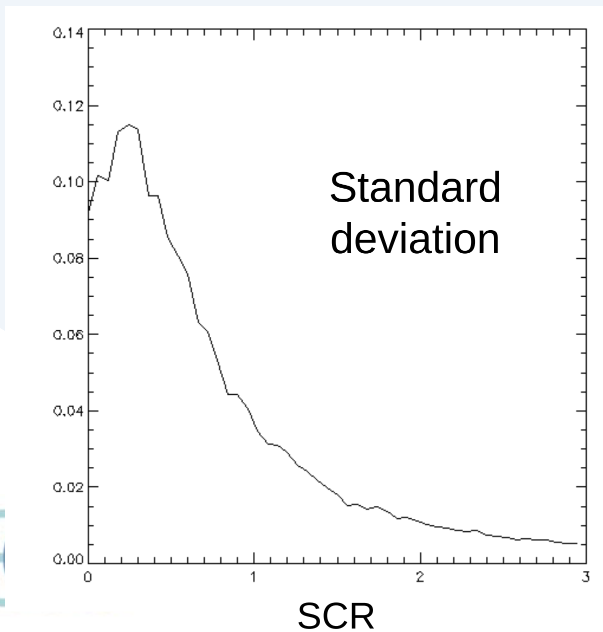
$$\underline{k}_3 = [0, 0, k_3]^T$$

$$[P] = \begin{bmatrix} \langle |k_1|^2 \rangle & 0 & 0 \\ 0 & \langle |k_2|^2 \rangle & 0 \\ 0 & 0 & \langle |k_3|^2 \rangle \end{bmatrix}$$

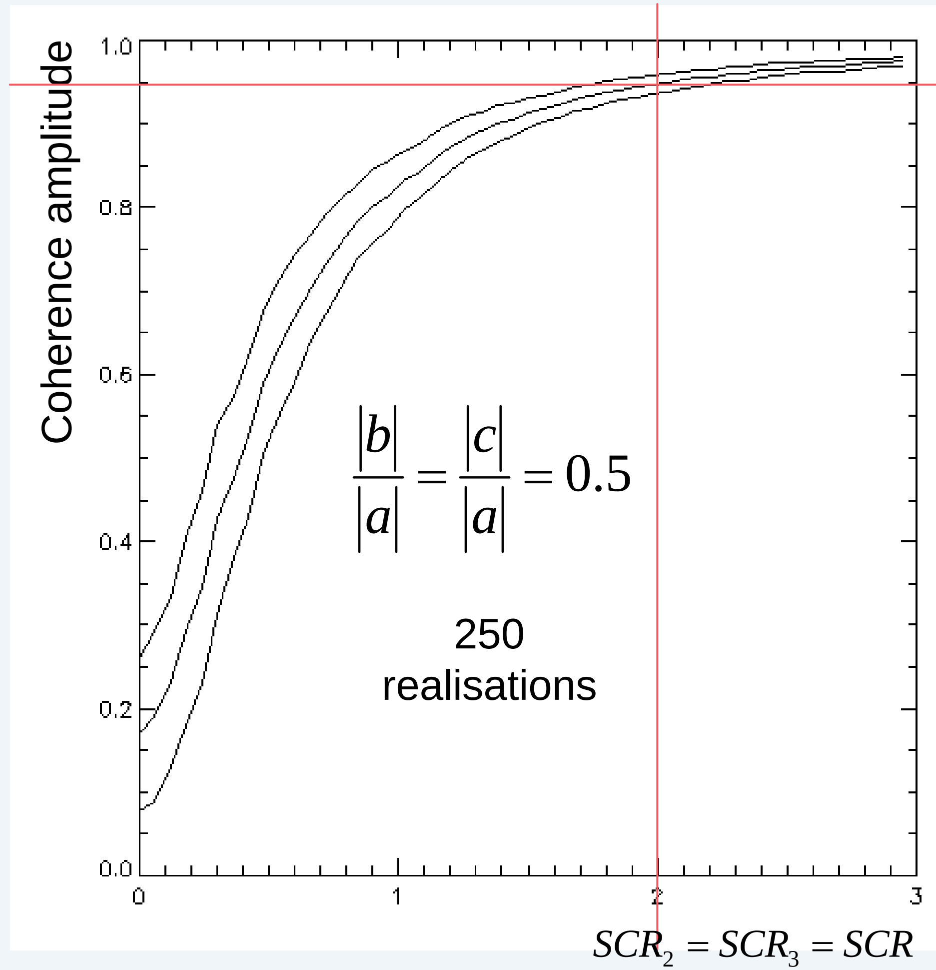
edinburgh
earth
observatory

Coherence: random variable

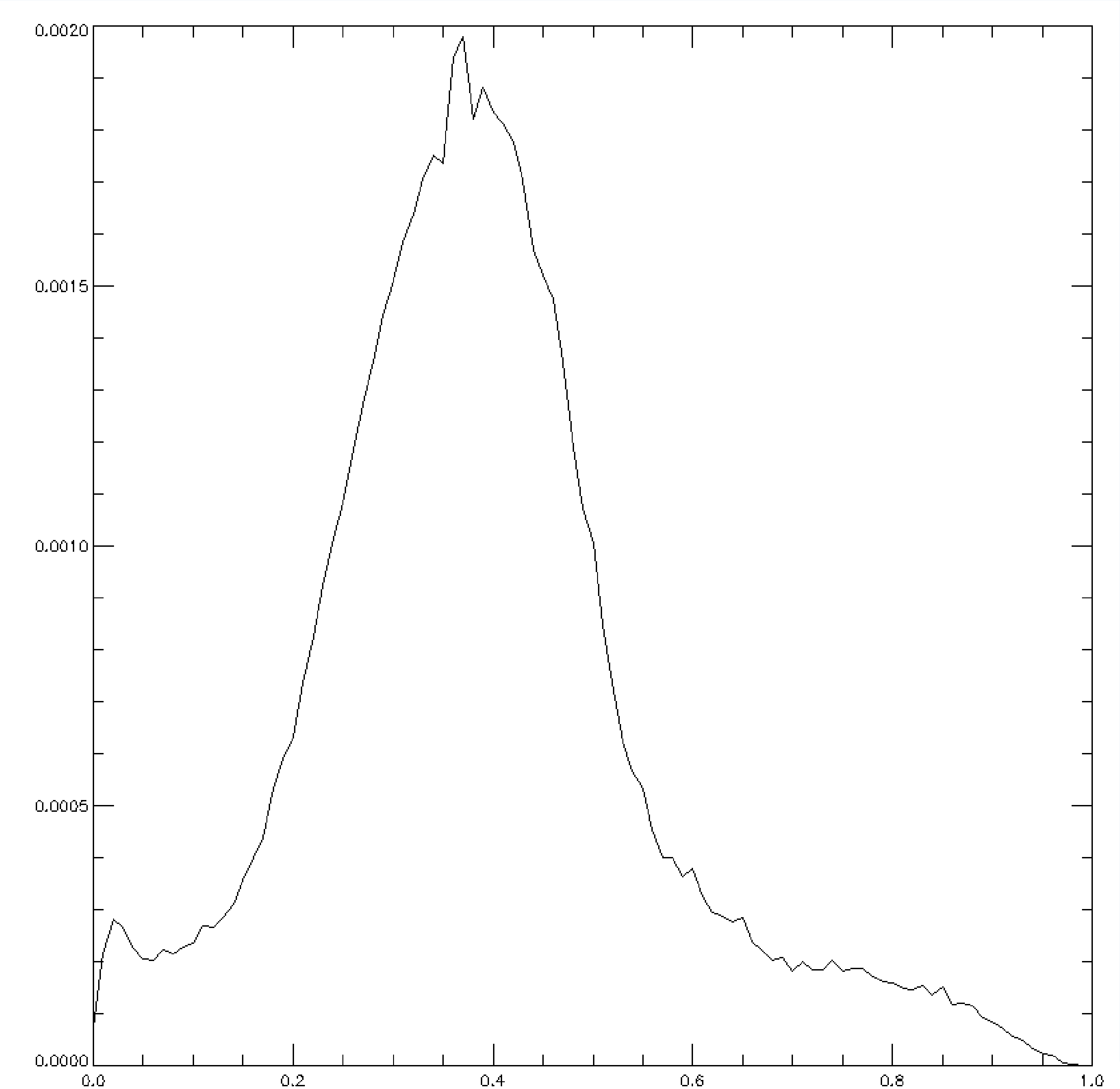




5x5



Entropy estimation



edinburgh earth observatory