POLINSAR AT LOW FREQUENCY AND IONOSPHERIC EFFECTS

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Ionopheric Effect

- Dispersive effect
  - Propagation time depends on the TEC and on the frequency. This creates a distortion of the chirp.
- Spatial variation of the TEC
  - Effect similar to trajectory disturbances $\Rightarrow$ extensive experience on very high resolution processing
  - Needs to be validated with representative 2-D phase screens
- Faraday rotation

Faraday rotation

$$M_{\Omega} = \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \begin{pmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{pmatrix} \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix}$$

$$M_{HH} = S_{HH}\cos^2\Omega - S_{VV}\sin^2\Omega$$

$$M_{VV} = S_{VV}\cos^2\Omega - S_{HH}\sin^2\Omega$$

$$M_{HV} = S_{HV} + (S_{HH} + S_{VV})\sin\Omega\cos\Omega$$

$$M_{VH} = S_{HV} - (S_{HH} + S_{VV})\sin\Omega\cos\Omega$$

$$M_{VH} + M_{HV} = 2S_{HV}$$

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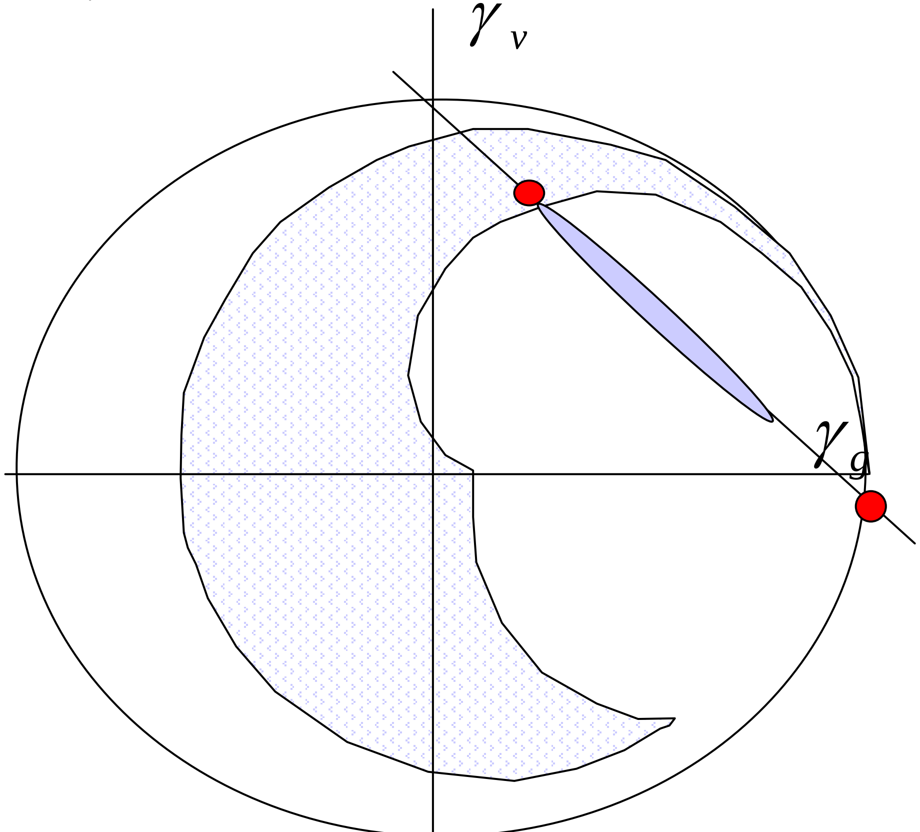
Standard RVoG inversion (*)

$$\gamma_S = \frac{\langle S_1 S_2^* \rangle}{\sqrt{\langle S_1 S_1^* \rangle \langle S_2 S_2^* \rangle}}$$



$$\gamma_V = e^{i\varphi_0} \frac{\int_0^{h_v} e^{2\frac{\sigma_x z}{\cos\theta}} e^{i k_z z} dz}{\int_0^{h_v} e^{2\frac{\sigma_x z}{\cos\theta}} dz}$$

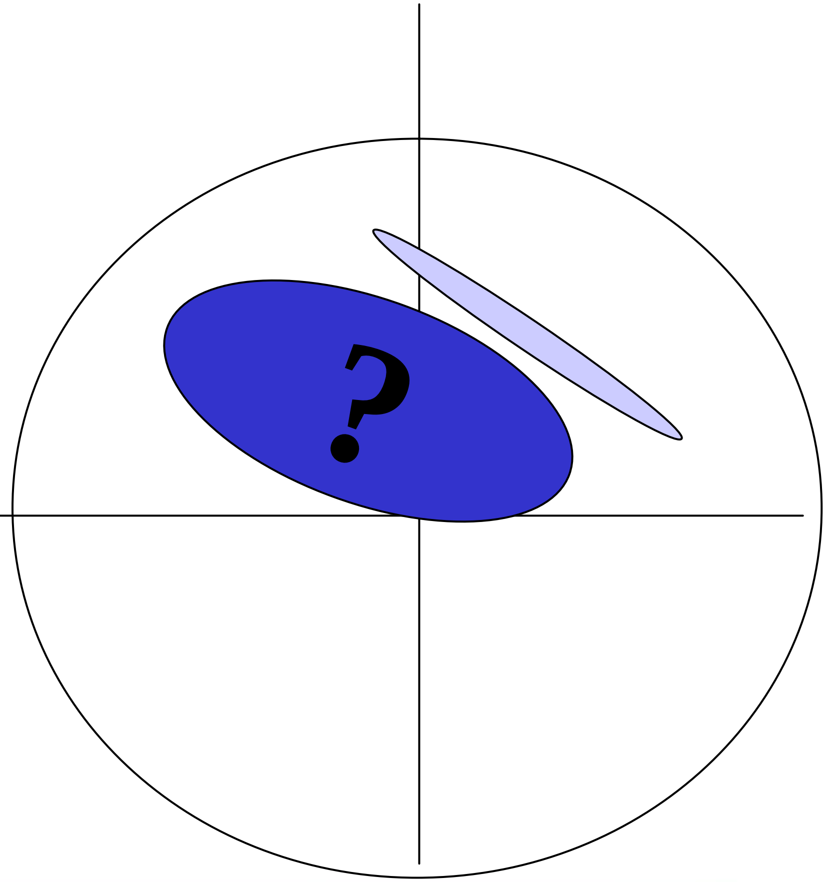
↓

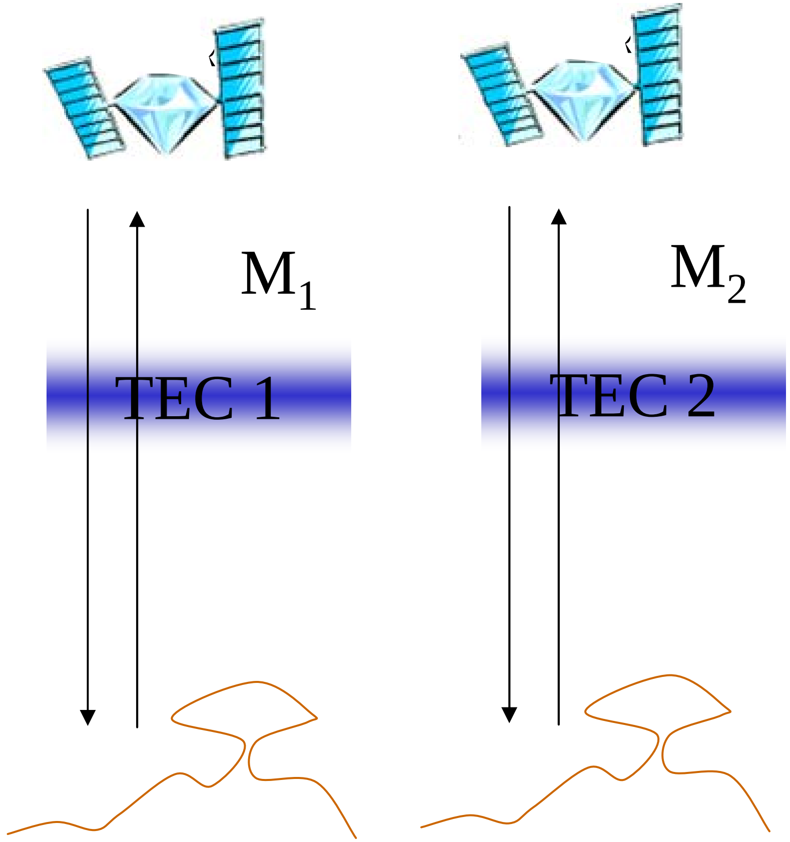
h σ_x

** Cloude and Papathanassiou*

Standard RVoG inversion + ionosphere

$$\gamma_M = \frac{\langle M_1 M_2^* \rangle}{\sqrt{\langle M_1 M_1^* \rangle \langle M_2 M_2^* \rangle}}$$

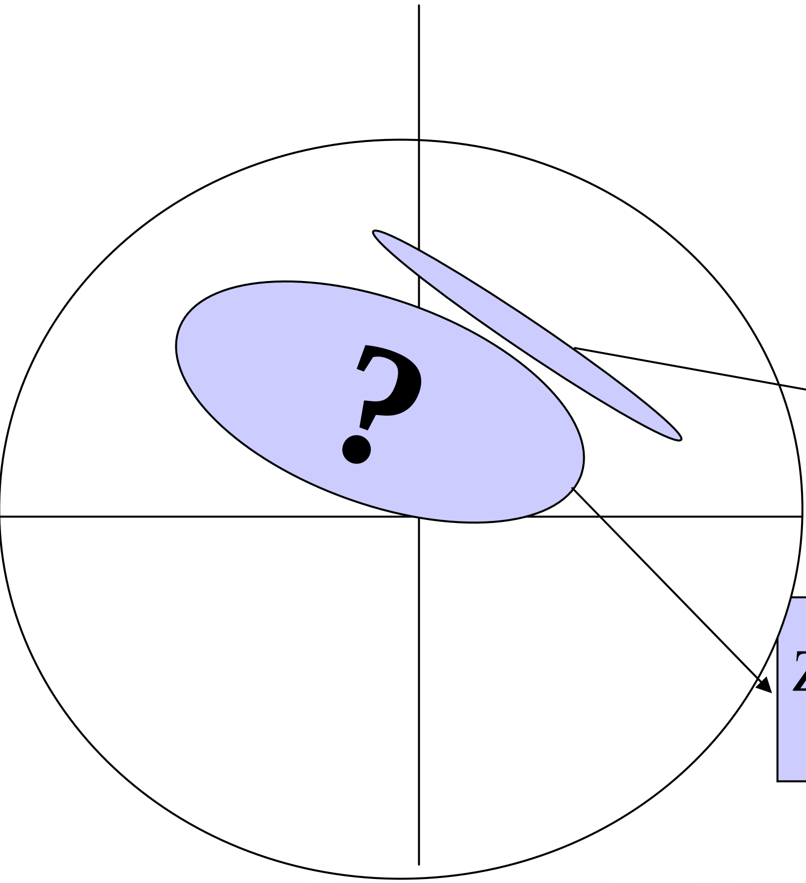




The set of interferometric coherences

$$E_{1RT} = k_R^T S_1 k_T \qquad E_{2RT} = k_R^T S_2 k_T$$

$$\gamma_{RT} = \frac{\left\langle E_{1RT} E_{2RT}^{*} \right\rangle}{\sqrt{\left\langle E_{1TRT} E_{1RT}^{*} \right\rangle \left\langle E_{2RT} E_{2RT}^{*} \right\rangle}}$$



$$Z = \{\gamma_{RT} \ \ for \ all \ k_R \ and \ k_T\}$$

= {the set of all interferometric coherences}

$$Z_{\Omega_1 \Omega_2} = \{\gamma_{RT}(\Omega_1, \Omega_2) \ \ for \ all \ k_R \ and \ k_T\}$$

Two cases

$$Z_{\Omega_1\Omega_2} = \left\{ \gamma_{RT}(\Omega_1, \Omega_2) \;\; for \;\; all \; k_R \;\; and \;\; k_T \right\}$$

$$\Omega_1 = \Omega_2$$

$$Z_{\Omega_1\Omega_2} = \left\{ \gamma_{RT}(\Omega_1, \Omega_1) \right\}$$

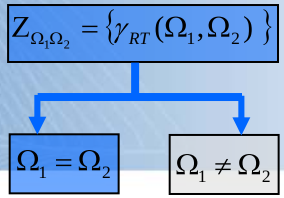
Full Pol/ Compact Pol

$$\Omega_1 \neq \Omega_2$$

$$Z_{\Omega_1\Omega_2} = \left\{ \gamma_{RT}(\Omega_1, \Omega_2) \right\}$$

Full Pol/ Compact Pol

Assuming identical ionospheres



$$\Omega_1 = \Omega_2 = \Omega$$

$$E_{1RT} = k_R^T R_\Omega S_1 R_\Omega k_T \qquad E_{2RT} = k_R^T R_\Omega S_2 R_\Omega k_T$$

For exemple

$$k_R = k_T = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\Omega = 0°$ $\qquad S_{HH}$

$\Omega \neq 0°$ $\qquad M_{HH} = S_{HH} \cos^2 \Omega - S_{VV} \sin^2 \Omega$

$$\gamma_\Omega(k_R, k_T) \neq \gamma_0(k_R, k_T)$$

Assuming identical ionospheres

$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1,\Omega_2)\}$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

$$E_{1RT} = k_R^T R_\Omega S_1 R_\Omega k_T$$

$$E_{1RT} = h_R^T S_1 h_T$$

$$E_{2RT} = k_R^T R_\Omega S_2 R_\Omega k_T$$

$$E_{2RT} = h_R^T S_2 h_T$$

Same measured signal

Faraday rotation	No Faraday rotation
$k_T = R_{-\Omega} h_T$	$h_T = R_\Omega k_T$
$k_R = R_\Omega h_R$	$h_R = R_{-\Omega} k_R$

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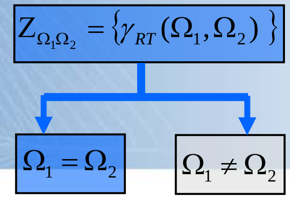

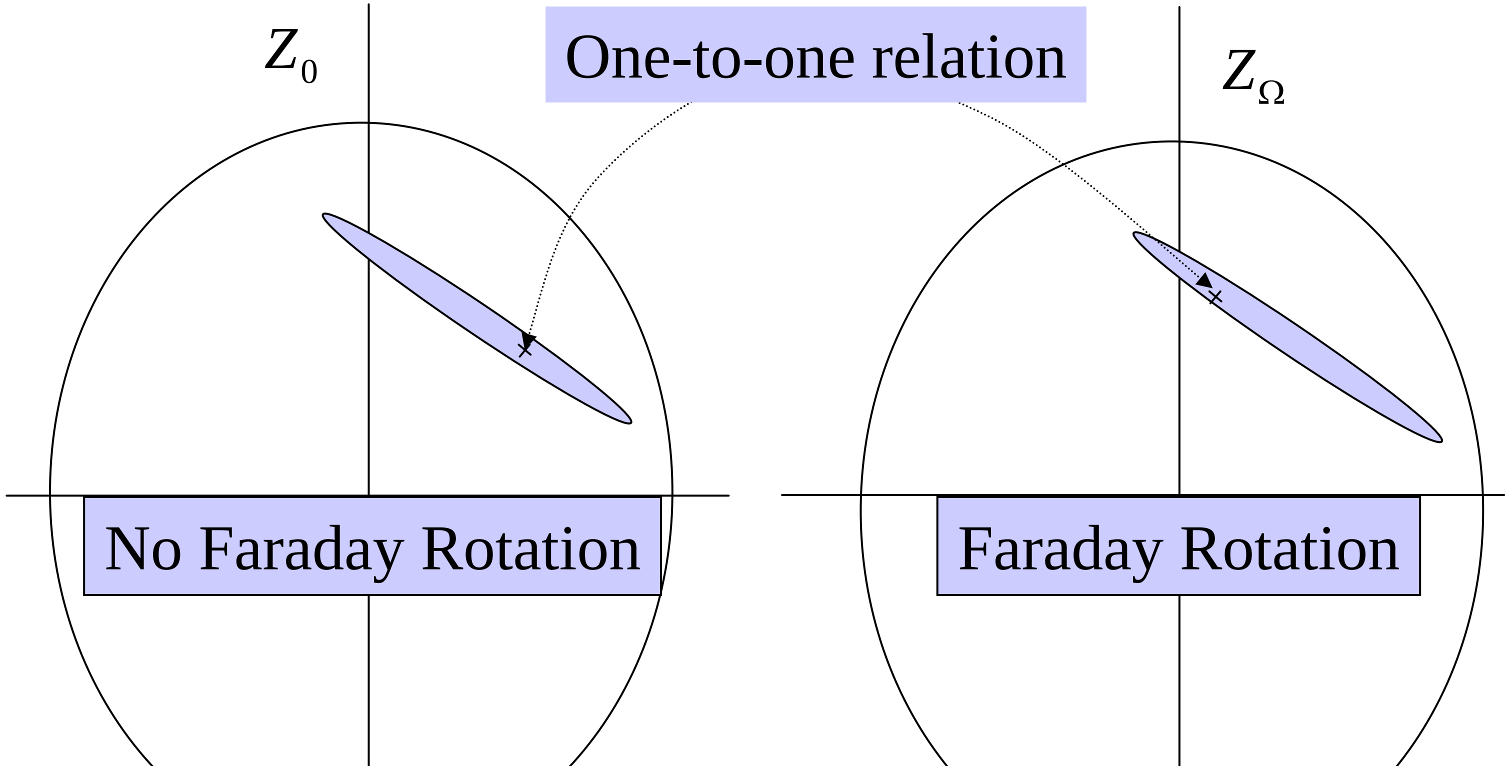

$$\gamma_\Omega(k_R, k_T) \neq \gamma_0(k_R, k_T)$$

$$\forall (k_R, k_T), \exists (h_R, h_T) \text{ such that } \gamma_\Omega(k_R, k_T) = \gamma_0(h_R, h_T)$$

The interferometric coherence set is invariant

No effect of ionosphere

Full

Assuming identical ionospheres

$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1,\Omega_2)\}$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

$$\Omega_1 = \Omega_2$$

$$\gamma_\Omega(k_R, k_T) \neq \gamma_0(k_R, k_T)$$

$\forall (k_R, k_T), \exists (h_R, h_T)$ such that $\gamma_0(k_R, k_T) = \gamma_\Omega(h_R, h_T)$

The interferometric coherence set is invariant

No effect of ionosphere

$Z_\Omega = Z$ ⇒ **The set of interferometric coherences is invariant**

The inversion can proceed without any adjustement except the selection of the ground. For that, use $M_{VH} + M_{HV} = 2S_{HV}$

Compact PolInSAR & same ionosphere

$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1,\Omega_2)\}$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

- The compact polarimetry mode:
  - $\pi/2$ mode: 1 circular transmit and 2 independent receive polarizations: (RR,RL) or (RH, RV)
- Circular on transmit at lower frequency is essential
  - The single polarization on receive will be rotated through the ionosphere; To insure the invariance of polarization at the surface level, circular polarization is the only choice
- Synthesis can be done on receive:

$$E_{1R} = \frac{1}{\sqrt{2}} k_R^T S_1 \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

Without ionosphere

$$E_{1R} = \frac{e^{-j\Omega}}{\sqrt{2}} k_R^T R_\Omega S_1 \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

With ionosphere

Assuming identical ionospheres

$$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1,\Omega_2)\}$$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

$$E_{1RC} = e^{-j\Omega} k_R^T R_\Omega S_1 \begin{pmatrix} 1 \\ -j \end{pmatrix} \qquad E_{2RC} = e^{-j\Omega} k_R^T R_\Omega S_2 \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

$$\gamma_\Omega = \frac{\langle E_{1RC} E_{2RC}^* \rangle}{\sqrt{\langle E \quad E \quad ^* \rangle \langle E \quad E \quad ^* \rangle}}$$

This is not the case if the transmit polarisation is not circular !!!

$$E_{1RC} = h_R^T \quad S_1 \begin{pmatrix} 1 \\ -j \end{pmatrix} \qquad h_R = e^{-j\Omega} R_{-\Omega} k_R \; ; \; k_R = e^{j\Omega} R_\Omega h_R$$

$$Z_\Omega = Z$$

The set of interferometric coherence is invariant

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Identical Faraday rotation

$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1, \Omega_2)\}$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

- Full polarimetry
  - The set of interferometric coherences is globally invariant
  - No effect on the inversion

- Compact polarimetry
  - If the transmit polarization is circular, the set of interferometric coherence is globally invariant
  - no effect on the inversion

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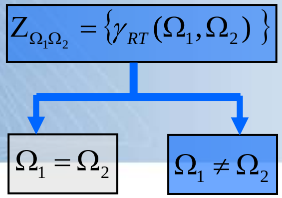
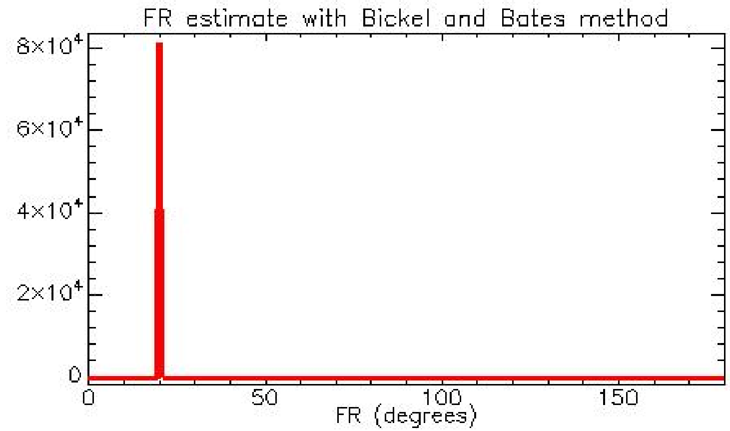
Different ionospheres



- Differential Faraday rotation
- Correction of the data prior to PolInSAR inversion
  - Full polarimetric case: Bickell and Bates, Freeman



  - Compact pol: more later
- What is the required accuracy of the correction?

Assuming a different ionosphere on co-pol

$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1,\Omega_2)\}$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

- Notion of differential ionosphere
  - Assume no FR on acquisition 1

HH or VV

$$\gamma_\Omega = \frac{\langle E_1 E_2^* \rangle}{\sqrt{\langle E_1 E_1^* \rangle \langle E_2 E_2^* \rangle}}$$

$$E_1 = S_{HH}$$

$$E_2 = S_{HH} \cos^2 \Omega - S_{VV} \sin^2 \Omega$$

$$\langle E_1 E_2^* \rangle = \cos^2 \Omega |S_{HH}|^2 - \sin^2 \Omega \langle S_{HH} S^*_{VV} \rangle$$

$$|\gamma_\Omega| > |\cos 2\Omega| \gamma_0$$

- $\Omega < 2°$,
  - Small loss of coherence = 2%
  - Small error on the interf. phase < 2°

Assuming a different ionosphere

$$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1,\Omega_2)\}$$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

- Assume no FR on acquisition 1

RR or LL

$$\gamma_\Omega = \frac{\langle E_1 E_2^{*}\rangle}{\sqrt{\langle E_1 E_1^{*}\rangle\langle E_2 E_2^{*}\rangle}}$$

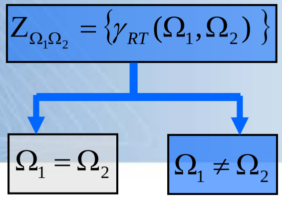
$$E_1 = S_{RR}$$

$$E_2 = S_{RR}$$

$$\gamma_\Omega = \gamma_0$$

Invariant with FR

Assuming a different ionosphere



- Notion of differential ionosphere
  - Assume no FR on acquisition 1

RL

$$\gamma_\Omega = \frac{\langle E_1 E_2^* \rangle}{\sqrt{\langle E_1 E_1^* \rangle \langle E_2 E_2^* \rangle}}$$

$$E_1 = S_{RL}$$

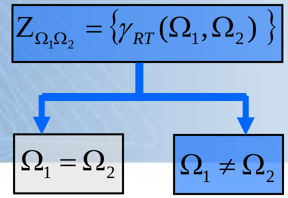
$$E_2 = e^{-2j\Omega} S_{RL}$$

$$\langle E_1 E_2^* \rangle = \cos^2 \Omega |S_{HH}|^2 - \sin^2 \Omega \langle S_{HH} S^*_{VV} \rangle$$

$$\gamma_\Omega = e^{2j\Omega} \gamma_0$$

- $\Omega < 2°$,
  - no loss of coherence

Different ionospheres for PolInSAR



- Correction of the differential Faraday rot. to within 2°
  - Bickell and Bates, Freeman…

- Apply PolInSAR inversion on corrected data

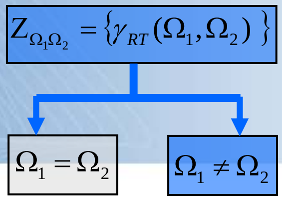
- Make good use of 3 FR invariant coherences

$$M_{VH} + M_{HV} = 2S_{HV}$$

$$M_{RR} = S_{RR}$$

$$M_{LL} = S_{LL}$$

Assuming different ionospheres



- Compact PolInSAR

$$\begin{pmatrix} S_{RH} \\ S_{RV} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix} \begin{pmatrix} 1 \\ -j \end{pmatrix}$$

- Only one Faraday rotation invariant coherence

$$M_{RR} = S_{RR}$$

- We know that Faraday rotation will lower the coherence: The two polarisation states are not matched

Assuming a different ionophere

$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1, \Omega_2)\}$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$

Therefore, the correct correction of FR will maximize the interferometric coherence

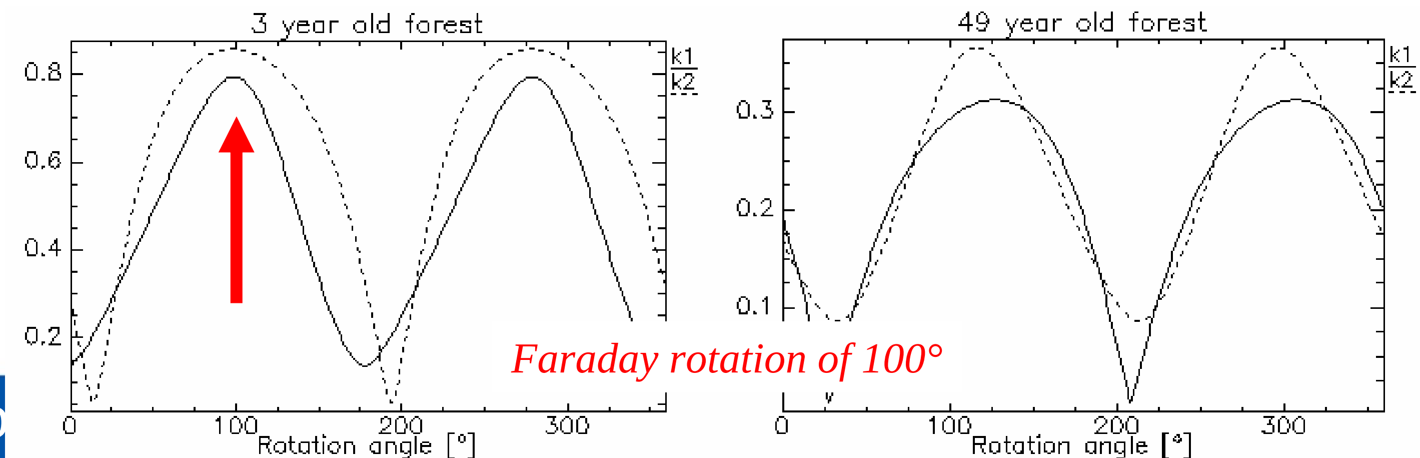
M_{1RH}

M_{2RH}

FR correction

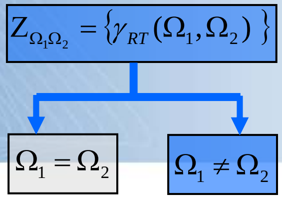
$$C_{2\Omega} = e^{j\Omega} \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T R_{-\Omega} \begin{pmatrix} M_{2RH} \\ M_{2RV} \end{pmatrix}$$

$$\gamma_\Omega = \frac{\langle M_{1RH} C_{2\Omega}^* \rangle}{\sqrt{\langle M_{1RH} M_{1RH}^* \rangle \langle C_{2\Omega} C_{2\Omega}^* \rangle}}$$

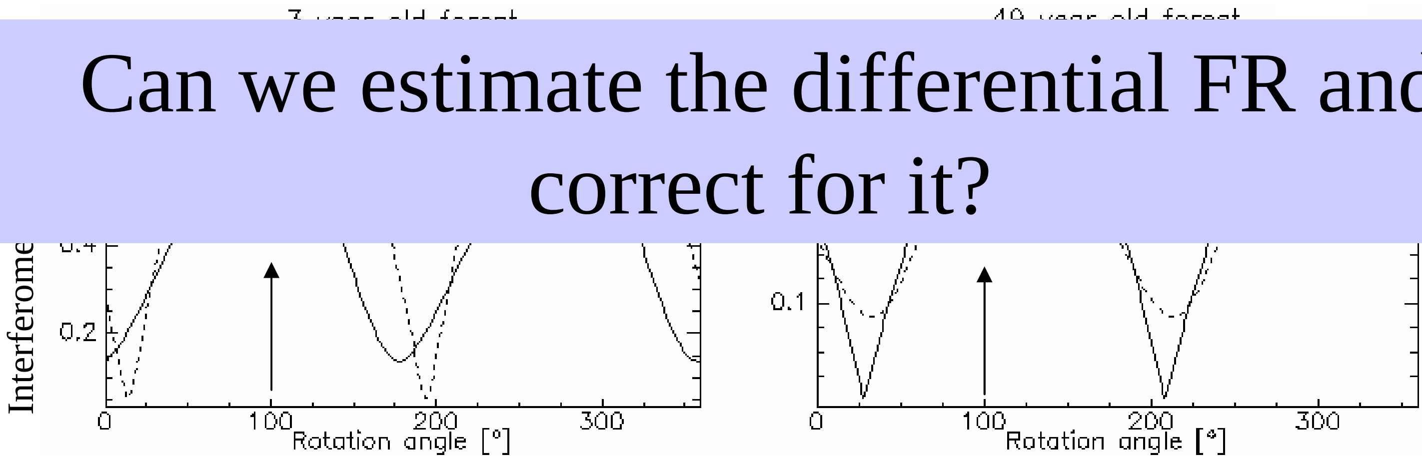


Comp

PolInSAR and ionophere: Large differential FR?



Can we estimate the differential FR and correct for it?



Variation of the coherence with respect to a FR correction on the second acquisition

Simulation over Airborne data

$\Omega = 0°$ on Day 1

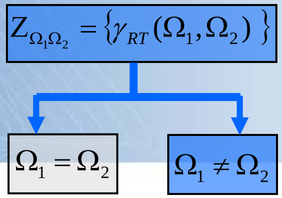
$\Omega = 100°$ on Day 2

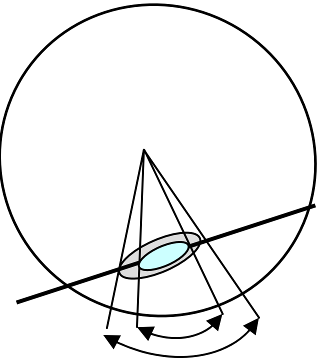
We maximize the coherence by correcting

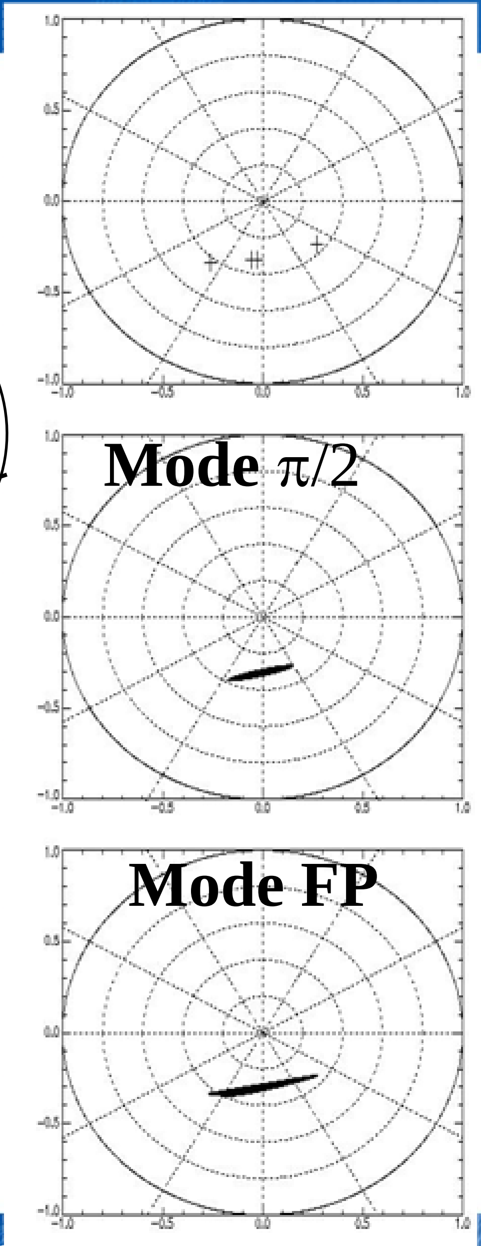
Yes with an accuracy better than 5°

Dubois-Fernandez et al. "*The compact polarimetry alternative …* ", IEEE TGRS October 2008

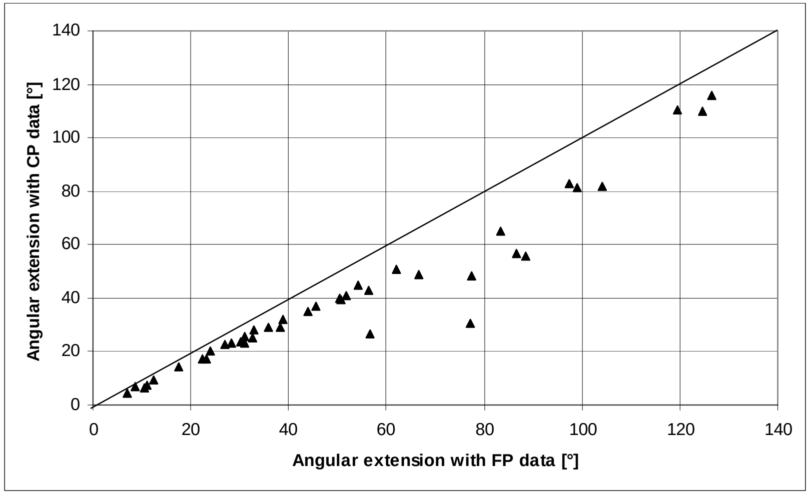
Compact PolInSAR inversion



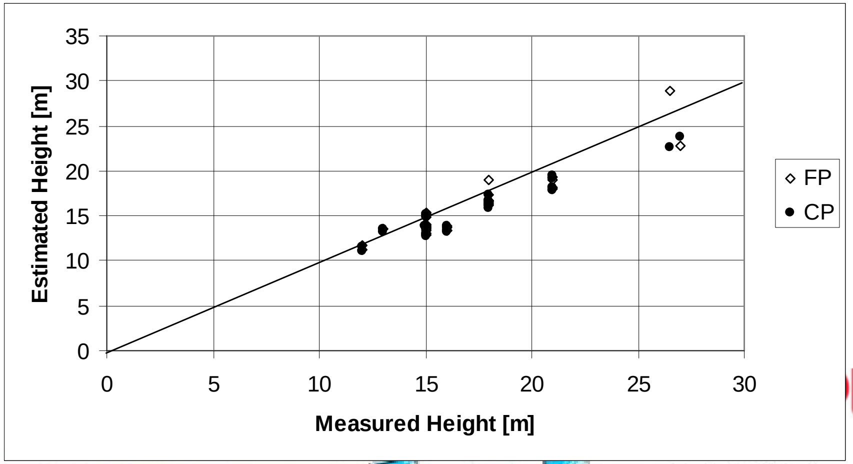




Angular sector



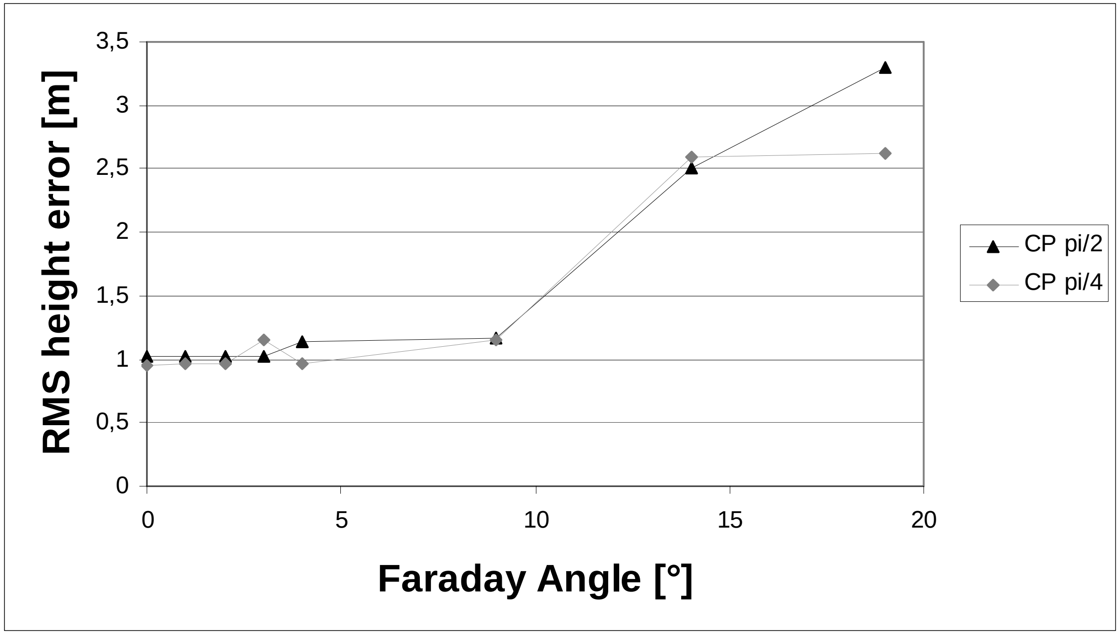
Inversion results



Compact PolInSAR inversion

$Z_{\Omega_1\Omega_2} = \{\gamma_{RT}(\Omega_1, \Omega_2)\}$

$\Omega_1 = \Omega_2$ | $\Omega_1 \neq \Omega_2$



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Conclusions

- Identical ionospheres
  - Invariance of the set of interferometric coherences when the ionospheres are identical on both measurements
    - Full polarimetry and compact polarimetry (Circular transmit)
- Different ionospheres
  - 3 invariant coherences with Faraday rotation for FP
  - 1 invariant coherence with CP
  - Full polarimetry
    - Correct the two datasets prior to data analysis
  - Compact polarimetry
    - Correct for the differential FR by maximizing the interferometric coherences over all linear polarisations