A Joint Density of Interferometric and/or Polarimetric Images: Application to Change Detection

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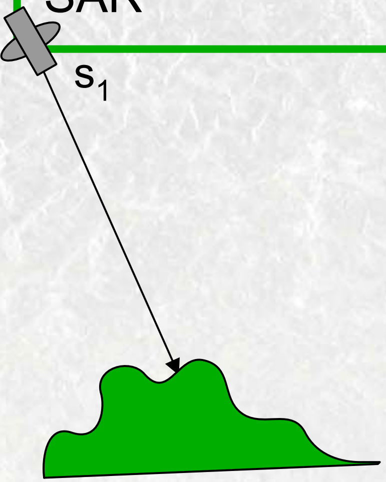
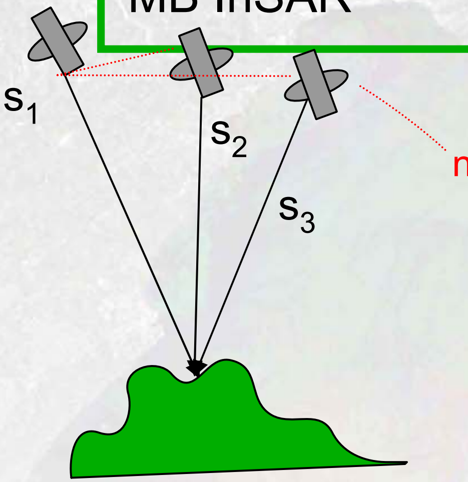
Motivation :

- Multi-channel SAR Systems Model
- $p(A_{11},A_{22})$:Definition of joint distribution of two POLSAR images
- Scalar change detection measure on Multi-channel SAR system regarding KL-divergence
- Shortly review on two well-known statistics regarding multi-channel SAR system
- Examples on real and simulated data
- Conclusion and discussion

SAR



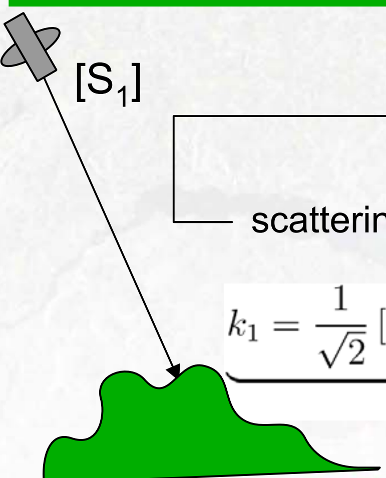
MB InSAR



Σ:= coherency matrix

$$\Sigma = \begin{bmatrix} s_1 s_1^* & s_1 s_2^* \\ s_2 s_1^* & s_2 s_2^* \end{bmatrix}$$

Pol-SAR

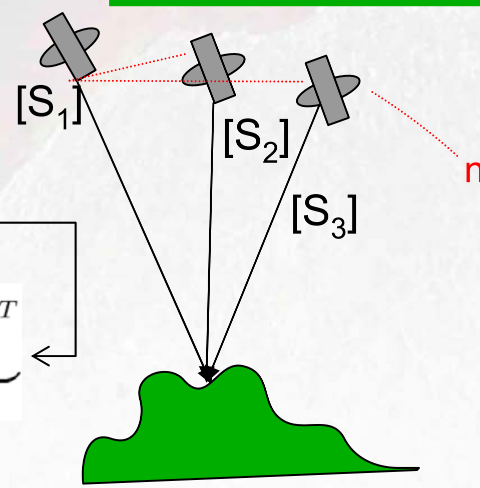


$$[S_1] = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix}$$

scattering matrix ↕ scattering vector

$$k_1 = \underbrace{\frac{1}{\sqrt{2}} [S_{HH} + S_{VV},\ S_{VV} - S_{HH},\ 2S_{HV}]^T}_{!pauli\ basis}$$

MB Pol-InSAR



$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

$$\Sigma_{11} = k_1 k_1^*$$

$$\Sigma_{12} = k_1 k_2^*$$

$$\Sigma_{22} = k_2 k_2^*$$

Characterization of Temporal Polarimetric and/or Interferometric Data

Multi-channel SAR systems are generaly considered as multicomponent Gaussian circular process whose number of component is equal to the number of polarimetric and interferometric channels of the system.

Example: The pdf of on intensity, PolSAR, and InSAR images are obtained by determining the marginal pdfs of the PolInSAR image pdf.

$$p(k_1) = \int p(\vec{k}_1, \vec{k}_2) d\vec{k}_2$$ → PDF of PolSAR images

$$p(k_{1hh}) = \int p(k_{1hh}, k_{1hv}, k_{1vv}) dk_{1hv} dk_{1vv}$$ → PDF of single channel images

$$p(k_{1hh}, k_{2hh}) = \int p(k_{1hh}, k_{1hv}, k_{1vv}, k_{2hh}, k_{2hv}, k_{2vv}) dk_{1hv} dk_{1vv} dk_{2hv} dk_{2vv}$$ → PDF of interferometric pair

Characterization of Temporal Polarimetric and/or Interferometric Data

target vector $k = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}_{2m\times 1}$ with covariance matrix $\Sigma_{2m\times 2m}$

Since the covariance matrix $\Sigma_{2m\times 2m}$ is unknown, it is estimated from *n* independent samples known as looks. Then estimated *2m*x*2m* covariance matrix A is characterized by *2m*–variate Wishart distribution with n degrees of freedom with the condition of n ≥ 2m.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

Using the well known results:

$$A_{11.2} = A_{11} - A_{12}A_{22}^{-1}A_{21} \rightarrow W_n(n-m, \Sigma_{11.2})$$

$$A_{12} \mid A_{22} \rightarrow N_{m,m}(\Sigma_{12}\Sigma_{22}^{-1}A_{22}, \Sigma_{11.2} \otimes A_{22})$$

$$A_{12}A_{22}^{-1}A_{21} \mid A_{22} \rightarrow W_m(m, \Sigma_{11.2}, \Sigma_{12}\Sigma_{22}^{-1}A_{22}\Sigma_{22}^{-1}\Sigma_{21}\Sigma_{11.2}^{-1})$$

$$\Rightarrow p(A_{11}, A_{22})$$

Characterization of Temporal Polarimetric and/or Interferometric Data

$$p(A_{11},A_{22}) = \frac{n^{2mn}\left|A_{11}\right|^{n-m}\left|A_{22}\right|^{n-m} etr\left(-n\,\frac{\Sigma_{11}^{-1}A_{11}+\Sigma_{22}^{-1}A_{22}}{I_m - P^2}\right)}{\left|\Sigma_{11}\right|^{n}\left|\Sigma_{22}\right|^{n}\left|I_m - P^2\right|^{n}}\;\underbrace{{}_0\tilde{F}_1(n;n^2 A_{11}^{1/2}\Sigma_{11.2}^{-1}\Sigma_{12}\Sigma_{22}^{-1}A_{22}\Sigma_{22}^{-1}\Sigma_{21}\Sigma_{11.2}^{-1}A_{11}^{1/2})}$$

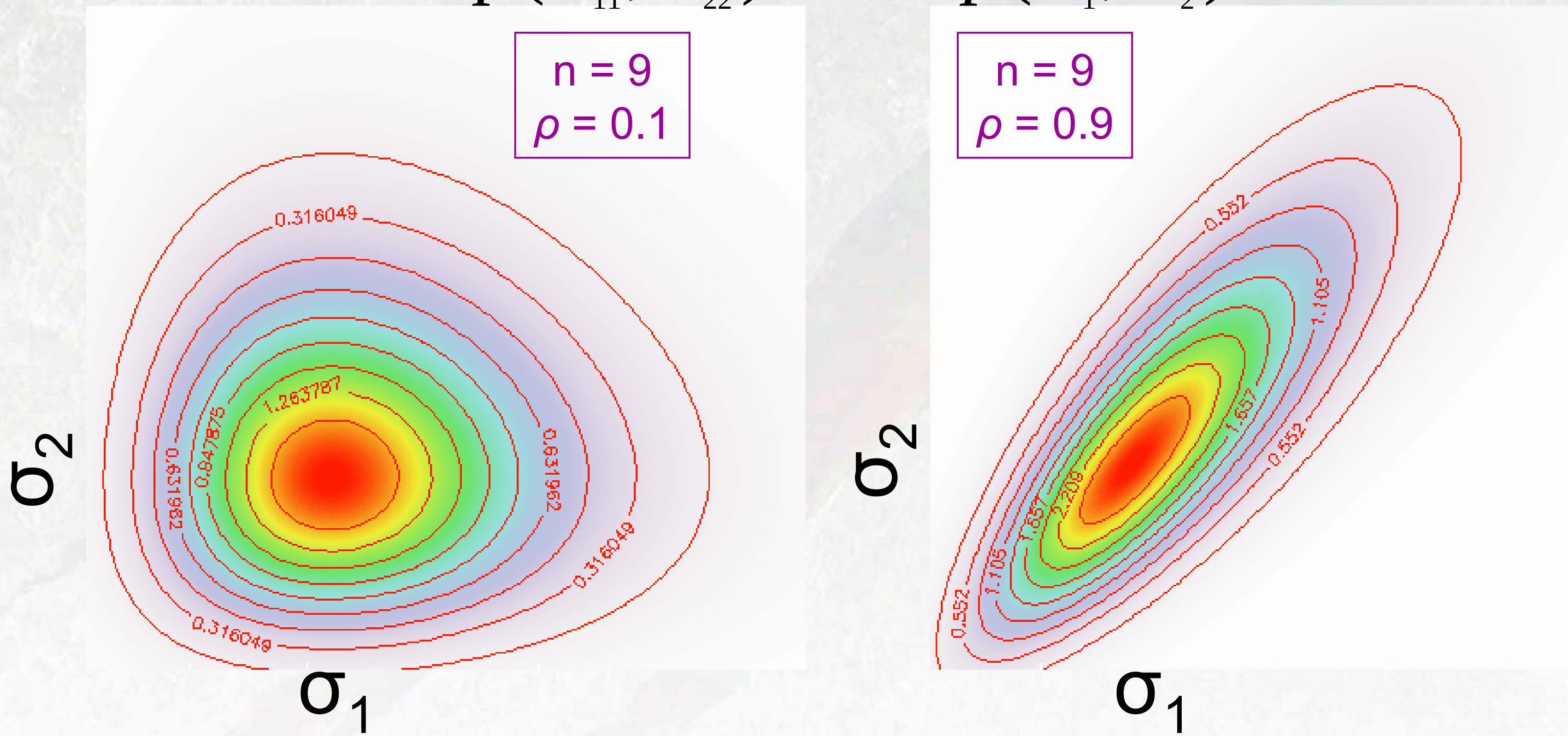
determinant of the *m*x*m* matrix including a modified Bessel fuction

- In the case of decorrelation, $p(A_{11},A_{22})$ converges to multiplication of two wishart distribution
- It is a phase preserving joint density of complex Wishart matrices with arbitrary correlation
- Regarding hermitian multiplication, for m = 1 there is no phase information. In the condition of m ≥ 1 phase information is preserved.
- m = 1 → $p(A_{11},A_{22})$ converges to a bivariate gamma distribution

[1] Bivariate Gamma Distributions for Image Registration and Change Detection, IEEE Trans. on Image Processing, vol. 16, no.7, July 2007.
[2] Glacier Velocity Monitoring by Maximum Likelihood Texture Tracking, IEEE Trans. on Geoscience and Remote Sensing, vol 8, Feb. 2009.

Characterization of Temporal Polarimetric and/or Interferometric Data

$$p(A_{11}, A_{22}) \stackrel{m=1}{\Rightarrow\Rightarrow} p(\sigma_1, \sigma_2)$$



comparision of theoretical pdf with histogram of two gamma distribution esa

Time varying behaviour of the system:

- Distance measurement without any assumption concerning their independence
- Well-known and very powerful change detection algorithm regarding second order statistics take into account the determinant of the PolSAR images.

$$D_n(\Sigma_{11},\Sigma_{22},\Sigma_{12}) = \int \log\left[\frac{p(A_{11},A_{22})}{p(A_{11})\,p(A_{22})}\right] p(A_{11},A_{22})\,d\vec{A}$$

$A_{11} \rightarrow W_m(n,\Sigma_{11})$ $A_{22} \rightarrow W_m(n,\Sigma_{22})$ are densities of the m^2 complex element vector obtained by stacking the columns of A_{11} and A_{22}.

For the sake of clarity, KL-divergence test is the measurement of the distance between two pdfs including correlated and uncorrelated pdf of temporal data set.

Scalar change detection measure on Multidimensional SAR System :

- To define a convenient scalar parameter for change detection regardless of which system configuration it is has been the subject of recent studies.
- KL-divergence is a efficient in comparing imagining systems with different channel number.
- !! As expected, a convenient change detection measure should decrease when considers a subset of the measurement, as the case for the KL-divergence test.

KL-divergence test:

$$D_n(\Sigma_{11},\Sigma_{22},\Sigma_{12}) = E[\log({}_0\tilde{F}_1(n,t))] - tr(-\frac{2nP}{I-P}) - n\log(|I-P|)$$

with the condition of $0 < P < I_m$ which means that both P and $I_m - P$ must be positive definite.

Scalar change detection measure on Multidimensional SAR System :

$$D_n(\Sigma_{11},\Sigma_{22},\Sigma_{12}) = E[\log({}_0\tilde{F}_1(n,t))] - tr(-\frac{2nP}{I-P}) - n\log(|I-P|)$$

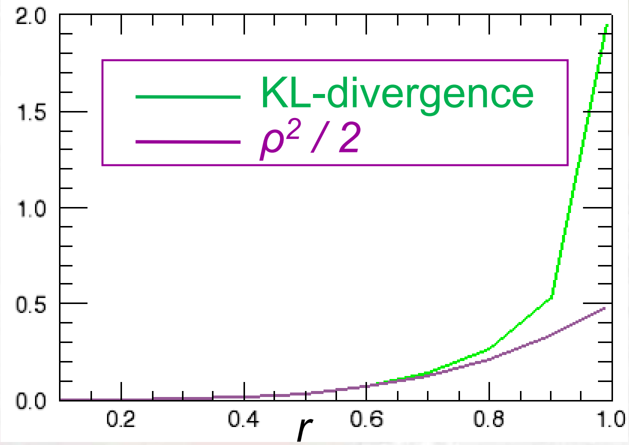
D_n will decrease regarding the dimension of the multi-channel SAR system

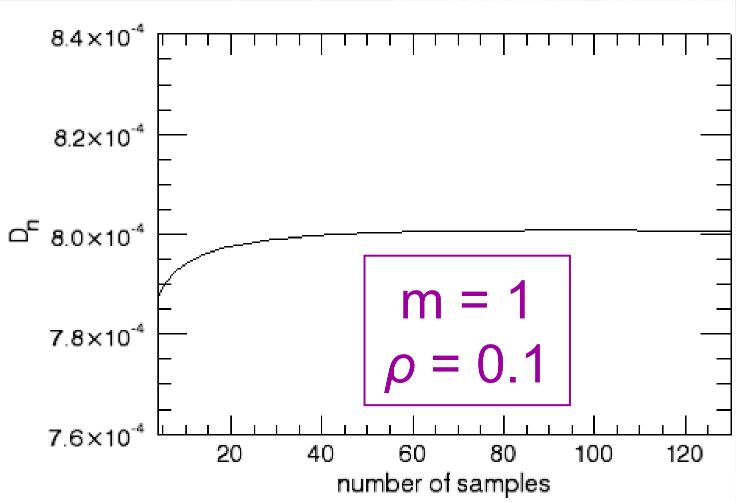
- It is interesting to see that KL-divergence is related to one to one transformation of correlation between independent samples.
- Consequently, *Dn* and *P* should provide similar performance for image registration and change detection. OR Is there are some advantage to apply the proposed technique?

Scalar change detection measure on Multidimensional SAR System :

Asymptotic behavior analysis:

m =1 : correlation between two amplitude images





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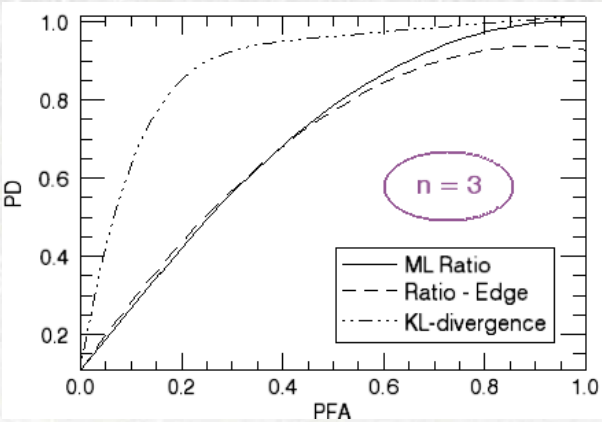
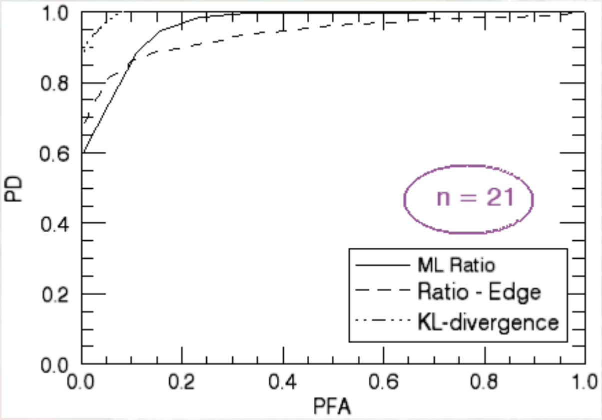
A comparison of change detection statistics in POLSAR images:

ROC curves





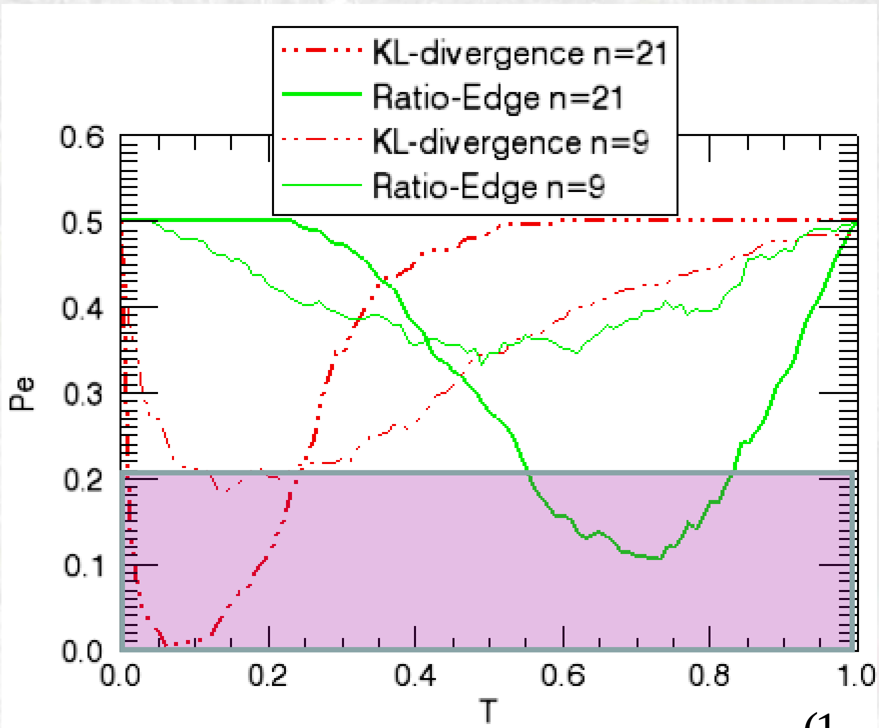
$$\frac{|A_{11}|^n |A_2|^n}{|A_{11} + A_{22}|^{2n}} > \alpha$$

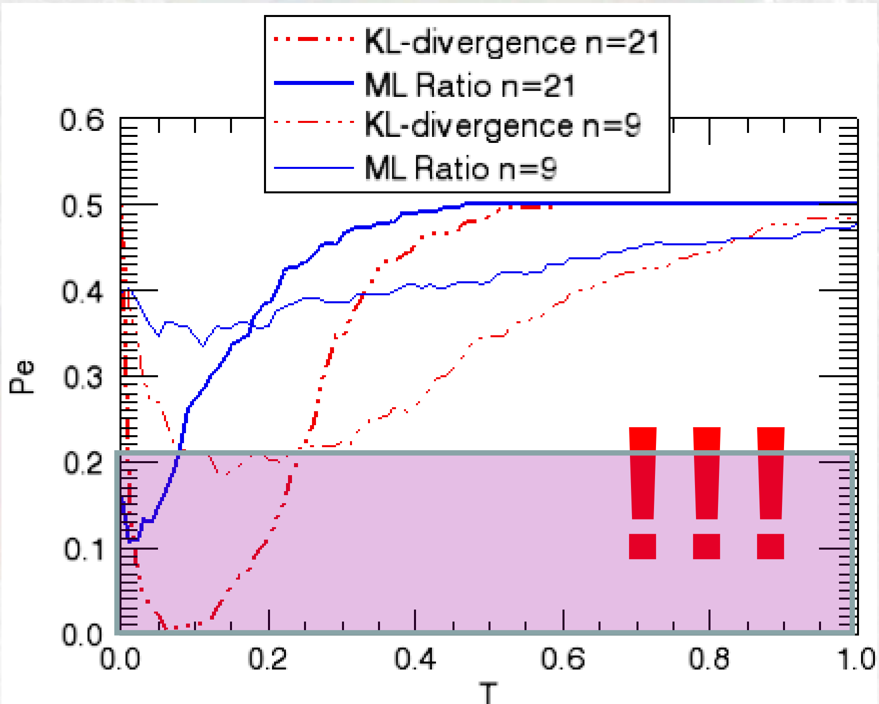
Information Theory-Based Approach for Contrast Analysis in Polarimetric and/or Interferometric SAR Images, IEEE Trans. on Geoscience and Remote Sensing, vol 46, no 8, Aug. 2008.

$$n\log|A_{11}| + n\log|A_{22}| - 2n\log|A_{11} + A_{22}| > \alpha$$ → ML Ratio !!! → KL-divergence

$$re = \frac{|A_{11}|}{|A_{22}|} \quad if\ re > 1, \quad re = \frac{1}{re}$$ → Ratio - Edge

Probability of error of theROCcurves

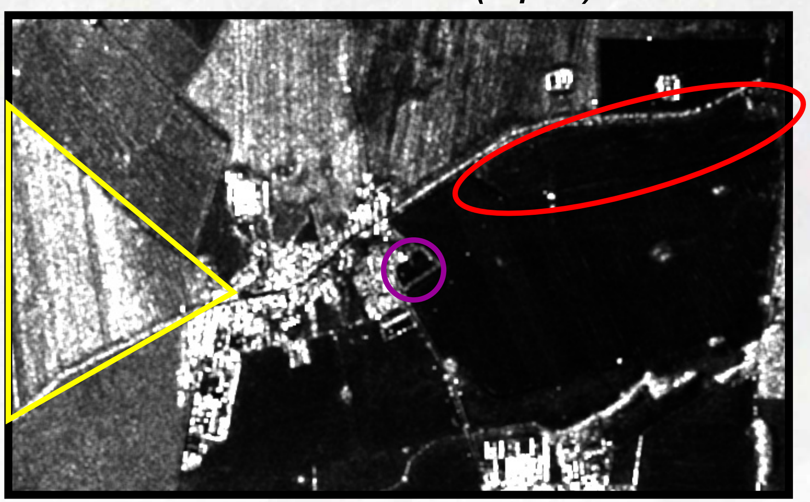




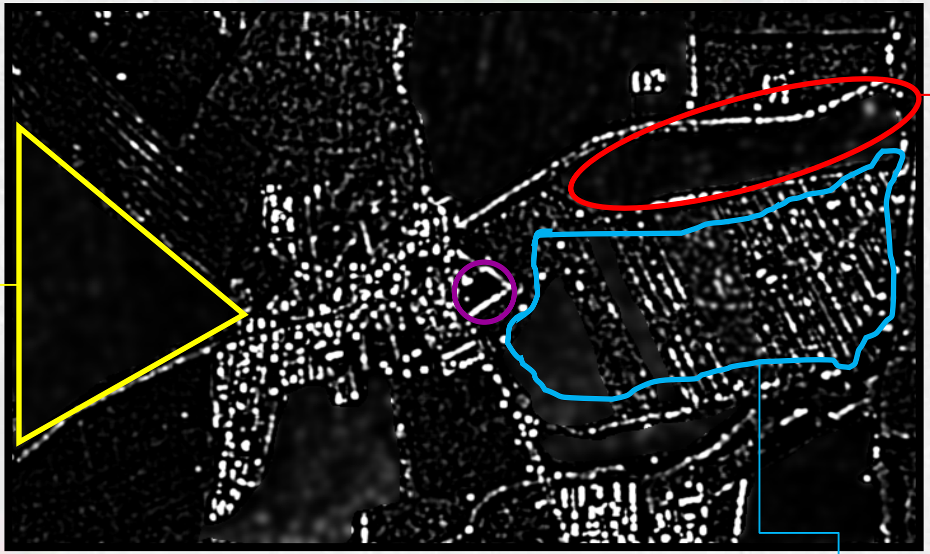
$$Pe = \frac{(1 - PD + PFA)}{2}$$

- There are more values of the threshold *T* satisfying the condition of Pe ≤ 0.2.
- The threshold is easier to be adjusted with the proposed method.

time 1 (April)



time 2 (August)



KL-divergence

Conclusion and discussian:

- It has been demonstrated that KL-divergence test is an appropriate change detection paremeter for multi-channel SAR system regardless of the dimension of the system.
- For a fixed image configuration, KL-divergence is a decreasing measurement of the number of channels.
- For m>1, the proposed approach condisers polarimetric information. For m=1, ony amplitude information is considerd.

Future work:

- The proposed joint distribution of temporal polarimetric data will be applied to polarimetric image registration, removal of bias, temporal scattering mechanism analysis, new polarimetric basis calculation ?....
- Since the proposed technique is sensible for small changes, its performance will be tested with other well-known change detection algorithms on real data.
- KL-divergence test with

$$D_n = \int \log\left[\frac{p(A_{12} \mid A_{22})}{p(A_{11})}\right] p(A_{11} \mid A_{22})\, d\overrightarrow{A_{12}}$$

will also be tested.

Thanks for attention....