ABSTRACT

Modelling of a cloud of cylinders can be interesting within the framework of SAR images of natural media: forest, crops, and so on. Similar types of models and descriptions are used in optics and light scattering for optical media such as skin or colloidal solutions. In this paper, several polarimetric parameters are investigated on this type of targets. These parameters are dedicated to the study of natural media, such as the ones obtained by the Cloude-Pottier decomposition, or given by the Lu and Chipman decomposition which is commonly used in optical polarimetry. The first one has been developed for the monostatic case, whereas the second one is more dedicated to forward scattering.

In order to be able to understand and to compare them, these both decompositions are tested for the range of bistatic angles varying from 0 (backscattering case) to π (forward scattering case). In order to control the parameters of the target, we have proposed a simple geometrical model of a cloud of cylinders with a given orientation distribution. This approach is validated both comparing it to the fields modelled by COSMO (infinite cylinder approximation) and by a comparison to the indoor measurement of a group of cylinders.

We are thus able to study the variation of bistatic polarimetric parameters. Among others, it is pointed out that the Lu and Chipman decomposition is correct for the forward direction (bistatic angle higher than $\frac{\pi}{2}$) but need to be adapted for backscattering configuration (or bistatic angle lower than $\frac{\pi}{2}$). Difficulties in the interpretation of alpha in the bistatic context are also discussed.

1 INTRODUCTION

This work is in the spirit of [2] and [6], which deals with polarimetric tools usually dedicated to optics and applied to SAR images. All the polarimetric parameters used in optics can be calculated and applied in radar imaging, in the case of the natural targets. So it seemed natural to study at first theoretically a simple target, in order to improve our understanding of bistatic polarimetry. A set of cylinders is ideal to study the combination of several mechanisms, because a cylinder is an canonical element representing a diattenuator (different attenuations on the eigenpolarizations), and the random volume model is commonly used to describe natural media (forest in radar, skin cells in optics, etc.) In this paper, we propose a simple modelling of a cloud of cylinders with a given distribution of orientations. To achieve the maximum of literal calculations, we chose a truncated uniform angular distribution. For simplicity reasons, we consider the case of the normal incidence, and of an azimuthal bistatic angle, varying from the monostatic configuration to the forward scattering configuration. The first case is well known in radar, whereas the second case is frequently met in optics. This configuration allows to study how the various polarimetric parameters evolve continuously from backscattering to forward scattering. This geometry is represented on figure 1.

In order to propose a model simple enough to be literally handled, we made the restrictive hypothesis of a cylinder whose radius is small compared to the wavelength. In the following section, we propose a simple model for the distribution of such a cloud of cylinders. In sections 3 and 4, we study the results of two classical polarimetric decompositions for the natural targets:
the Cloude Pottier decomposition which is traditionally used in radar [1], and the Lu and Chipman decomposition [3], which is traditionally devoted to optical polarimetry.

The section 3 is interested in both particular cases of the backscattering and the forward scattering which can be literally solved, whereas the section 4 deals with a configuration with a bistatic angle $\varphi$. In this last case, some of the decomposition results are numeric.

Sections 5 and 6 are dedicated to the validation of the model: the section 5 compares the resulting simulated RCS and polarimetric parameters to those simulated by a more sophisticated model based on the infinite cylinder approximation (COSMO), whereas section 6 compares the results to the measurement performed in an anechoic chamber on a set of nails representing cylinders.

The last section will allow us to summarize the theoretical deductions brought by this model and to consider following theoretical studies.

2 PROPOSITION OF A SIMPLE GEOMETRICAL MODELLING

In this section, we introduce a simple model which enables to compute the polarimetric parameters according to the bistatic angle, for the target which has been described above. This model relies on the main assumption that the radius of the cylinder is very small compared to the wavelength. In this case:

- We suppose that the bistatic return from a vertical cylinder is uniform in the transverse horizontal plane.

- In order to take into account a rotation of the cylinder, we first express the modification which affects the polarization axis between the incidence plane and the diffusion plane. This transformation implies a factor $\cos \varphi$ between the horizontal vector of the incident wave $H_0$ and the horizontal vector defined for the scattered wave $H$ (Figure 2). We apply this projection to the Sinclair matrix of the cylinder obtained after rotation in the monostatic case.

- We take into account an energetic factor $\sqrt{1 - \sin^2 \theta \sin^2 \varphi}$ which is representative for the projection of the section of the cylinder in wave plane of the scattered wave.

This leads to the equation:

$$S = \sqrt{1 - \sin^2 \theta \sin^2 \varphi} \mathbf{P}_\varphi \mathbf{R}_\theta \begin{pmatrix} a \\ 0 \\ b \end{pmatrix} \mathbf{R}_\theta^T \quad (1)$$

where $\mathbf{R}_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ is the rotation matrix with the angle $\theta$ in the incident wave plane, $\begin{pmatrix} a \\ 0 \\ b \end{pmatrix}$ is the Sinclair matrix obtained for the vertical cylinder measured in the monostatic configuration, and $\mathbf{P}_\varphi = \begin{pmatrix} \cos \varphi & 0 \\ 0 & 1 \end{pmatrix}$ is a projection matrix which takes into account the transformation of the horizontal vector of the scattered wave plane, according to the horizontal vector $H_0$ of the incident wave plane, as described on figure 2.

The four elements of the Sinclair matrix can then be expressed in terms of $\theta$, and $\varphi$, the bistatic angle. They are represented on the figure 3.
3 ANALYSIS OF POLARIMETRIC DECOMPOSITIONS FOR THE BACKSCATTERING CASE AND FORWARD SCATTERING CASE

From the previous expression, we can easily express the coherence matrix of a cylinder directed by an angle $\theta$. Then, we consider an angular distribution centered around $\theta_0$ and uniform with an amplitude $\Delta\theta$.

Then we let:

$$P(\theta) = \begin{cases} \frac{1}{2\pi} & \text{if } \theta \in [\theta_0 - \frac{\Delta\theta}{2}, \theta_0 + \frac{\Delta\theta}{2}] \\ 0 & \text{otherwise} \end{cases}$$

The calculation of the matrix of coherence for the whole cloud is made in an incoherent way by supposing a random distribution of the positions of cylinders. Attenuation is not taken into account in this study. For the integration of the chosen angular distribution, we use the following formulae of integration:

$$\int_{-\pi}^{\pi} P(\theta) \cos(2\theta) d\theta = \cos(2\theta_0) \sin(\Delta\theta)$$

$$\int_{-\pi}^{\pi} P(\theta) \sin(2\theta) d\theta = \sin(2\theta_0) \sin(\Delta\theta)$$

These formulae allow us to find the coherence matrices written at the end of this article. We present the solutions of the matrix eigendecomposition for two particular cases: the backscattering case and the forward scattering case.

3.1 The Backscattering case

In the backscattering case, it is possible to show that eigenvalues and eigenvectors can be expressed following:

$$\lambda_0 = 0 \quad v_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda_1 = \frac{(a - b)^2}{2} (\Delta\theta - \frac{\sin(2\Delta\theta)}{2}) \quad v_1 = \begin{pmatrix} 0 \\ \sin(2\theta_0) \\ \cos(2\theta_0) \end{pmatrix}$$

$$\lambda_2 = \frac{\text{tr}(T') - \lambda_1 + \sqrt{(\text{tr}(T') - \lambda_1)^2 - 4 \det(T')}}{2} \quad v_2 = \begin{pmatrix} f(\Delta\theta) \\ g(\Delta\theta) \cos(2\theta_0) \\ -g(\Delta\theta) \sin(2\theta_0) \end{pmatrix}$$

$$\lambda_3 = \frac{\text{tr}(T') - \lambda_1 - \sqrt{(\text{tr}(T') - \lambda_1)^2 - 4 \det(T')}}{2} \quad v_3 = \begin{pmatrix} h(\Delta\theta) \\ k(\Delta\theta) \cos(2\theta_0) \\ -k(\Delta\theta) \sin(2\theta_0) \end{pmatrix}$$

3.2 The forward scattering case

The forward scattering case can be deduced from the backscattering case only using line and columns permutation. More precisely, the coherency matrix can be obtained using:

$$T_f = \Omega \quad T_b \quad \Omega^\dagger$$

$$\Omega = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j \\ 0 & 0 & j & 0 \end{pmatrix}$$

This matrix corresponds to the same eigenvalues than in the backscattering case, and eigenvectors can be obtained thanks to $\Omega$:

$$v_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$v_1 = \begin{pmatrix} \sin(2\theta_0) \\ 0 \\ j \cos(2\theta_0) \end{pmatrix}$$

$$v_2 = \begin{pmatrix} g(\Delta\theta) \cos(2\theta_0) \\ f(\Delta\theta) \\ 0 \\ -jg(\Delta\theta) \sin(2\theta_0) \end{pmatrix}$$

$$v_3 = \begin{pmatrix} h(\Delta\theta) \cos(2\theta_0) \\ k(\Delta\theta) \\ 0 \\ -k(\Delta\theta) \sin(2\theta_0) \end{pmatrix}$$

This means that all polarimetric parameters depending only on the eigenvalues (entropy, anisotropy...) are the same in the backscattering case and in the forward scattering case, whereas parameters based on eigenvectors are not (for example alpha).

4 ANALYSIS OF POLARIMETRIC DECOMPOSITIONS FOR THE GENERAL BISTATIC CASE

In this section we consider two classical polarimetric decompositions used for natural targets: The Cloude Pottier decomposition and the Lu and Chipman decomposition. The purpose is to see how the bistatic angle can influence the deduced parameters.
4.1 Expression of the coherency matrix

It is quite convenient to introduce these two parameters:

\[
D_\varphi = \frac{1 - \cos(\varphi)}{1 + \cos(\varphi)} \tag{13}
\]

and

\[
D_{a,b} = K = \frac{a - b}{a + b} \tag{14}
\]

which can be interpreted as two diattenuations, the first one coming from the bistatic transformation of the polarimetric incident basis, and the second one being the diattenuation of the cylinder. This last one has also been called "anisotropy" of the particle. With these parameters, the polarimetric scattering vector of one cylinder can be written following:

\[
\vec{V} = \begin{pmatrix} A \\ -B \\ -C \\ iCD_\varphi \end{pmatrix} \tag{15}
\]

where \( A = 1 - D_{a,b}D_\varphi \cos(\theta_0) \), \( B = D_\varphi - D_{a,b} \cos(\theta_0) \), and \( C = D_{a,b} \sin(\theta_0) \).

And the corresponding coherency matrix \( T = \vec{V}\vec{V}^\dagger \) is:

\[
T = \begin{pmatrix}
A^2 & -AB & -AC & -iD_\varphi AC \\
-AB & B^2 & BC & +iD_\varphi BC \\
-AC & BC & C^2 & +iD_\varphi C^2 \\
iD_\varphi AC & -iD_\varphi BC & -iD_\varphi C^2 & D_\varphi^2 C^2
\end{pmatrix} \tag{16}
\]

It would be also possible to express the coherency matrix of a cloud of such cylinders, by taking into account the integration over the truncated uniform distribution of angle, but this leads to a matrix that we did not manage to diagonalize literally. The following decompositions are then performed numerically.

4.2 Cloude-Pottier parameters

First we consider the case of \( \theta_0 = 0^\circ \), and \( \Delta \theta = 10^\circ \). This corresponds to the case of a volume which is almost vertically oriented. In the bistatic case, \( H \) and \( \alpha \) can be easily extended.

As expected, entropy \( H \) remains closed to 0 whereas \( \alpha \) is closed to \( \frac{\pi}{2} \). (\( H \approx 0.02 \) and \( \alpha \approx 45.2^\circ \))

For an intermediate angular distribution \( \Delta \theta = 45^\circ \), we now observe \( H \) varying between 0.15 and 0.25, and alpha angle lies between 47\(^\circ\) and 51\(^\circ\).

Finally, we consider the case of the "random volume", described using \( (\Delta \theta = 180^\circ) \)

\( \alpha \) evolves from 46\(^\circ\) to 67\(^\circ\) whereas \( H \) lies between 0.35 and 0.65, being minimum around \( \varphi = \frac{\pi}{2} \).

All these cases illustrate that there is a strong variation of \( H \) and \( \alpha \) when \( \varphi \) is varying. Moreover, \( \alpha \) is not symmetric on variable \( \varphi \). This can be explained...
by our model: the contribution of bistatic angle $\varphi$ can be expressed as an orthogonal change of basis of the coherency matrix $T$. In this case, eigenvalues (then $H$) are the same ones, but eigenvectors are different.

4.3 Lu-Chipman parameters

The three precedent cases are also investigated in the light of the Lu and Chipman decomposition. We summarized the basic ideas of this decomposition, omitting details that can be found in the reference [3]. This algorithm decomposes a Mueller matrix into a sequence of three matrix factors: a diattenuator followed by a retarder then followed by a depolarizer. For each Mueller matrix the three following parameters can be obtained: retardance $R$, diattenuation $D$ and depolarization $\delta$.

- **the diattenuator** only alters the amplitudes of the component of the electric field. Any diagonal and real Sinclair matrix corresponds to diattenuator, for example the Sinclair matrix of a canonical metallic cylinder.

- **the retarder**’s effect upon polarization states is equivalent to a rotation on the Poincaré sphere. It has a constant intensity transmittance independent of the incident polarization state, but causes different phase changes for its eigenpolarizations.

- **the depolarizer** can transform the input polarized or partial polarized light into less polarized light. A random volume is a pure depolarizer.

The results of this decomposition are tested on the three precedent cases: $\Delta \theta = 10^\circ$ on figure 7, $\Delta \theta = 45^\circ$ on figure 4.3 and $\Delta \theta = 90^\circ$ on figure 4.3. On these figures, it appears that Lu and Chipman decomposition presents at least two problems in the general bistatic case:

- First, the extracted parameters are all dependant on $\varphi$, specially depolarisation in the case of a vertically oriented volume.

- Second, retardance suffers from a discontinuity around $\varphi = \frac{\pi}{2}$, which is due to the extraction formulae which is proposed in the forward scattering case, and is not adapted for the cases $\varphi < \frac{\pi}{2}$.

These problems arise principally because the matrix multiplication is noncommutative, and then the order of the elements proposed in the decomposition is influencing the obtained results. Matrix multiplication being noncommutative, the three factor matrix decomposition from Lu and Chipman can result in six products of $M_D$, $M_R$ and $M_\Delta$. [4] as well as [5] have shown that they can be grouped into two families, each having a canonical form called ”forward” (Eq. 17) or ”reverse” (Eq. 18) decomposition.
\[ M = M_\Delta M_R M_D \]  

(17)

\[ M = M_D M_R M_\Delta \]  

(18)

Both the forward and the reverse decompositions can be successfully applied if the observed medium is homogeneous and if there is no specific order in which the phenomena such as depolarization occurs.

In the previous part of our work we have been using the forward canonical form, but our results did not meet our expectations: according to our model, a bistatic setting should have an effect mainly on the diattenuator (and thus \( M_D \)). Our Sinclair matrix written in Eq. 1 is indeed a canonical diattenuator but the forward decomposition lead to an unforeseen behavior for the depolarizer and the retarder that both vary with the bistatic angle \( \varphi \).

![Figure 10: Result of the Lu and Chipman reverse decomposition on our simulation](image)

We applied the reverse decomposition on our simulation, as described in [5] and the results were much closer from what we initially expected. The variation of the depolarizer vanished, the bistatic angle variation being isolated in the diattenuator as expected. This is very interesting, if our model is strong enough to be used in less specific cases, the reverse Lu and Chipman decomposition may be a very useful tool to remove the bistatic components from a Mueller Matrix. While the Cloude Pottier decomposition may be unable to extract the bistatic angle influence and suffers from the variation of \( H \) and \( \alpha \).

5 COMPARISON TO COSMO

5.1 Description of COSMO

"COSMO" models the vegetation as a set of discrete cylinders representing tree components. Electric fields scattered by each element are added coherently and attenuation can be computed using Foldy-Lax approximation. COSMO has been widely tested and validated in its monostatic version [7], and the fields scattered by a single dielectric cylinder have also been validated in the bistatic one. The fields scattered by a single cylinder are simulated using the infinite cylinder approximation. In order to validate our simplified model using COSMO, two steps are performed:

- first we compare the absolute values of the Sinclair matrix components for all possible inclinations \( \theta \) and all bistatic angles \( \varphi \).
- Then we compare results of two classical polarimetric decompositions on a set of cylinders with a given angular distribution: the Cloude Pottier decomposition and the Lu and Chipman decomposition. In this case we do not take into account attenuation, and we just add coherently the fields scattered by a set of cylinders with this angular distribution.

5.2 Comparisons

The result of the simulation of the RCS in terms of \( \varphi \) and \( \theta \) is shown on figure 11 and can be compared to previous figure 3. Results match very well. It shows that dependency of the cylinder RCS on these both parameters is very well reproduced.

![Figure 11: Representation of \( |H_h|, |H_v|, |V_h| \) and \( |V_v| \) simulated by our simplified model in terms of bistatic angle \( \varphi \) and cylinder orientation \( \theta \)](image)

Then the resulting Lu and Chipman parameters against the bistatic angle are compared on figure 12. Once again, the comparison between COSMO simulation and our simplified model on figure7 is very good, except concerning the retardance around \( \varphi \): this can be explained by the fact that COSMO considers only dielectric cylinders and no metallic cylinders.

The resulting Cloude-Pottier parameters simulated by COSMO are very close to the parameters simulated by our model.
6 COMPARISON TO INDOOR MEASUREMENT

6.1 Description of Measurement

BABI is an indoor coherent bistatic RCS measurement facility in ONERA, The French Aerospace Lab. The frequency range of operation can extend from 600 MHz to 40 GHz. It allows to acquire the coefficient of complex bistatic reflection (amplitude and phase) between 4 and 196 degrees for targets whose dimensions are of the order of one meter.

Full polarimetric bistatic acquisitions are made from 1 GHz to 3 GHz with a constant step of 15 MHz.

Emission and Reception antenna are carried on trolleys. These trolleys are pulled by engines which assure a reproducibility of location of antennas better than $10^{-2}$ degree. There move on a circular rail of radius 5.5m. Targets are mounted on a turning polyfoam column.

Actually, pure backscattering and forward direction configurations are not practically available: because of the isolation required between both emission and reception antennas, the pure monostatic case could not be achieved and consequently no measurement has been collected for bistatic angles lower than $4^\circ$. In addition, for the pure specular case, the measurements are noised by the strong response of the transmitter, as it is aligned with the receiver.

The target under study is a set of metallic nails 7 cms high and 3 mm in diameter. These nails are driven in a polystyrene plate following an angular distribution with an amplitude of $10^\circ$ around the vertical. This target is pictured on figure 13.

6.2 Comparisons

Figure 14 gives the resulting Lu and Chipman parameters computed on data acquired on the chamber, whereas figure 15 gives the resulting Cloude-Pottier parameters on the same target. These figures can be compared to figures 7 and 4.2. They show the ability of our simplified model to explain the trends of polarimetric parameters variations with the bistatic angle.

7 CONCLUSION

The simplified modelling of a cloud of cylinders with a truncated uniform angular distribution has raised some problems on the use of classical polarimetric decompositions in the bistatic configuration:

- For the Cloude Pottier decomposition, there is a variation of both alpha and entropy with the bistatic angle $\varphi$ which may lead to errors if the
parameters are used in classification following the usual methods. The interpretation of $\alpha$ angle is not meaningful in a general bistatic configuration as alpha isn’t even symmetric around $\frac{\pi}{2}$.

- For the Lu and Chipman decomposition: Diattenuation and Depolarization vary significantly for $\varphi$ values close from $\frac{\pi}{2}$. There exist a strong retardance discontinuity for $\varphi = \frac{\pi}{2}$.

This paper has also pointed out that applying polarimetric tools dedicated to optics can also be interesting in the context of SAR polarimetry, and specially in a bistatic setting. In the future we are planning to deepen our analysis using optical polarimetric data.

REFERENCES


ANNEX

Here are given the different expressions of coherency matrix in terms of $\theta_0$ and $\Delta \theta$, the model parameters:
Backscattering case:

\[
T = \begin{pmatrix}
1 & D_{a,b} \cos(2\theta_0) \sin_c(\Delta \varphi) & -D_{a,b} \sin(2\theta_0) \sin_c(\Delta \varphi) & 0 \\
D_{a,b} \cos(2\theta_0) \sin_c(\Delta \varphi) & D_{a,b}^2 \left(1 + \cos(4\theta_0) \sin_c(2\Delta \varphi)\right) & -D_{a,b}^2 \sin(4\theta_0) \sin_c(2\Delta \varphi) & 0 \\
-D_{a,b} \sin(2\theta_0) \sin_c(\Delta \varphi) & -D_{a,b}^2 \sin(4\theta_0) \sin_c(2\Delta \varphi) & D_{a,b}^2 \left(1 - \cos(4\theta_0) \sin_c(2\Delta \varphi)\right) & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (19)

Forward scattering case:

\[
T = \begin{pmatrix}
1 & D_{a,b} \cos(2\theta_0) \sin_c(\Delta \varphi) & 0 & iD_{a,b} \sin(2\theta_0) \sin_c(\Delta \varphi) \\
D_{a,b} \cos(2\theta_0) \sin_c(\Delta \varphi) & D_{a,b}^2 \left(1 + \cos(4\theta_0) \sin_c(2\Delta \varphi)\right) & 0 & iD_{a,b}^2 \sin(4\theta_0) \sin_c(2\Delta \varphi) \\
0 & 0 & 0 & 0 \\
-iD_{a,b} \sin(2\theta_0) \sin_c(\Delta \varphi) & -iD_{a,b}^2 \sin(4\theta_0) \sin_c(2\Delta \varphi) & 0 & D_{a,b}^2 \left(1 - \cos(4\theta_0) \sin_c(2\Delta \varphi)\right)
\end{pmatrix}
\] (20)