CALIBRATION OF PALSAR POLARIMETRIC SAR DATA

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• **In this presentation, we will:**
  – Describe a new approach to estimate Faraday rotation from Polarimetric SAR data
  – Method takes full account of all system distortions and is based on analysis of cross-talk terms
  – Show results are comparable with other techniques, incl. GPS
  – Confirm using PalSAR data that with enough FR, new approach allows for full polarimetric calibration without corner reflectors
  – Establish the system cross-talk for PalSAR to be ~ -40 dB
  – Compare FR estimates using the new approach with ground-based GPS measurements, including a correction for vertical TEC variations using the International Reference Ionosphere
  – Introduce a 3-D model of the ionosphere that assimilates ground-based and space-based GPS measurements (e.g COSMIC, CHAMP, GRACE)
• Model predictions of FR based on TEC, magnetic field

\[ \Omega = \frac{K}{f^2} \int_0^h NB \cos \psi \sec \theta_0 \, dh \]

Mean Faraday Rotation at L-Band, April, GMT = 12:00

Moderate Sunspot activity
R=20

High Sunspot activity
R=160

• Proposed pre-rotation of transmitted wave to adjust for expected FR
• Note \( \Omega = 0 \) crossing at Equator
• Introduced at ASAR 2003
• System Model:

\[
\begin{pmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{pmatrix}
= A(r, \theta) e^{j\phi}
\begin{pmatrix}
1 & \delta_2 \\
\delta_1 & f_1
\end{pmatrix}
\begin{pmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
S_{hh} & S_{vh} \\
S_{hv} & S_{vv}
\end{pmatrix}
\begin{pmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
1 & \delta_3 \\
\delta_4 & f_2
\end{pmatrix}
+ \begin{pmatrix}
N_{hh} & N_{vh} \\
N_{hv} & N_{vv}
\end{pmatrix}
\]

\[M = A e^{\phi} R^T R_F S_F T + N\]

• Assume cross-talk is negligible or stable enough to allow routine correction using pre-determined values =>

\[
\begin{pmatrix}
M'_{hh} & M'_{vh} \\
M'_{hv} & M'_{vv}
\end{pmatrix}
= A(r, \theta) e^{j\phi}
\begin{pmatrix}
1 & 0 \\
0 & f_1
\end{pmatrix}
\begin{pmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
S_{hh} & S_{vh} \\
S_{hv} & S_{vv}
\end{pmatrix}
\begin{pmatrix}
\cos \Omega & \sin \Omega \\
-\sin \Omega & \cos \Omega
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & f_2
\end{pmatrix}
+ \begin{pmatrix}
N_{hh} & N_{vh} \\
N_{hv} & N_{vv}
\end{pmatrix}
\]

• Balance channels using approach described in (Freeman, 2004) and estimate Faraday Rotation (FR) using (e.g. Bickel and Bates, 1965):

\[
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
= \begin{bmatrix}
1 & j \\
j & 1
\end{bmatrix}
\begin{bmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{bmatrix}
\begin{bmatrix}
1 & j \\
j & 1
\end{bmatrix}
\]

\[
\Omega = \frac{1}{4} \arg \left( Z_{12} Z_{21}^* \right)
\]

• Use any target with non-zero HV to resolve ambiguities in FR
‘Quegan method’: $\Omega=0$, $\delta$'s are small

- System Model (no Faraday Rotation):
  \[
  \begin{pmatrix}
  M_{hh} & M_{vh} \\
  M_{hv} & M_{vv}
  \end{pmatrix} = A(r, \theta) e^{i \phi} \begin{pmatrix}
  1 & \delta_2 / f_1 \\
  \delta_1 & 1
  \end{pmatrix} \begin{pmatrix}
  1 & 0 \\
  f_1 & S_{hh} & S_{vh} & 1 & 0
  \end{pmatrix} \begin{pmatrix}
  1 & \delta_3 \\
  \delta_4 / f_1 & f_2 / f_1
  \end{pmatrix} + \begin{pmatrix}
  N_{hh} & N_{vh} \\
  N_{hv} & N_{vv}
  \end{pmatrix}
  \]

- Assumptions are*:
  \[|\delta_i|^2 \ll 1, \text{ for } i = 1 \text{ to } 4; \ |f_1| \sim 1; \ |f_2| \sim 1; \]

  \[S_{hv} = S_{vh}; \ \langle S_{hh} S_{hv}^* \rangle = \langle S_{vh} S_{hv}^* \rangle = 0; \ \rho_{hhvv^*} \neq 1; \ \langle S_{hv} S_{hv}^* \rangle \neq 0 \]

- Solve for five system parameters: $f_1 / f_2$, $\delta_1$, $\delta_2 / f_1$, $\delta_3$, $\delta_4 / f_1$

- Additional information is needed to solve for $f_1$ and $f_2$ to complete the relative calibration by defining each term in the distortion matrix

- This is often obtained from the signature of an external target with known HH/VV ratio such as a corner reflector (but it can also be obtained from receive channel cal-signals)

*Similar approaches are described in Klein and Freeman (1991) and Freeman et al (1992)
Moriyama et al’s PalSAR results

Comparison between Rio Branco and Tomakomai (2/3)

• Cross-talk

**Significance:** System cross-talk is established to be < -35 dB (evaluated when/where $\Omega \sim 0$)

Courtesy Prof. T. Moriyama, Nagasaki U.
Moriyama et al’s PalSAR results

Comparison between Rio Branco and Tomakomai (3/3)

- Faraday rotation angle estimated by Freeman method

Significance: i) FR is close to zero near equator (as predicted); ii) Even small (1-2 deg) variations in $\Omega$ can be detected
Case I: $\Omega$, $\delta$'s are small

- Start with the full System Model:

$$
\begin{pmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{pmatrix} = A(r, \theta) e^{j\phi} \begin{pmatrix} 1 & \delta_2 \\ \delta_1 & f_1 \end{pmatrix} \begin{pmatrix} \cos \Omega & \sin \Omega \\ -\sin \Omega & \cos \Omega \end{pmatrix} S_{hh} S_{vh} \begin{pmatrix} 1 & \delta_3 \\ \delta_4 & f_2 \end{pmatrix} + \begin{pmatrix} N_{hh} \\ N_{hv} \end{pmatrix}
$$

- When $\Omega$ is small, $\cos \Omega \approx 1$, $\sin \Omega \approx \Omega$

- If $\delta$'s are also small, we can neglect $(\delta \Omega)$ product terms, so:

$$
\begin{pmatrix} 1 & \delta_2 \\ \delta_1 & f_1 \end{pmatrix} \begin{pmatrix} \cos \Omega & \sin \Omega \\ -\sin \Omega & \cos \Omega \end{pmatrix} \approx \begin{pmatrix} 1 & \Omega \\ \delta_1 - \Omega f_1 & f_1 \end{pmatrix}
$$

- This is identical in form to Quegan's method which should yield:

$$
F = f_1 / f_2; \quad \delta'_1 = \delta_1 - \Omega f_1; \quad \delta'_2 = (\Omega + \delta_2) / f_1
$$

- There are 7 unknowns, with only five equations, so additional information is needed to complete the calibration.

$$
\delta'_3 = \delta_3 + \Omega f_2; \quad \delta'_4 = (\delta_4 - \Omega) / f_2
$$

[Similarly for the right-hand side]
Case Ia: $\Omega$ is small and $\delta$’s $\Rightarrow 0$

For small $\Omega$, $\delta$’s system model can be expressed as:

$$\begin{pmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{pmatrix} = A(r, \theta) e^{j\phi} \begin{pmatrix} 1 & \Omega + \delta_2 \\ \delta_1 - \Omega f_1 & f_1 \end{pmatrix} \begin{pmatrix} S_{hh} & S_{vh} \\ S_{hv} & S_{vv} \end{pmatrix} \begin{pmatrix} 1 & \delta_4 - \Omega \\ \delta_3 + \Omega f_2 \end{pmatrix} + \begin{pmatrix} N_{hh} & N_{vh} \\ N_{hv} & N_{vv} \end{pmatrix}$$

Now let $\delta$’s $\Rightarrow 0$:

$$\begin{pmatrix} M_{hh} & M_{vh} \\ M_{hv} & M_{vv} \end{pmatrix} = A(r, \theta) e^{j\phi} \begin{pmatrix} 1 & \Omega / f_1 \\ -\Omega f_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & f_1 \end{pmatrix} \begin{pmatrix} S_{hh} & S_{vh} \\ S_{hv} & S_{vv} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ f_1 & -\Omega / f_1 \end{pmatrix} + \begin{pmatrix} N_{hh} & N_{vh} \\ N_{hv} & N_{vv} \end{pmatrix}$$

Identical in form to Quegan’s method with:

$$\delta_1 = -\Omega f_1; \quad \delta_2 = \Omega / f_1; \quad \delta_3 = \Omega f_2; \quad \delta_4 = -\Omega / f_2$$

Therefore, for small $\Omega$, and zero $\delta$’s,

- explains Moriyama et al’s results
- Quegan approach confuses $\Omega$ with $\delta$’s
- can solve these expressions for $f_1$, $f_2$, $\Omega$

This will not be the case near solar max, when $\Omega$ is expected to increase.

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$\delta$ Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5°</td>
<td>-41 dB</td>
</tr>
<tr>
<td>1°</td>
<td>-35 dB</td>
</tr>
<tr>
<td>2°</td>
<td>-29 dB</td>
</tr>
<tr>
<td>3°</td>
<td>-25 dB</td>
</tr>
<tr>
<td>6°</td>
<td>-20 dB</td>
</tr>
<tr>
<td>10°</td>
<td>-15 dB</td>
</tr>
</tbody>
</table>
For small $\Omega$, $\delta$’s system model can be expressed as:

\[
\begin{pmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{pmatrix} = A(r, \vartheta) \ e^{j(\Theta + \delta)}
\begin{pmatrix}
1 & \frac{(\Omega + \delta_2)}{f_1} \\
\vartheta & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & S_{hh} & S_{vh} \\
0 & f_1 & S_{hv} & S_{vv}
\end{pmatrix}
\begin{pmatrix}
1 & 0 & \frac{\delta_3 + \Omega f_2}{f_1} \\
\frac{\delta_4 - \Omega}{f_1} & f_2 / f_1
\end{pmatrix} +
\begin{pmatrix}
N_{hh} & N_{vh} \\
N_{hv} & N_{vv}
\end{pmatrix}
\]

which is identical in form to Quegan’s method, an application of which should yield:

\[
F = f_1 / f_2; \quad \delta_1 = \delta_1 - \Omega f_1; \quad \delta_2 = \frac{(\Omega + \delta_2)}{f_1}
\]

\[
\delta_3 = \delta_3 + \Omega f_2; \quad \delta_4 = \frac{(\delta_4 - \Omega)}{f_1}
\]

For radar antennas whose cross-talk is symmetric on transmit and receive*, i.e.

\[
\delta_1 = \delta_3 \quad \text{and} \quad \delta_2 = \delta_4
\]

Thus

\[
f_1 = \sqrt{\frac{(\delta_3 - \delta_2)}{(\frac{\delta_3 - \delta_4}{F})}} \frac{F}{F+1}; \quad \Omega = \frac{f_1}{2} (\frac{\delta_2 - \delta_4}{F}); \quad f_2 = \frac{f_1}{F};
\]

\[
\delta_1 = \frac{[\delta_1 + \delta_3 - \Omega (f_2 - f_1)]}{2}; \quad \delta_2 = \frac{f_1 (\delta_2 + \delta_4)}{2}
\]

Only restriction is that $\Omega \neq 0$ and $f_2 \neq -f_1$.

*In monostatic radar systems, all passive and most active antennas are reciprocal.

=> Can solve for all five system distortion terms $f_1, f_2, \Omega, \delta_1$ and $\delta_2$, without additional information from an external target!
HH mag.  
(Note that the sign of derived FR is flipped to compare with the result from another group.)

FR: histogram (top) and mean values vs. range (middle) and azimuth (bottom) distance

May 14, 2008  
Pi: Ionospheric Specifications Using SAR and GPS
Case II: $\Omega$, $\delta$'s are small and cross-talk is symmetric

- **Results - from PalSAR data**

<table>
<thead>
<tr>
<th></th>
<th>Washington, DC (01/06/07)</th>
<th>Comparison ([1] and [2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>f_1</td>
<td>_{}$</td>
</tr>
<tr>
<td>$\text{Arg}(f_1)$</td>
<td>24.2°</td>
<td>-1.9°</td>
</tr>
<tr>
<td>$</td>
<td>f_2</td>
<td>_{}$</td>
</tr>
<tr>
<td>$\text{Arg}(f_2)$</td>
<td>0.9°</td>
<td>21.8°</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>2.6°</td>
<td>4.0°</td>
</tr>
<tr>
<td>$</td>
<td>\delta_1</td>
<td>_{}$</td>
</tr>
<tr>
<td>$</td>
<td>\delta_2</td>
<td>_{}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Atacama Desert (02/07/07)</th>
<th>Comparison ([1] and [2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>f_1</td>
<td>_{}$</td>
</tr>
<tr>
<td>$\text{Arg}(f_1)$</td>
<td>6.5°</td>
<td>1.9°</td>
</tr>
<tr>
<td>$</td>
<td>f_2</td>
<td>_{}$</td>
</tr>
<tr>
<td>$\text{Arg}(f_2)$</td>
<td>-16°</td>
<td>21.8°</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>-0.9°</td>
<td>-0.8°</td>
</tr>
<tr>
<td>$</td>
<td>\delta_1</td>
<td>_{}$</td>
</tr>
<tr>
<td>$</td>
<td>\delta_2</td>
<td>_{}$</td>
</tr>
</tbody>
</table>


[2] $\Omega$-values Derived from GPS estimates of TEC, corrected for PalSAR altitude/viewing geometry
• Results for $|f_1|/|f_2|$ are consistent with an average value of 1.5

• Results for $\text{arg}(f_1/f_2)$ are consistent with expected value ($23^\circ$)
Results

- Results for $|f_1|$, $|f_2|$ are consistent with expected values for $|FR|>1$
- Results for smaller FR angles are noisy (small signal problem)

- Results for $\arg(f_1)$ and $\arg(f_2)$ are consistent with expected values when $|FR|>1$
- Results for smaller FR angles are noisier
• Results for cross-talk are < -35 dB and are on average closer to -40 dB

• Results suggest consistent cross-talk phase for |FR|>1

• Spread is relatively small (+/- 30° and +/- 50°)
For small $\Omega$, $\delta$’s system model can be expressed as:

$$
\begin{pmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{pmatrix} = A(r, \theta) \ e^{i\phi} 
\begin{pmatrix}
1 & \Omega + \delta_2 \\
\delta_1 - \Omega f_1 & f_1
\end{pmatrix}
\begin{pmatrix}
S_{hh} & S_{vh} \\
S_{hv} & S_{vv}
\end{pmatrix}
\begin{pmatrix}
1 & \delta_3 + \Omega f_2 \\
\delta_4 - \Omega & f_2
\end{pmatrix}
+ 
\begin{pmatrix}
N_{hh} & N_{vh} \\
N_{hv} & N_{vv}
\end{pmatrix}
$$

To calibrate (ignoring additive noise and system-level amplitude and phase $A(r, \theta)$ and $e^{i\phi}$):

$$
\begin{pmatrix}
\hat{S}_{hh} & \hat{S}_{vh} \\
\hat{S}_{hv} & \hat{S}_{vv}
\end{pmatrix} = \frac{1}{\beta}
\begin{pmatrix}
f_1 & -(\Omega + \delta_2) \\
-(\delta_1 - \Omega f_1) & 1
\end{pmatrix}
\begin{pmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{pmatrix}
\begin{pmatrix}
f_2 & -(\delta_3 + \Omega f_2) \\
\delta_4 - \Omega & 1
\end{pmatrix}
$$

where $\beta = \left[f_1 - (\delta_1 - \Omega f_1)(\Omega + \delta_2)\right]\left[f_2 - (\delta_1 + \Omega f_2)(\delta_2 - \Omega)\right]$

If cross-talk can be considered negligible:

$$
\begin{pmatrix}
\hat{S}_{hh} & \hat{S}_{vh} \\
\hat{S}_{hv} & \hat{S}_{vv}
\end{pmatrix} \approx \frac{1}{\beta'}
\begin{pmatrix}
f_1 & -\Omega \\
\Omega f_1 & 1
\end{pmatrix}
\begin{pmatrix}
M_{hh} & M_{vh} \\
M_{hv} & M_{vv}
\end{pmatrix}
\begin{pmatrix}
f_2 & -\Omega f_2 \\
\Omega & 1
\end{pmatrix}
$$

where $\beta' = f_1 f_2 \left(1 + \Omega^2\right)^2$
Phase Advance or Time Delay:

$$\phi = \frac{2\pi}{c} \cdot \frac{40.3}{f} \int_{r_t}^{r_v} n_e ds$$

$$= \frac{8.45 \times 10^{-7}}{f} \text{TEC}$$

Faraday Rotation:

$$\Omega = \frac{K}{f^2} \int_{r_t}^{r_v} n_e B_0 \cos \theta ds$$

- $n_e$ – electron density
- $B_0$ – ambient mag. field
- $\theta$ – angle between $k$ and $B_0$
- $K = 2.365 \times 10^4$ (in MKS units)

Dual-Freq. Phase Combination:

$$\Phi_t = \Phi_2 - \Phi_1 = (1 - f_1^2/f_2^2) \frac{40.3}{f_1^2} \text{TEC} + \frac{c}{f_1} N_1 + \frac{c}{f_2} N_2 + \delta + \varepsilon$$

($\Phi$ in units of length)

Rate of TEC (ROT) Change:

$$\text{ROT} = \frac{\text{TEC}(t + \Delta t) - \text{TEC}(t)}{\Delta t}$$

Rate of TEC Index (ROTI):

$$\text{ROTI} = \sigma(\text{ROT})_{\Delta t}$$
December 19, 2008
Pi et al.: Imaging the Ionosphere Using SAR & GPS
• Faraday rotation is derived from polarimetric measurements made using ALOS L-band PALSAR over Alaska at 07:28 UT on 4/1/2007.

• The size of the Image scene is about ~28 x 62 km².

• 1D FR in the azimuth dir: mean FR in the range dimension is shown as a function of latitude. Noise of about 0.5 degrees is seen in the data before smoothing.

• 2D image: Features at scales < 110 x 197 m² are smoothed or removed to reduce noise.

• The tilted strip seems aligned with the ambient magnetic inclination contour.

• The FR image shows not only ionospheric gradient but also curvature within the radar scene in both azimuth and range directions.

(220.243°, 61.959°) (220.796°, 62.027°)
Magnetic Inclination

US/UK World Magnetic Chart -- Epoch 2000
Inclination - Main Field (I)

Pi et al.: Imaging the Ionosphere Using SAR & GPS (Fall AGU)
A Series of ALOS Scenes over Alaska

Using SAR & GPS
• Multiple strips of enhanced Faraday rotation aligned with magnetic inclination contours are observed in a single path over polar region by PALSAR in a polarimetric mode.
• FR structures as small as 0.1~0.2 degrees are identified after smoothing to reduce noise.
• FR Discontinuity between the images raise possible calibration (by JAXA) or processing issues.

Pi et al.: Imaging the Ionosphere Using SAR & GPS (Fall AGU)
Global Assimilative Ionospheric Model

- Time dependent
- 3D grid in a magnetic frame

- Numerically solving plasma continuity and momentum equations
- Finite volume on a fixed Eulerian grid
- Hybrid explicit-implicit time integration scheme

Multiple ions: O+, H+, He+, NO+, O2+, N+

Driving Forces → Physics Model → Obs. Operator

Kalman Filter → 4DVAR

Global and regional modeling by solving plasma hydrodynamic equations

Pi et al.: GAIM
Faraday rotation is retrieved from ALOS/PalSAR data and using the GPS-based techniques, respectively, along the ALOS orbit track.

- SAR-FR (blue) is obtained using the new calibration technique.
  - IRI is used to estimate the suborbital to total TEC ratio.
  - IGRF is used to specify magnetic field.

- GIM-TEC (black) used in FR estimation is also plotted as a reference.

- GAIM-FR (3D) estimation is also being processed.

**GPS-GIM TEC is rescaled and shifted to fit in the plot.**
Conclusions

- New approach to estimate FR gives results that are comparable with other techniques
- Technique takes full account of system distortions
- With enough FR, full polarimetric calibration without corner reflectors is possible
- Exception when $\Omega < 1^\circ$ (equivalent to a x-talk of $\sim -35$ dB), new algorithm for full calibration appears SNR-limited
- PalSAR system cross-talk is $< -35$ dB and probably $\sim -40$ dB
- FR estimates from new algorithm and GPS-based estimates corrected for vertical profile are consistent
  $$\Rightarrow$$ GPS-based FR estimates can be used as a first order correction for future spaceborne SARs