EQUIVALENCE OF DIFFERENT FORMAT RADAR POLARIMETRIC DATA FOR COHERENCY MATRIX ESTIMATION

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OUTLINE

- Introduction
- Coherency Matrix **T** Definition
- Data Formats
- Coherency Matrix Estimation Algorithms
- Equivalence Demonstration
- Real Data Analysis
- Conclusions

INTRODUCTION

- Hermitian Coherency matrix is largely employed for many Pol-SAR application :

a) Speckle Filtering
b) Detection Studies
c) Classification
d) Inversion Studies
e) Interferometry

- Multi look effect have been recently investigated.
- Signal formats considered for coherency matrix estimation are heterogeneous:

a) Time format (multilook averaging processing)
b) Spatial format (Boxcar filtering)
c) Hybrid format (multilook + Spatial filter)

- **Goal:** Coherency matrix estimation obtained from different four data formats are identical
- Validation is performed by real data analysis, in ISAR scenario

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DEFINITION OF THE TARGET COHERENCY MATRIX

- The target 4x4 coherency matrix **T** has been defined by S. R. Cloude extending the Wiener definition of wave 2x2 coherency matrix :

$$\mathbf{S} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \rightarrow \mathbf{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} b = S_{HH} + S_{VV} & r = S_{HH} + S_{VV} & g = S_{HV} + S_{VH} & n = jS_{HV} - jS_{VH} \end{bmatrix}^T$$

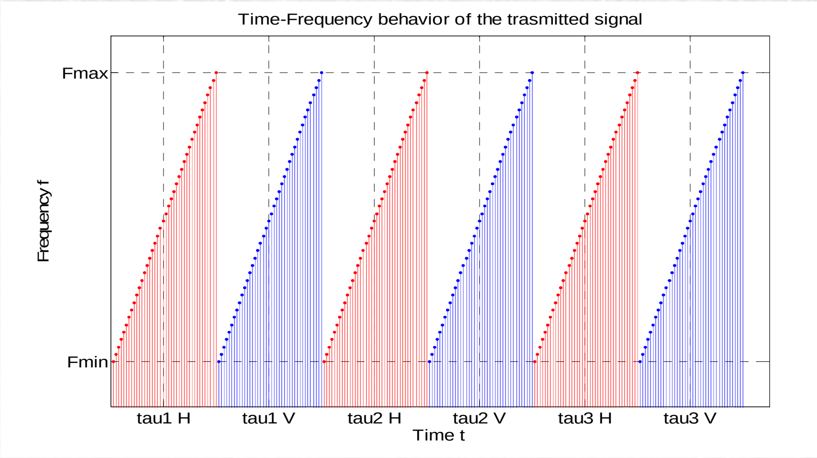
$$\mathbf{T} = \mathbf{k}\mathbf{k}^H = \begin{bmatrix} |b|^2 & br^* & bg^* & bn^* \\ rb^* & |r|^2 & rg^* & rn^* \\ gb^* & gr^* & |g|^2 & gn^* \\ nb^* & nr^* & ng^* & |n|^2 \end{bmatrix}$$

- Polarimetric signals, can be considered random processes, and **T** elements can be estimated by introducing a time averaging operator $\langle \ldots \rangle$

$$\overline{\mathbf{T}} = \langle \mathbf{T} \rangle = \frac{1}{(b-a)} \int_a^b \mathbf{T} dt \qquad \langle \mathbf{T} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathbf{T}(t_n)$$

ASSUMPTIONS

A) Transmitted Signal: Stepped Frequency Waveforms

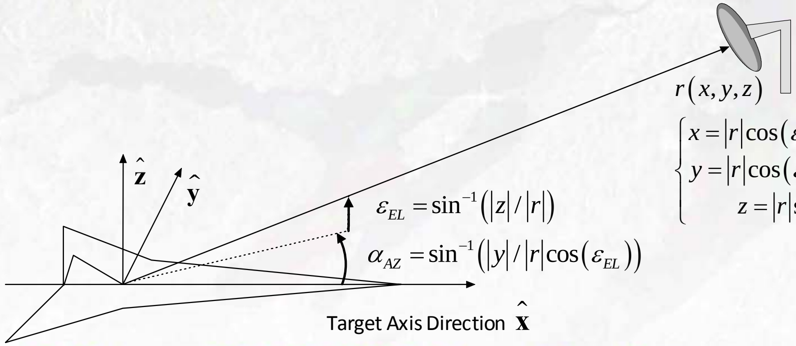


$$f_n = f_0 + n\Delta f$$

$$1 \le n \le N$$

$$\langle \mathbf{T} \rangle = \frac{1}{N}\sum_{n=1}^{N} \mathbf{T}\left(f_n\right) =$$

B) Uniform Azimuth sampling and radial motion compensated signals



$$\begin{cases} x = |r|\cos(\varepsilon_{EL})\cos(\alpha_{AZ}) \\ y = |r|\cos(\varepsilon_{EL})\sin(\alpha_{AZ}) \\ z = |r|\sin(\varepsilon_{EL}) \end{cases}$$

$$\alpha_{az}\left(\tau_k\right) = \alpha_0 + k\Delta\alpha$$

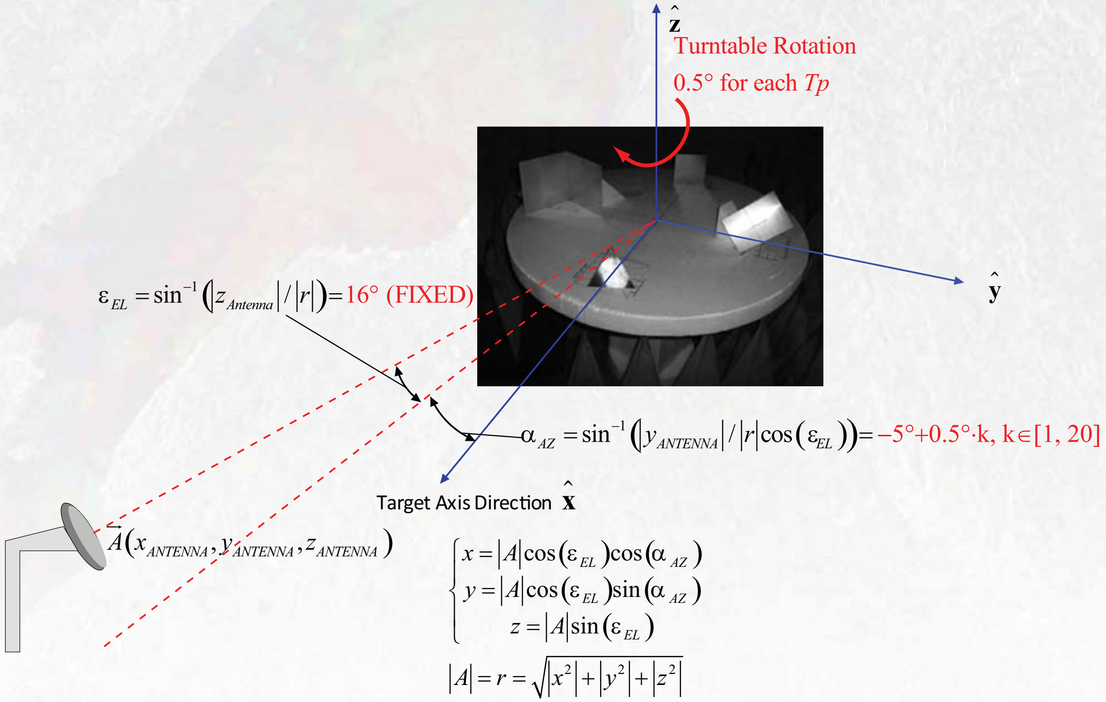
$$\varepsilon_{El}\left(\tau_k\right) = \varepsilon_0$$

Cross Range imaging is synthesized by Fast Fourier Transform

EXPERIMENTAL REFERENCE DATA

Pol-ISAR UWB System in Controlled Environment:
4 Combination of elementary targets over Polystyrene Turntable in Anechoic Chamber (CEPAMiR Adelaide)

RADAR PARAMETERS	
Central Frequency	f0 =10 GHz
Frequency Step	Δf = 50 MHz
Tx frequencies for Sweep	N = 80
Tx Sweeps for Pol.	K = 20
Azimuth Angle	+- 5°
Azimut Step	δα = 0.5°
Tx/Rx Pol.	H,V
Range Resolution	Δr = 3.75cm
Cross Range Resolution	Δcr = 9 cm
Elevation Angle	16°



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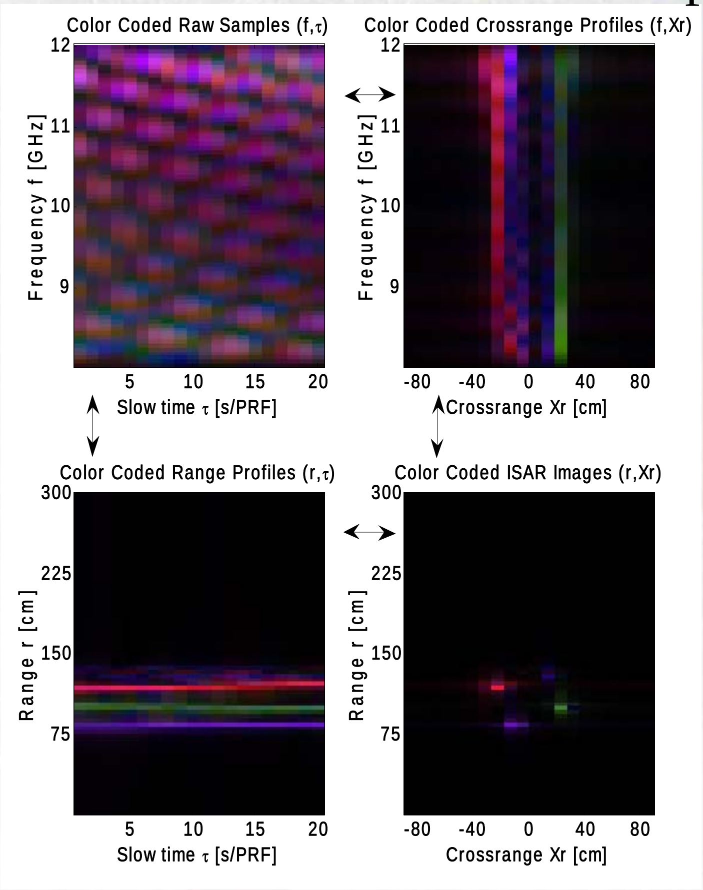
POLARIMETRIC DATA FORMATS

Format-I : *frequency / time*

$$S_{TR}(f_n,\tau_k)=\begin{bmatrix} S_{TR}(f_1,\tau_1) & S_{TR}(f_1,\tau_2) & . & . & S_{TR}(f_1,\tau_K)\\ S_{TR}(f_2,\tau_1) & S_{TR}(f_2,\tau_2) & . & . & S_{TR}(f_2,\tau_K)\\ . & . & . & . & .\\ . & . & . & . & .\\ S_{TR}(f_N,\tau_1) & S_{TR}(f_N,\tau_2) & . & . & S_{TR}(f_N,\tau_K)\end{bmatrix}$$

$1\le n\le N; 1\le k\le K; T=H,V; R=H,V$



Format-IV : *frequency / crossrange*

$$S_{TR}(f_n,Xr_j)=DFT_\tau\left(S_{TR}(f_n,Xr_j)\cdot U(f_n,Xr_j)\right)=\frac{1}{\sqrt{K}}\sum_{j=1}^{K}S_{TR}(f_n,Xr_j)e^{j2\pi(k-1)(j-1)/K}\Big|_{1\le k\le K}=$$

$$=\begin{bmatrix} S_{TR}(f_1,Xr_1) & S_{TR}(f_1,Xr_2) & . & . & S_{TR}(f_1,Xr_K)\\ S_{TR}(f_2,Xr_1) & S_{TR}(f_2,Xr_2) & . & . & S_{TR}(f_2,Xr_K)\\ . & . & . & . & .\\ . & . & . & . & .\\ S_{TR}(f_N,Xr_1) & S_{TR}(f_N,Xr_2) & . & . & S_{TR}(f_N,Xr_K)\end{bmatrix}$$

$1\le i\le N; 1\le j\le K; T=H,V; R=H,V$

Format-II : *range / time*

$$S_{TR}(r_i,\tau_k)=IDFT_f\left(S_{TR}(f_n,\tau_k)\right)=\frac{1}{\sqrt{N}}\sum_{n=1}^{N}S_{TR}(f_n,\tau_k)e^{j2\pi(n-1)(i-1)/N}\Big|_{1\le i\le N}=$$

$$\begin{bmatrix} S_{TR}(r_1,\tau_1) & S_{TR}(r_1,\tau_2) & . & . & S_{TR}(r_1,\tau_K)\\ S_{TR}(r_2,\tau_1) & S_{TR}(r_2,\tau_2) & . & . & S_{TR}(r_2,\tau_K)\\ . & . & . & . & .\\ . & . & . & . & .\\ S_{TR}(r_N,\tau_0) & S_{TR}(r_N,\tau_1) & . & . & S_{TR}(r_N,\tau_K)\end{bmatrix}$$

$1\le i\le N; 1\le k\le K; T=H,V; R=H,V$

Format-III : *range / crossrange*

$$S_{TR}(r_i,Xr_j)=DFT_\tau\left(S_{TR}(r_i,\tau_j)\cdot U(r_i,\tau_j)\right)=\frac{1}{\sqrt{K}}\sum_{j=1}^{K}S_{TR}(r_i,\tau_k)e^{j2\pi(k-1)(j-1)/K}\Big|_{1\le k\le K}=$$

$$=\begin{bmatrix} S_{TR}(r_1,Xr_1) & S_{TR}(r_1,Xr_2) & . & . & S_{TR}(r_1,Xr_K)\\ S_{TR}(r_2,Xr_1) & S_{TR}(r_2,Xr_2) & . & . & S_{TR}(r_2,Xr_K)\\ . & . & . & . & .\\ . & . & . & . & .\\ S_{TR}(r_N,Xr_1) & S_{TR}(r_N,Xr_2) & . & . & S_{TR}(r_N,Xr_K)\end{bmatrix}$$

$1\le i\le N; 1\le j\le K; T=H,V; R=H,V$

$$\begin{cases}\text{Blue}=S_{HH}+S_{VV}\\ \text{Red}=S_{HH}-S_{VV}\\ \text{Green}=S_{HV}+S_{HV}\end{cases}$$

COHERENCY MATRIX ESTIMATION ALGORITHMS

1. *Projection of polarimetric channels on Pauli scattering vector* **k**
2. *Outer Product of scattering vectors (every pixel is represented by a Rank-1* **T** *matrix)*
3. *2D average operation*

$$\text{ALGO-I}: \langle \textit{frequency} / \textit{time} \rangle$$

$$\overline{\mathbf{T}}_I = \langle \mathbf{T} \rangle = \frac{1}{NK}\sum_{n=1}^{N}\sum_{k=1}^{K}\mathbf{T}(f_n,\tau_k)$$

$$\mathbf{T}(f_n,\tau_k) = \mathbf{k}_I(f_n,\tau_k)\,\mathbf{k}_I(f_n,\tau_k)^H$$

$$\mathbf{k}_I = \frac{1}{\sqrt{2}}\begin{bmatrix} b_I = S_{HH}(f_n,\tau_k) + S_{VV}(f_n,\tau_k) \\ g_I = S_{HH}(f_n,\tau_k) - S_{VV}(f_n,\tau_k) \\ r_I = S_{HV}(f_n,\tau_k) + S_{VH}(f_n,\tau_k) \\ n_I = jS_{HV}(f_n,\tau_k) - jS_{VH}(f_n,\tau_k) \end{bmatrix}$$

$$1 \le n \le N; 1 \le k \le K$$

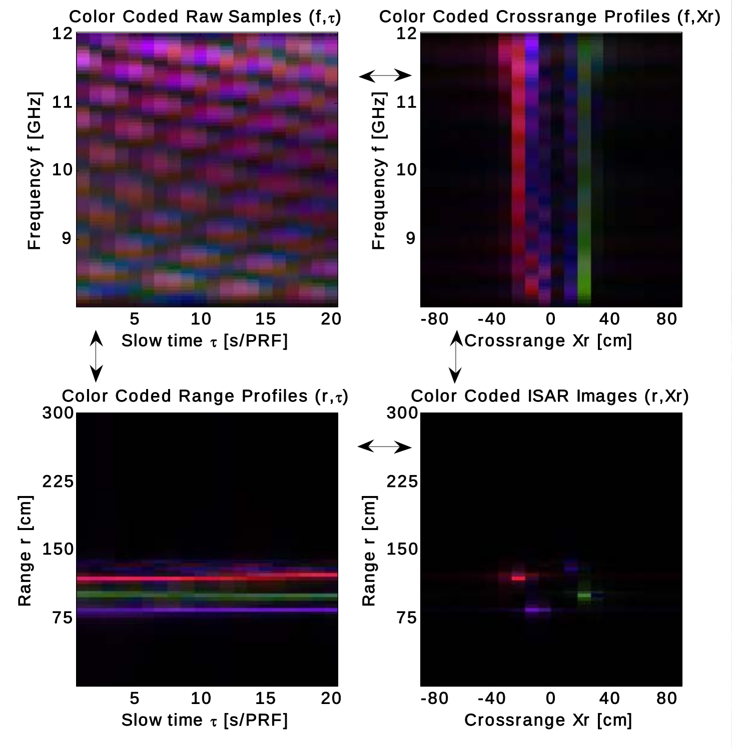
$$\text{ALGO-II}: \langle \textit{range} / \textit{time} \rangle$$

$$\overline{\mathbf{T}}_{II} = \langle \mathbf{T} \rangle = \frac{1}{NK}\sum_{i=1}^{N}\sum_{k=1}^{K}\mathbf{T}(r_i,\tau_k)$$

$$\mathbf{T}(r_i,\tau_k) = \mathbf{k}_{II}(r_i,\tau_k)\,\mathbf{k}_{II}(r_i,\tau_k)^H$$

$$\mathbf{k}_{II} = \frac{1}{\sqrt{2}}\begin{bmatrix} b_{II} = S_{HH}(r_i,\tau_k) + S_{VV}(r_i,\tau_k) \\ r_{II} = S_{HH}(r_i,\tau_k) - S_{VV}(r_i,\tau_k) \\ g_{II} = S_{HV}(r_i,\tau_k) + S_{HV}(r_i,\tau_k) \\ n_{II} = jS_{HV}(r_i,\tau_k) - jS_{HV}(r_i,\tau_k) \end{bmatrix}$$

$$1 \le i \le N; 1 \le k \le K$$



$$\begin{cases} \text{Blue} = S_{HH} + S_{VV} \\ \text{Red} = S_{HH} - S_{VV} \\ \text{Green} = S_{HV} + S_{HV} \end{cases}$$

$$\text{ALGO-IV}: \langle \textit{frequency} / \textit{crossrange} \rangle$$

$$\overline{\mathbf{T}}_{III} = \langle \mathbf{T} \rangle = \frac{1}{NK}\sum_{n=1}^{N}\sum_{j=1}^{K}\mathbf{T}(f_n,Xr_j)$$

$$\mathbf{T}(r_i,Xr_j) = \mathbf{k}_{IV}(f_n,Xr_j)\,\mathbf{k}_{III}(f_n,Xr_j)^H$$

$$\mathbf{k}_{IV} = \frac{1}{\sqrt{2}}\begin{bmatrix} b_{IV} = S_{HH}(f_n,Xr_j) + S_{VV}(f_n,Xr_j) \\ r_{IV} = S_{HH}(f_n,Xr_j) - S_{VV}(f_n,Xr_j) \\ g_{IV} = S_{HV}(f_n,Xr_j) + S_{VH}(f_n,Xr_j) \\ n_{IV} = jS_{HV}(f_n,Xr_j) - jS_{VH}(f_n,Xr_j) \end{bmatrix}$$

$$1 \le i \le N; 1 \le j \le K$$

$$\text{ALGO-III}: \langle \textit{range} / \textit{crossrange} \rangle$$

$$\overline{\mathbf{T}}_{III} = \langle \mathbf{T} \rangle = \frac{1}{NK}\sum_{i=1}^{N}\sum_{j=1}^{K}\mathbf{T}(r_i,Xr_j)$$

$$\mathbf{T}(r_i,Xr_j) = \mathbf{k}_{III}(r_i,Xr_j)\,\mathbf{k}_{III}(r_i,Xr_j)^H$$

$$\mathbf{k}_{III} = \frac{1}{\sqrt{2}}\begin{bmatrix} b_{III} = S_{HH}(r_i,Xr_j) + S_{VV}(r_i,Xr_j) \\ r_{III} = S_{HH}(r_i,Xr_j) - S_{VV}(r_i,Xr_j) \\ g_{III} = S_{HV}(r_i,Xr_j) + S_{VH}(r_i,Xr_j) \\ n_{III} = jS_{HV}(r_i,Xr_j) - jS_{VH}(r_i,Xr_j) \end{bmatrix}$$

$$1 \le i \le N; 1 \le j \le K$$

EQUIVALENCE BETWEEN RA

$ALGO\text{-}I$ for a single column:

$$\overline{\mathbf{T}_I}^{k=\bar{k}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{T}\left(f_n,\tau_{k=\bar{k}}\right) = \frac{1}{N}\sum_{n=1}^{N}\mathbf{T}\left(f_n\right)$$

$ALGO\text{-}II$ for a single column

$$\overline{\mathbf{T}_{II}}^{k=\bar{k}} = \left\langle \mathbf{T}\right\rangle = \frac{1}{N}\sum_{i=1}^{N}\mathbf{T}\left(r_i,\tau_{k=\bar{k}}\right) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{T}\left(r_i\right)$$

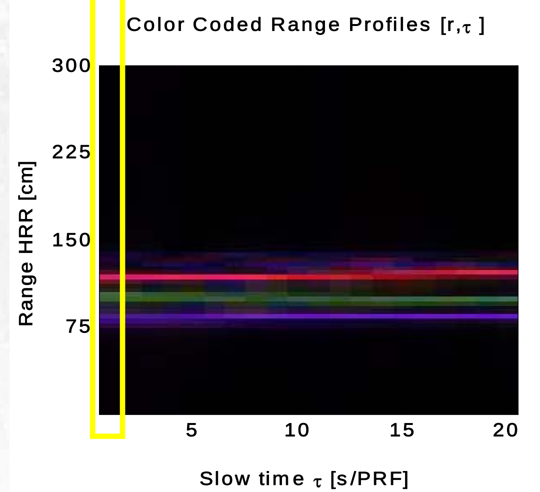
$$\overline{\mathbf{T}_{II}}^{k=\bar{k}} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{T}\left(r_i,\tau_{k=\bar{k}}\right) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{T}\left(r_i\right)$$

$$\frac{1}{N}\begin{bmatrix} \sum_{i=1}^{N}\left|b(r_i)\right|^2 & \sum_{i=1}^{N}b(r_i)r(r_i)^* & \sum_{i=1}^{N}b(r_i)g(r_i)^* & \sum_{i=1}^{N}b(r_i)n(r_i)^* \\ \sum_{i=1}^{N}r(r_i)b(r_i)^* & \sum_{i=1}^{N}\left|r(r_i)\right|^2 & \sum_{i=1}^{N}r(r_i)g(r_i)^* & \sum_{i=1}^{N}r(r_i)n(r_i)^* \\ \sum_{i=1}^{N}g(r_i)b(r_i)^* & \sum_{i=1}^{N}g(r_i)r(r_i)^* & \sum_{i=1}^{N}\left|g(r_i)\right|^2 & \sum_{i=1}^{N}g(r_i)n(r_i)^* \\ \sum_{i=1}^{N}n(r_i)b(r_i)^* & \sum_{i=1}^{N}n(r_i)r(r_i)^* & \sum_{i=1}^{N}n(r_i)g(r_i)^* & \sum_{i=1}^{N}\left|n(r_i)\right|^2 \end{bmatrix} \begin{cases} b(r_i) = S_{HH}\left(r_i,\tau_{k=\bar{k}}\right) + S_{VV}\left(r_i,\tau_{k=\bar{k}}\right) \\ r(r_i) = S_{HH}\left(r_i,\tau_{k=\bar{k}}\right) - S_{VV}\left(r,\tau_{k=\bar{k}i}\right) \\ g(r_i) = S_{HV}\left(r_i,\tau_{k=\bar{k}}\right) + S_{VH}\left(r_i,\tau_{k=\bar{k}}\right) \\ n(r_i) = jS_{HV}\left(r_i,\tau_{k=\bar{k}}\right) - jS_{VH}\left(r_i,\tau_{k=\bar{k}}\right) \end{cases}$$

$\Updownarrow_{PARSEVAL}$ $\Updownarrow_{PARSEVAL}$

$$= \frac{1}{N}\begin{bmatrix} \sum_{n=1}^{N}\left|B(f_n)\right|^2 & \sum_{n=1}^{N}B(f_n)R(f_n)^* & \sum_{n=1}^{N}B(f_n)G(f_n)^* & \sum_{i=i}^{N}B(f_n)N(f_n)^* \\ \sum_{n=1}^{N}R(f_n)B(f_n)^* & \sum_{n=1}^{N}\left|R(f_n)\right|^2 & \sum_{n=1}^{N}R(f_n)G(f_n)^* & \sum_{n=1}^{N}R(f_n)N(f_n)^* \\ \sum_{n=1}^{N}G(f_n)B(f_n)^* & \sum_{n=1}^{N}G(f_n)R(f_n)^* & \sum_{n=1}^{N}\left|G(f_n)\right|^2 & \sum_{n=1}^{N}G(f_n)N(f_n)^* \\ \sum_{n=1}^{N}N(f_n)B(f_n)^* & \sum_{n=1}^{N}N(f_n)R(f_n)^* & \sum_{n=1}^{N}N(f_n)G(f_n)^* & \sum_{n=1}^{N}\left|R(f_n)\right|^2 \end{bmatrix} \begin{cases} B(f_n) = DFT\left[b(r_i)\right] \\ R(f_n) = DFT\left[r(r_i)\right] \\ G(f_n) = DFT\left[g(r_i)\right] \\ N(f_n) = DFT\left[n(r_i)\right] \end{cases} =$$

$$= \frac{1}{N}\sum_{n=1}^{N}\mathbf{T}\left(f_n\right) = \frac{1}{N}\sum_{n=1}^{N}\mathbf{T}\left(f_n, r_{k=\bar{k}}\right) = \overline{\mathbf{T}_I}^{k=\bar{k}}$$



The two above sentences are equivalent by applying the Bessel –Parseval Theorem to the matrices elements. By averaging *K* columns of estimates of ALGO-I and ALGO-II, **identical coherency matrix are obtained.**

$$\overline{\mathbf{T}}_{II} = \frac{1}{K}\sum_{k=1}^{K}\overline{\mathbf{T}_{II}}^{k=\bar{k}} = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{N}\sum_{i=1}^{N}\mathbf{T}\left(r_i,\tau_{k=\bar{k}}\right) = \frac{1}{K}\sum_{k=1}^{K}\frac{1}{N}\sum_{n=1}^{N}\mathbf{T}\left(f_b,\tau_{k=\bar{k}}\right) =$$

$$\frac{1}{KN}\sum_{k=1}^{K}\sum_{n=1}^{N}\mathbf{T}\left(f_b,\tau_k\right) = \overline{\mathbf{T}}_I$$

EQUIVALENCE BETWEEN CROSS-R

ALGO - II for a fixed range bin

$$\overline{\mathbf{T}}_{II}^{\,i=\bar{i}} = \langle \mathbf{T} \rangle = \frac{1}{K}\sum_{k=1}^{K} \mathbf{T}\left(r_{i=\bar{i}}, \tau_k\right) = \frac{1}{K}\sum_{k=1}^{K} \mathbf{T}\left(\tau_k\right)$$

ALGO - III for a fixed range bin

$$\overline{\mathbf{T}}_{III}^{\,i=\bar{i}} = \frac{1}{K}\sum_{j=1}^{K} \mathbf{T}\left(r_{i=\bar{i}}, Xr_j\right) = \frac{1}{K}\sum_{j=1}^{K} \mathbf{T}\left(Xr_j\right)$$

$$\overline{\mathbf{T}_{II}}^{\,i=\bar{i}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{T}\left(r_{i=\bar{i}}, \tau_k\right) = \frac{1}{K}\sum_{k=1}^{K} \mathbf{T}\left(\tau_k\right)$$

$$\frac{1}{K}\begin{bmatrix} \sum_{k=1}^{K}\left|b(\tau_k)\right|^2 & \sum_{k=1}^{K} b(\tau_k) r(\tau_k)^* & \sum_{k=1}^{K} b(\tau_k) g(\tau_k)^* & \sum_{k=1}^{K} b(\tau_k) n(\tau_k)^* \\ \sum_{k=1}^{K} r(\tau_k) b(\tau_k)^* & \sum_{k=1}^{K}\left|r(\tau_k)\right|^2 & \sum_{k=1}^{K} r(\tau_k) g(\tau_k)^* & \sum_{k=1}^{K} r(\tau_k) n(\tau_k)^* \\ \sum_{k=1}^{K} g(\tau_k) b(\tau_k)^* & \sum_{k=1}^{K} g(\tau_k) r(\tau_k)^* & \sum_{k=1}^{K}\left|g(\tau_k)\right|^2 & \sum_{k=1}^{K} g(r_i) n(\tau_k)^* \\ \sum_{k=1}^{K} n(\tau_k) b(r_i)^* & \sum_{k=1}^{K} n(\tau_k) r(\tau_k)^* & \sum_{k=1}^{K} n(\tau_k) g(\tau_k)^* & \sum_{k=1}^{K}\left|n(\tau_k)\right|^2 \end{bmatrix} \quad \begin{cases} b(\tau_k) = S_{HH}\left(r_{i=\bar{i}}, \tau_k\right) + S_{VV}\left(r_{i=\bar{i}}, \tau_k\right) \\ r(\tau_k) = S_{HH}\left(r_{i=\bar{i}}, \tau_k\right) - S_{VV}\left(r_{i=\bar{i}}, \tau_k\right) \\ g(\tau_k) = S_{HV}\left(r_{i=\bar{i}}, \tau_k\right) + S_{VH}\left(r_{i=\bar{i}}, \tau_k\right) \\ n(\tau_k) = jS_{HV}\left(r_{i=\bar{i}}, \tau_k\right) - jS_{VH}\left(r_{i=\bar{i}}, \tau_k\right) \end{cases}$$

$$\Updownarrow_{PARSEVAL} \qquad \Updownarrow_{PARSEVAL}$$

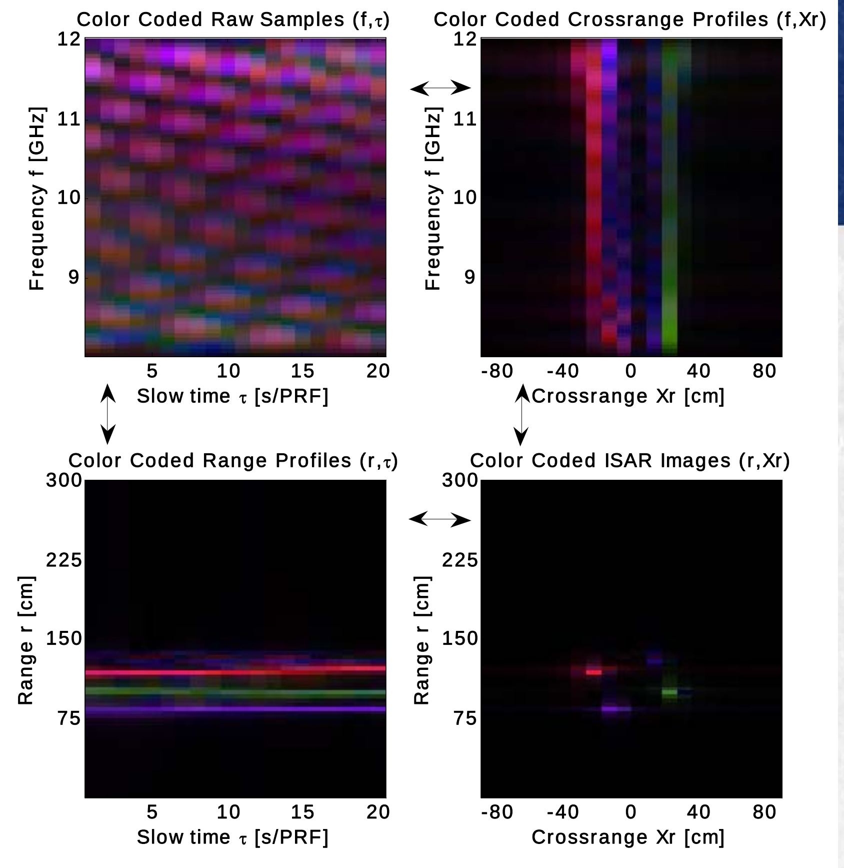
$$= \frac{1}{K}\begin{bmatrix} \sum_{k=1}^{K}\left|B(Xr_k)\right|^2 & \sum_{k=1}^{K} B(Xr_k) R(Xr_k)^* & \sum_{k=1}^{K} B(Xr_k) G(Xr_k)^* & \sum_{k=1}^{K} B(Xr_k) N(Xr_k)^* \\ \sum_{k=1}^{K} R(Xr_k) B(Xr_k)^* & \sum_{k=1}^{K}\left|R(Xr_k)\right|^2 & \sum_{k=1}^{K} R(Xr_k) G(Xr_k)^* & \sum_{k=1}^{K} R(Xr_k) N(Xr_k)^* \\ \sum_{k=1}^{K} G(Xr_k) B(Xr_k)^* & \sum_{k=1}^{K} G(Xr_k) R(Xr_k)^* & \sum_{k=1}^{K}\left|G(Xr_k)\right|^2 & \sum_{k=1}^{K} G(Xr_k) N(Xr_k)^* \\ \sum_{k=1}^{K} N(Xr_k) B(Xr_k)^* & \sum_{k=1}^{K} N(Xr_k) R(Xr_k)^* & \sum_{k=1}^{K} N(Xr_k) G(Xr_k)^* & \sum_{k=1}^{K}\left|N(Xr_k)\right|^2 \end{bmatrix} \quad \begin{cases} B(Xr_k) = DFT\left[b(\tau_k)\right] \\ R(Xr_k) = DFT\left[r(\tau_k)\right] \\ G(Xr_k) = DFT\left[g(\tau_k)\right] \\ N(Xr_k) = DFT\left[n(\tau_k)\right] \end{cases} =$$

$$= \frac{1}{K}\sum_{k=1}^{K} \mathbf{T}\left(Xr_k\right) = \frac{1}{K}\sum_{k=1}^{K} \mathbf{T}\left(r_{i=\bar{i}}, Xr_k\right) = \overline{\mathbf{T}_{III}}^{\,i=\bar{i}}$$

The two above sentences are equivalent by applying the Bessel –Parseval Theorem to the matrices elements.
By averaging *N* rows of estimates of ALGO-II and ALGO-III, **identical coherency matrix are obtained**.

$$\overline{\mathbf{T}}_{II} = \frac{1}{N}\sum_{\bar{i}=1}^{N} \overline{\mathbf{T}}_{II}^{\,i=\bar{i}} = \frac{1}{N}\sum_{\bar{i}=1}^{N} \frac{1}{K}\sum_{k=1}^{K} \mathbf{T}\left(r_{i=\bar{i}}, \tau_k\right) = \frac{1}{N}\sum_{\bar{i}=1}^{N} \frac{1}{K}\sum_{j=1}^{K} \mathbf{T}\left(r_{i=\bar{i}}, X\tau_j\right) = \frac{1}{KN}\sum_{i=1}^{K}\sum_{j=1}^{N} \mathbf{T}\left(r_i, X\tau_j\right) = \overline{\mathbf{T}}_{III}$$

EQUIVALENCE BETWEEN THE FOUR DATA FORMATS



$$\overline{\mathbf{T}} = \langle \mathbf{T} \rangle = \frac{1}{(b-a)} \int_a^b \mathbf{T} dt$$

$\Updownarrow$ Time-Sampling

$$\langle \mathbf{T} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathbf{T}(t_n)$$

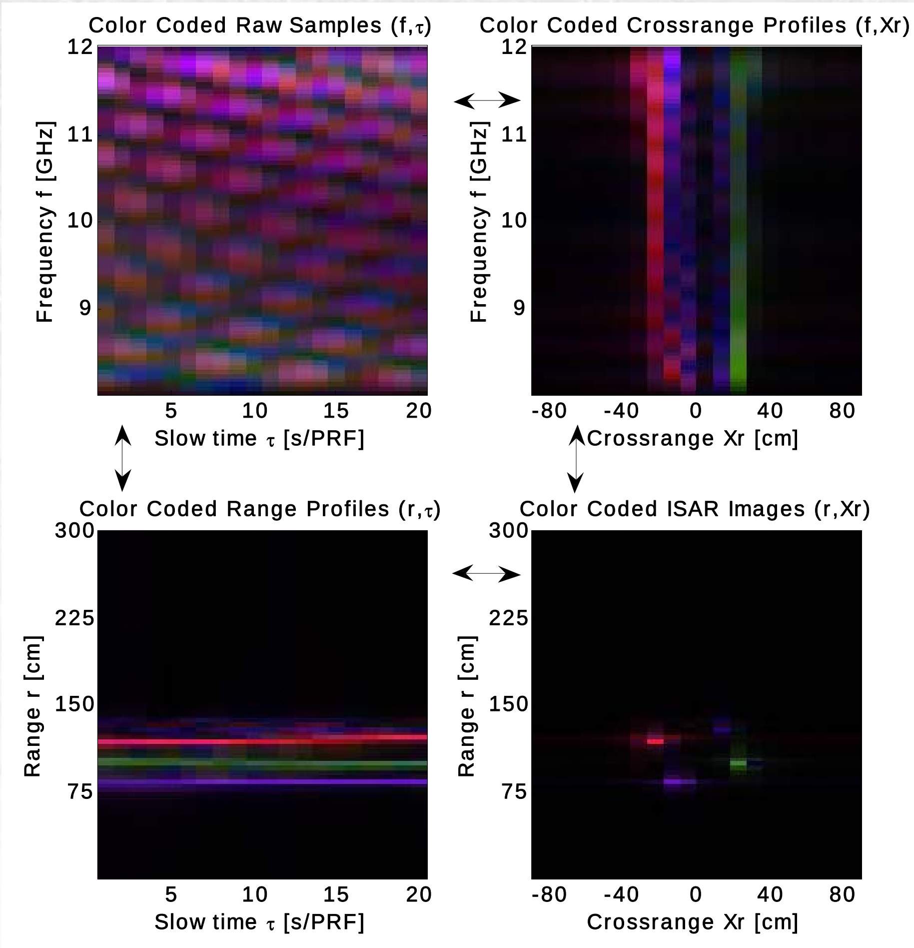
$\Updownarrow$ *Stepped Frequency Waveform*

$$\langle \mathbf{T} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathbf{T}(f_n) \quad \Leftrightarrow_{K-Average} \quad \overline{\mathbf{T}_{\mathrm{I}}} = \frac{1}{NK} \sum_{n=1}^{N} \sum_{k=1}^{K} \mathbf{T}(f_n, \tau_k) \quad \Leftrightarrow^{\alpha_{AZ}}_{MOTION} \wedge \Leftrightarrow_{PARSEVAL} \quad \overline{\mathbf{T}_{\mathrm{IV}}} = \frac{1}{NK} \sum_{n=1}^{K} \sum_{j=1}^{K} \mathbf{T}(f_n, Xr_j)$$

$\Updownarrow$ PARSEVAL $\qquad \Updownarrow$ PARSEVAL $\qquad \Updownarrow$ PARSEVAL

$$\langle \mathbf{T} \rangle = \frac{1}{N} \sum_{i=1}^{N} \mathbf{T}(r_i) \quad \Leftrightarrow_{K-Average} \quad \overline{\mathbf{T}_{\mathrm{II}}} = \frac{1}{NK} \sum_{i=1}^{N} \sum_{k=1}^{K} \mathbf{T}(r_i, \tau_k) \quad \Leftrightarrow^{\alpha_{AZ}}_{MOTION} \wedge \Leftrightarrow_{PARSEVAL} \quad \overline{\mathbf{T}_{\mathrm{III}}} = \frac{1}{NK} \sum_{i=1}^{N} \sum_{j=1}^{K} \mathbf{T}(r_i, Xr_j)$$

EXPERIMENTAL VALIDATION OF THE THEORY



$$\begin{cases} \text{Blue} = S_{HH} + S_{VV} \\ \text{Red} = S_{HH} - S_{VV} \\ \text{Green} = S_{HV} + S_{HV} \end{cases}$$

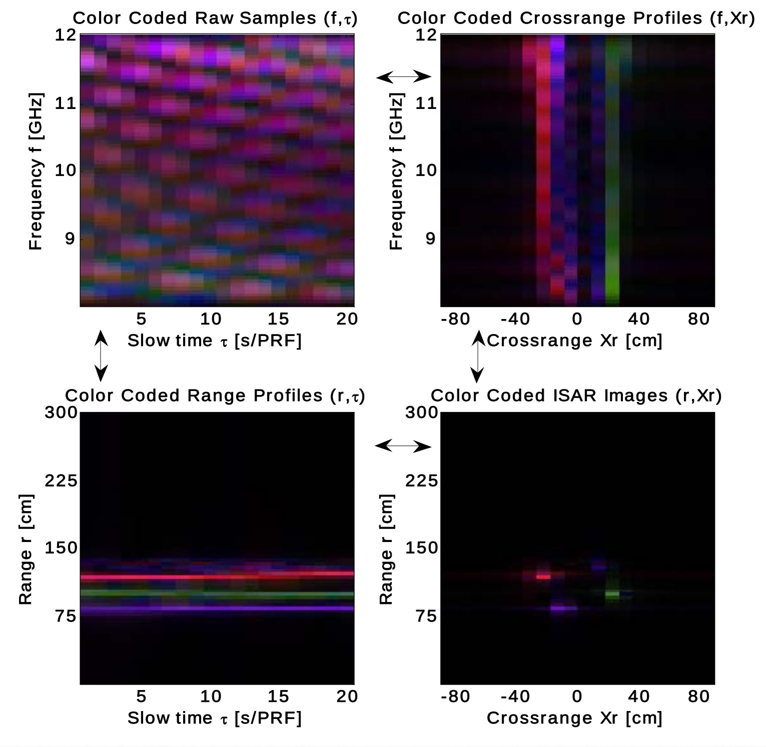
$$\overline{\mathbf{T}}_{ALGO-I} = \overline{\mathbf{T}}_{ALGO-II} = \overline{\mathbf{T}}_{ALGO-III} = \overline{\mathbf{T}}_{ALGO-IV}$$

$$\overline{\mathbf{T}} = \begin{bmatrix} 67 & -3-38i & 1+7i & 0 \\ -3+38i & 74 & -12-6i & 0 \\ 1-7i & -12+6i & 17 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} 10^{-2}$$

CONCLUSIONS:

Equivalence Between different domains of integration of the signals has been proved in the special case.

A relationship between temporal, spectral and spatial estimate has been traced



$$\langle t \rangle$$

$\Updownarrow$ *sampling*

$$\langle t_n \rangle$$

$\Updownarrow$ *SFW*

$$\langle f_n \rangle \Leftrightarrow_{K-average} \langle f_n, \tau_k \rangle \Leftrightarrow_{MOTION-MODEL} \wedge \Leftrightarrow_{PARSEVAL} \langle f_n, Xr_j \rangle$$

$\Updownarrow$ *PARSEVAL* $\qquad \Updownarrow$ *PARSEVAL* $\qquad \Updownarrow$ *PARSEVAL*

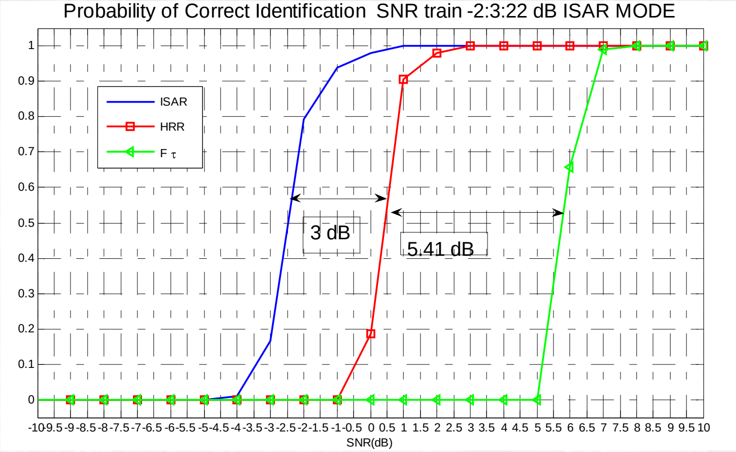
$$\langle r_i \rangle \Leftrightarrow_{K-average} \langle r_i, \tau_k \rangle \Leftrightarrow_{MOTION-MODEL} \wedge \Leftrightarrow_{PARSEVAL} \langle r_i, Xr_j \rangle$$

APPLICATION OF THE THEORY:

We have been extending the results of our classification scheme based on ICTD [1]:
[1] Paladini, Martorella, Berizzi, "Incoherent ISAR decomposition for target classification", Proc. Eurad 2008 Amsterdam.

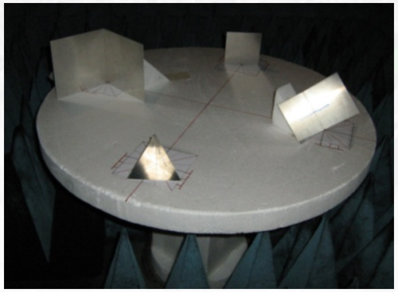
1)Training is made by windowed ISAR noisy images

2) Testing is made with the Pol-ISAR, Pol -HRR, and frequency/time data on AWGN



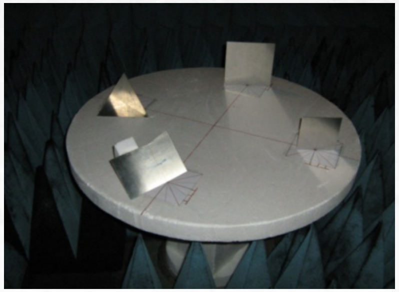
- Applying A=23 by B=10 pixel window ISAR provides greater performances.
- Other data formats work correctly but with lower performances in noisy scenario.
- In AWGN ISAR Gain = (NK)/(AB)
- N/A= 80/23 = 5.41 dB
- K/B= 20/10 = 3 dB

T1

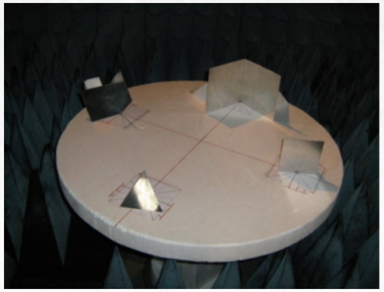


T2

T3



T4



Thank you For The Attention!

• Appendices: A review of Bessel-Parseval Theorem.

$Let,\ x(r_i), y(r_i),$ be two complex function of real discrete variable

the complex cross-correlation of the two signals can be evaluated on the range domain:

$$z(r_i) = \sum_{i=1}^{N} x(r_i) y(r_i)^*$$

The second term can be numericcally transformed by means of DFT:

$$Y(f_n) = DFT\left(y(r_i)\right) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} y(r_i)^* \, e^{j2\pi(i-1)(n-1)/N}{}_{1\le r\le N}$$

The transformation is reversible by the IDFT:

$$y(r_i) = IDFT\left(Y(f_n)\right) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} Y(f_n)^* \, e^{-j2\pi(i-1)(n-1)/N}{}_{1\le r\le N}$$

by substitution we obtain:

$$z(r_i) = \sum_{i=1}^{N} x(r_i) \left[\frac{1}{\sqrt{N}} \sum_{n=1}^{N} Y(f_n)^* e^{-j2\pi(i-1)(n-1)/N} {}_{1\le i\le N} \right]$$

by inverting the two series operators:

$$z(r_i) = \sum_{n=1}^{N} Y(f_n)^* \left[\frac{1}{\sqrt{N}} \sum_{i=1}^{N} x(r_i) e^{-j2\pi(i-1)(n-1)/N} {}_{1\le f\le N} \right] =$$

$$\sum_{n=1}^{N} Y(f_n)^* X(f_n) = \sum_{n=1}^{N} X(f_n) Y(f_n)^*$$

therefore

$$z(r_i) = \sum_{i=1}^{N} x(r_i) y(r_i)^* = \sum_{n=1}^{N} X(f_n) Y(f_n)^*$$