

# A Joint Density of Interferometric and/or Polarimetric Images: Application to Change Detection

Esra Erten, Andreas Reigber and Rafael Z. Schneider



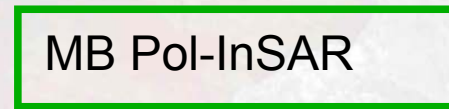
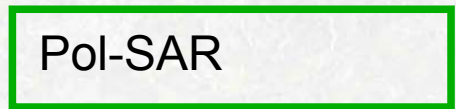
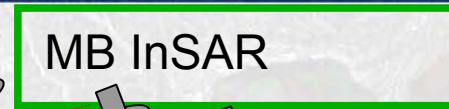
**Computer Vision and Remote Sensing, Technical University of Berlin,  
D-10587, Berlin, Germany.**

**Microwaves and Radar Institute, SAR Technology Department  
German Aerospace Centre (DLR)  
D-82234 Oberpfaffenhofen, Germany.**



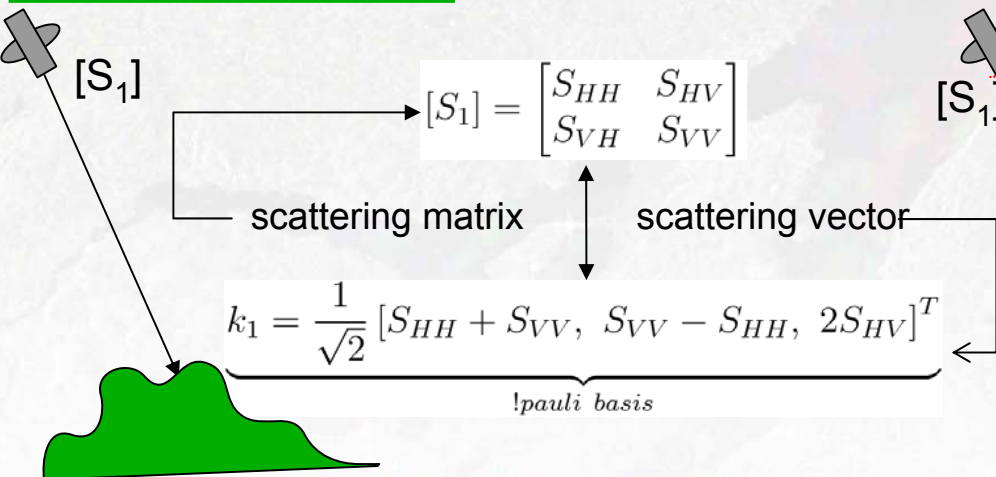
## Motivation :

- Multi-channel SAR Systems Model
- $p(A_{11}, A_{22})$  : Definition of joint distribution of two POLSAR images
- Scalar change detection measure on Multi-channel SAR system regarding KL-divergence
- Shortly review on two well-known statistics regarding multi-channel SAR system
- Examples on real and simulated data
- Conclusion and discussion



$\Sigma :=$  coherency matrix

$$\Sigma = \begin{bmatrix} S_1 S_1^* & S_1 S_2^* \\ S_2 S_1^* & S_2 S_2^* \end{bmatrix}$$



$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

$$\Sigma_{11} = k_1 k_1^*$$

$$\Sigma_{12} = k_1 k_2^*$$

$$\Sigma_{22} = k_2 k_2^*$$

## Characterization of Temporal Polarimetric and/or Interferometric Data

Multi-channel SAR systems are generally considered as multicomponent Gaussian circular process whose number of component is equal to the number of polarimetric and interferometric channels of the system.

**Example:** The pdf of on intensity, PolSAR, and InSAR images are obtained by determining the marginal pdfs of the PolInSAR image pdf.

$$p(k_1) = \int p(\vec{k}_1, \vec{k}_2) d\vec{k}_2 \longrightarrow \text{PDF of PolSAR images}$$

$$p(k_{1hh}) = \int p(k_{1hh}, k_{1hv}, k_{1vv}) dk_{1hv} dk_{1vv} \longrightarrow \text{PDF of single channel images}$$

$$p(k_{1hh}, k_{2hh}) = \int p(k_{1hh}, k_{1hv}, k_{1vv}, k_{2hh}, k_{2hv}, k_{2vv}) dk_{1hv} dk_{1vv} dk_{2hv} dk_{2vv} \longrightarrow \text{PDF of interferometric pair}$$

## Characterization of Temporal Polarimetric and/or Interferometric Data

target vector  $k = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}_{2m \times 1}$  with covariance matrix  $\Sigma_{2m \times 2m}$

Since the covariance matrix  $\Sigma_{2m \times 2m}$  is unknown, it is estimated from  $n$  independent samples known as looks. Then estimated  $2m \times 2m$  covariance matrix  $A$  is characterized by  $2m$ -variate Wishart distribution with  $n$  degrees of freedom with the condition of  $n \geq 2m$ .

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

Using the well known results:

$$A_{11.2} = A_{11} - A_{12}A_{22}^{-1}A_{21} \rightarrow W_n(n - m, \Sigma_{11.2})$$

$$A_{12} | A_{22} \rightarrow N_{m,m}(\Sigma_{12}\Sigma_{22}^{-1}A_{22}, \Sigma_{11.2} \otimes A_{22})$$

$$A_{12}A_{22}^{-1}A_{21} | A_{22} \rightarrow W_m(m, \Sigma_{11.2}, \Sigma_{12}\Sigma_{22}^{-1}A_{22}\Sigma_{22}^{-1}\Sigma_{21}\Sigma_{11.2}^{-1})$$

}  $p(A_{11}, A_{22})$

## Characterization of Temporal Polarimetric and/or Interferometric Data

$$p(A_{11}, A_{22}) = \frac{n^{2mn} |A_{11}|^{n-m} |A_{22}|^{n-m} \text{etr} \left( -n \frac{\Sigma_{11}^{-1} A_{11} + \Sigma_{22}^{-1} A_{22}}{I_m - P^2} \right)}{|\Sigma_{11}|^n |\Sigma_{22}|^n |I_m - P^2|^n} \underbrace{\tilde{F}_1(n; n^2 A_{11}^{1/2} \Sigma_{11.2}^{-1} \Sigma_{12} \Sigma_{22}^{-1} A_{22} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{11.2}^{-1} A_{11}^{1/2})}_{\text{determinant of the } mxm \text{ matrix including a modified Bessel function}}$$

determinant of the  $mxm$  matrix including a modified Bessel function

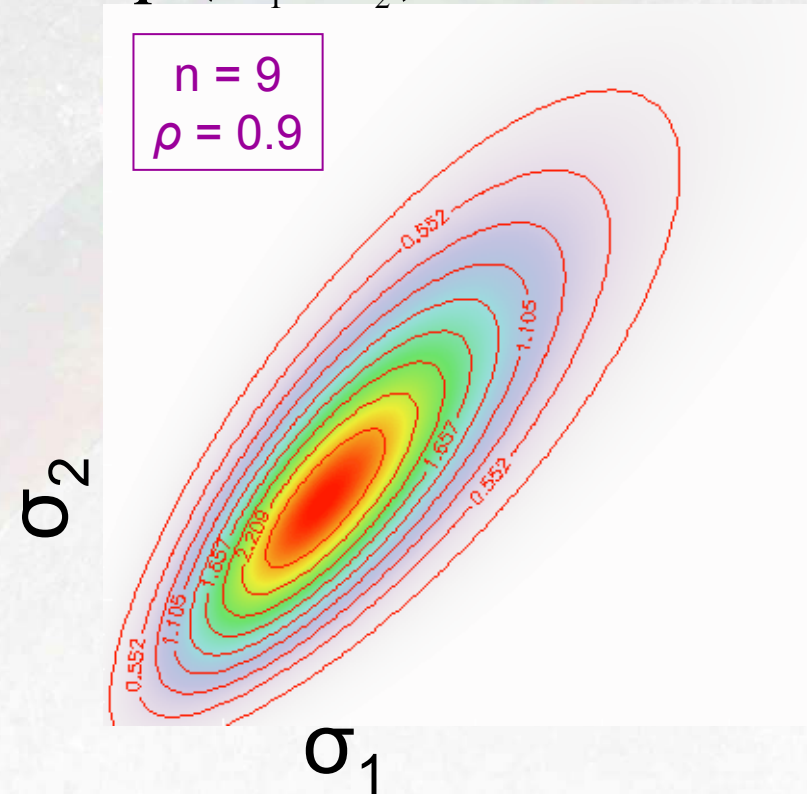
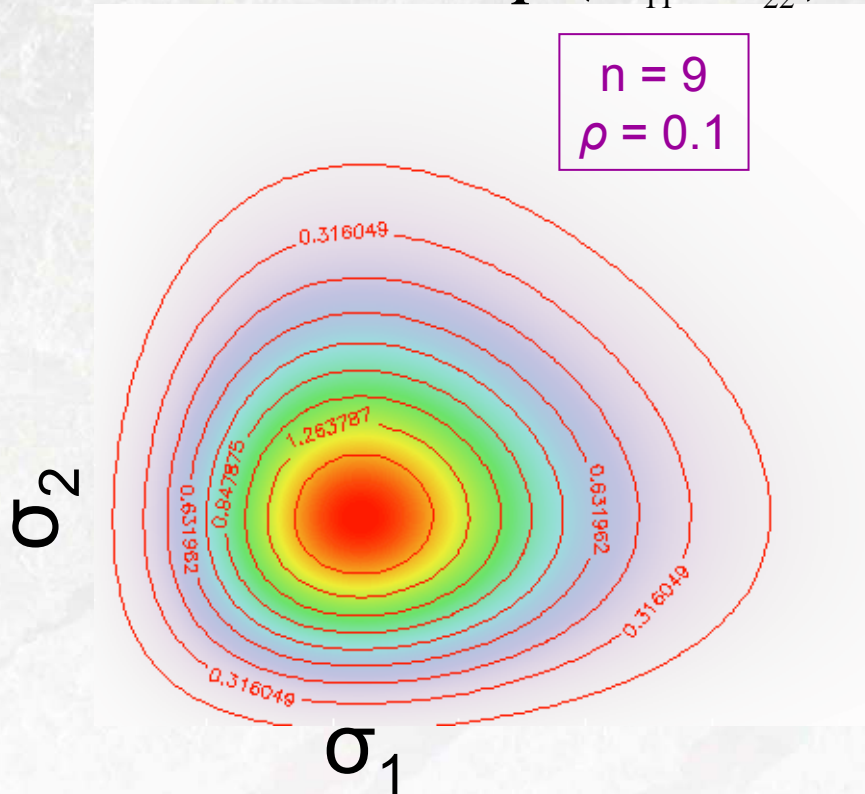
- In the case of decorrelation,  $p(A_{11}, A_{22})$  converges to multiplication of two wishart distribution
- It is a phase preserving joint density of complex Wishart matrices with arbitrary correlation
- Regarding hermitian multiplication, for  $m = 1$  there is no phase information. In the condition of  $m \geq 1$  phase information is preserved.
- $m = 1 \rightarrow p(A_{11}, A_{22})$  converges to a **bivariate gamma** distribution

[1] Bivariate Gamma Distributions for Image Registration and Change Detection, IEEE Trans. on Image Processing, vol. 16, no.7, July 2007.

[2] Glacier Velocity Monitoring by Maximum Likelihood Texture Tracking, IEEE Trans. on Geoscience and Remote Sensing, vol 8, Feb. 2009.

Characterization of Temporal Polarimetric and/or Interferometric Data

$$p(A_{11}, A_{22}) \xrightarrow{m=1} p(\sigma_1, \sigma_2)$$



comparison of theoretical pdf with histogram of two gamma distribution

## Time varying behaviour of the system:

- Distance measurement without any assumption concerning their independence
- Well-known and very powerful change detection algorithm regarding second order statistics take into account the determinant of the PolSAR images.

$$D_n(\Sigma_{11}, \Sigma_{22}, \Sigma_{12}) = \int \log \left[ \frac{p(A_{11}, A_{22})}{p(A_{11})p(A_{22})} \right] p(A_{11}, A_{22}) d\bar{A}$$

$A_{11} \rightarrow W_m(n, \Sigma_{11})$   $A_{22} \rightarrow W_m(n, \Sigma_{22})$  are densities of the  $m^2$  complex element vector obtained by stacking the columns of  $A_{11}$  and  $A_{22}$ .

For the sake of clarity, KL-divergence test is the measurement of the distance between two pdfs including correlated and uncorrelated pdf of temporal data set.



## Scalar change detection measure on Multidimensional SAR System :

- To define a convenient scalar parameter for change detection regardless of which system configuration it is has been the subject of recent studies.
- KL-divergence is a efficient in comparing imagining systems with different channel number.
- !! As expected, a convenient change detection measure should decrease when considers a subset of the measurement, as the case for the KL-divergence test.

KL-divergence test:

$$D_n(\Sigma_{11}, \Sigma_{22}, \Sigma_{12}) = E[\log({}_0\tilde{F}_1(n, t))] - tr\left(-\frac{2nP}{I - P}\right) - n \log(|I - P|)$$

with the condition of  $0 < P < I_m$  which means that both  $P$  and  $I_m - P$  must be positive definite.

## Scalar change detection measure on Multidimensional SAR System :

$$D_n(\Sigma_{11}, \Sigma_{22}, \Sigma_{12}) = E[\log({}_0\tilde{F}_1(n, t))] - \text{tr}\left(-\frac{2nP}{I-P}\right) - n \log(|I-P|)$$

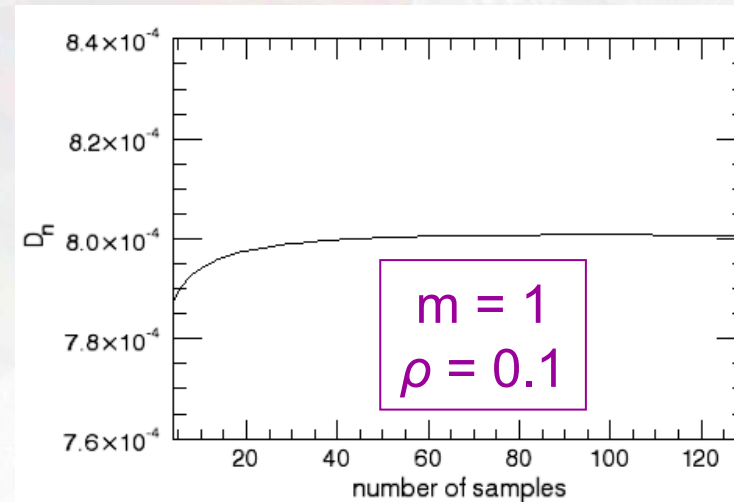
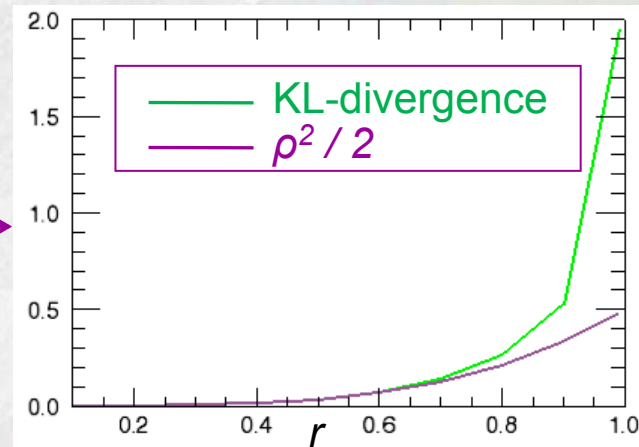
$D_n$  will decrease regarding the dimension of the multi-channel SAR system

- It is interesting to see that KL-divergence is related to one to one transformation of correlation between independent samples.
- Consequently,  $Dn$  and  $P$  should provide similar performance for image registration and change detection. **OR Is there are some advantage to apply the proposed technique?**

Scalar change detection measure on Multidimensional SAR System :

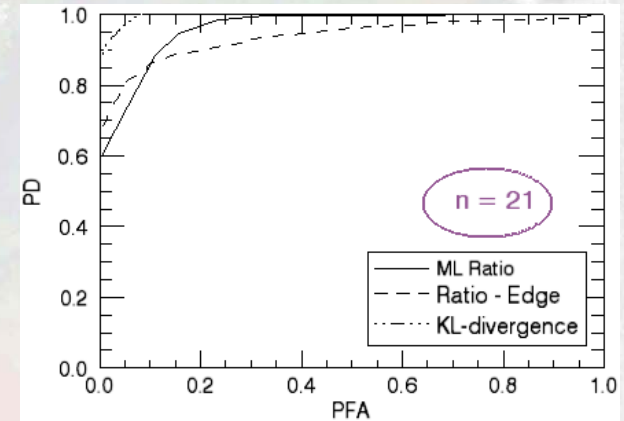
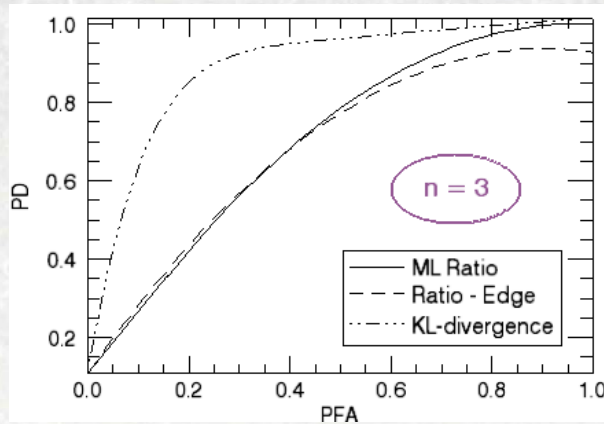
Asymptotic behavior analysis:

$m = 1$  : correlation between two amplitude images



A comparison of change detection statistics in POLSAR images:

ROC curves



$$\frac{|A_{11}|^n |A_{22}|^n}{|A_{11} + A_{22}|^{2n}} > \alpha$$

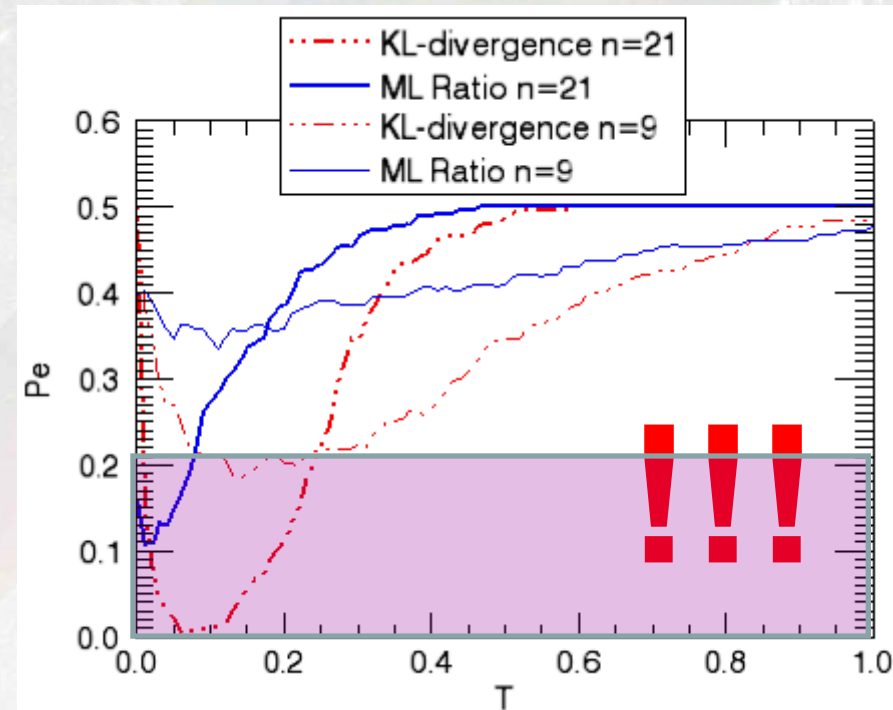
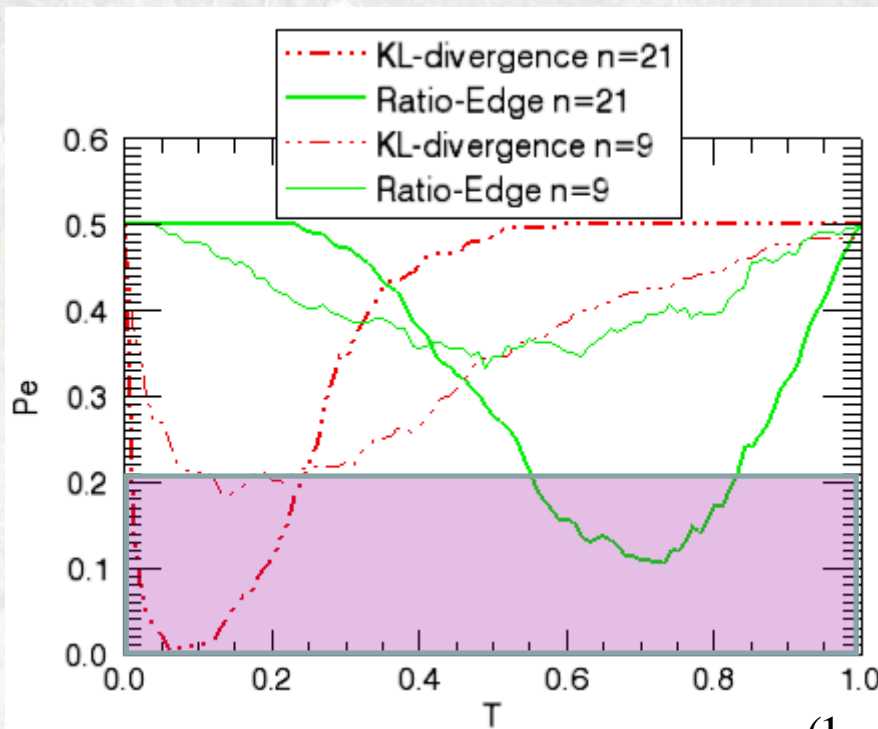
Information Theory-Based Approach for Contrast Analysis in Polarimetric and/or Interferometric SAR Images, IEEE Trans. on Geoscience and Remote Sensing, vol 46, no 8, Aug. 2008.



$$n \log |A_{11}| + n \log |A_{22}| - 2n \log |A_{11} + A_{22}| > \alpha \longrightarrow \text{ML Ratio !!!} \longrightarrow \text{KL-divergence}$$

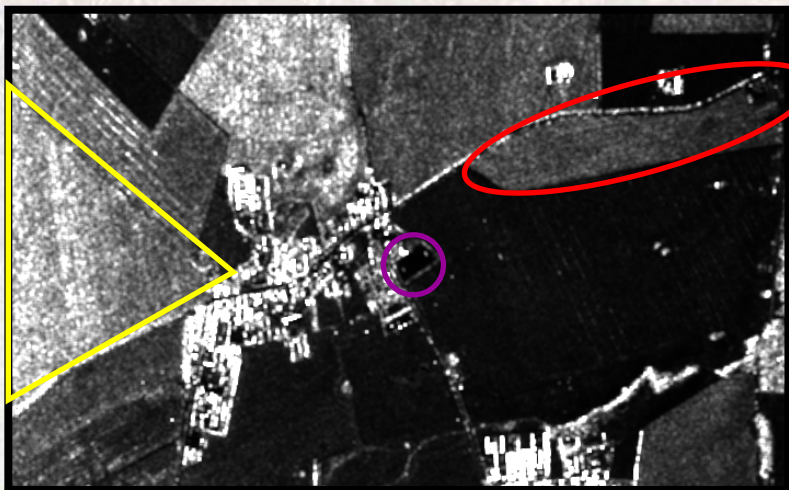
$$re = \frac{|A_{11}|}{|A_{22}|} \quad \text{if } re > 1, \quad re = \frac{1}{re} \quad \longrightarrow \text{Ratio - Edge}$$

Probability of error of the ROC curves

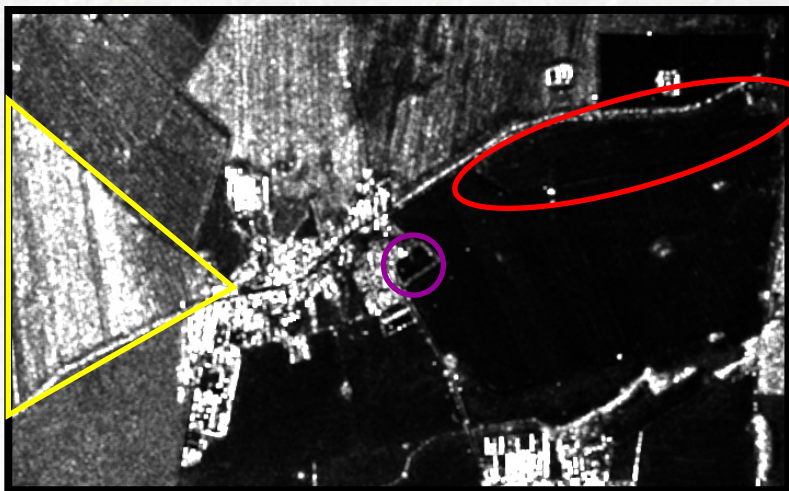


$$Pe = \frac{(1 - PD + PFA)}{2}$$

- There are more values of the threshold  $T$  satisfying the condition of  $Pe \leq 0.2$ .
- The threshold is easier to be adjusted with the proposed method.



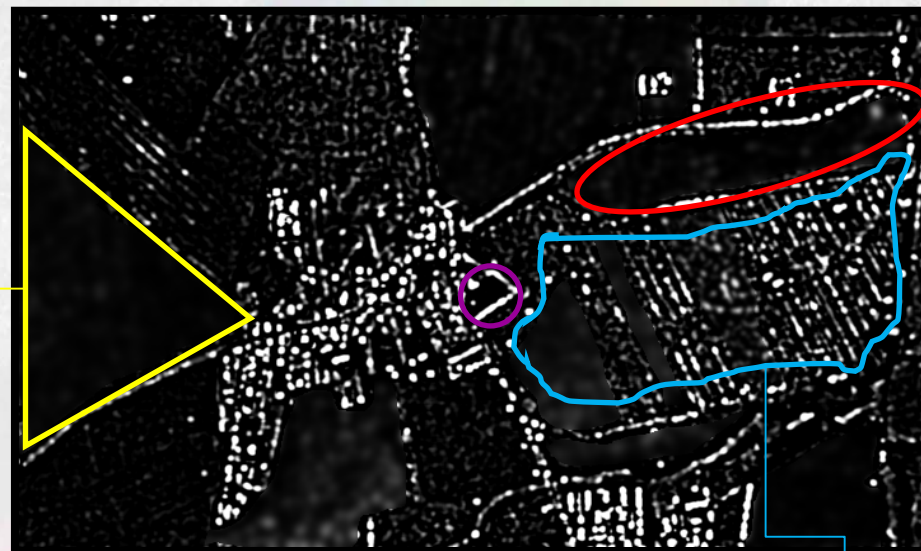
*time 1 (April)*



*time 2 (August)*

rapeseed

grassland



*KL-divergence*

wheat

## Conclusion and discussion:

- It has been demonstrated that KL-divergence test is an appropriate change detection parameter for multi-channel SAR system regardless of the dimension of the system.
- For a fixed image configuration, KL-divergence is a decreasing measurement of the number of channels.
- For  $m > 1$ , the proposed approach considers polarimetric information. For  $m = 1$ , only amplitude information is considered.

## Future work:

- The proposed joint distribution of temporal polarimetric data will be applied to polarimetric **image registration, removal of bias, temporal scattering mechanism analysis, new polarimetric basis calculation ?....**
- Since the proposed technique is sensible for small changes, its performance will be tested with other well-known change detection algorithms on real data.
- KL-divergence test with

will also be tested.

$$D_n = \int \log \left[ \frac{p(A_{12} | A_{22})}{p(A_{11})} \right] p(A_{11} | A_{22}) d\overline{A_{12}}$$

*Thanks for attention....*

