Multi-frequency Pol-InSAR signatures of a subpolar glacier

Jayanti Sharma, Irena Hajnsek, Kostas Papanastassiou
Microwaves and Radar Institute
Introduction: Modelling Coherence over Land Ice

Scattering models for land ice

- Modelling of Pol-InSAR data for land ice at relatively early stage of development
- No consensus on the principle mechanisms for volume scattering and their variation with frequency/glacier facie
- Most existing research models the backscattering coefficient ($\sigma^0$) rather than interferometric observables

Glacier geometry

- Begin with classic Pol-InSAR scattering models (Treuhaft2000, Cloude&Papathanassiou 2003) adapted to a glacier geometry, differs from vegetation scenarios in that:
  - At microwave wavelengths, glaciers can be considered infinitely thick
  - Surface scattering (if significant) occurs at the top of the volume
  - Refraction effects (changes angle of incidence and wavelength in the medium)
Data description

- **SVALEX (Svalbard Airborne Experiment)** SAR data campaign
- Conducted in April 2005 over Austfonna ice cap (Summit) and Etonbreen drainage basin (Glacier), Svalbard, Norway (~ 80°N, 24°E)
- Joint project between
  - **DLR-HR** (Microwaves and Radar Institute)
  - **AWI** (Alfred-Wegener Institute)

Corner reflector at summit

DLR’s E-SAR (left), AWI’s airborne platform
## E-SAR parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>X</th>
<th>L</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ [m]</td>
<td>0.031</td>
<td>0.23</td>
<td>0.86</td>
</tr>
<tr>
<td>$f0$ [GHz]</td>
<td>9.60</td>
<td>1.30</td>
<td>0.35</td>
</tr>
<tr>
<td>PRF [Hz/chan]</td>
<td>1000</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>Chirp BW [MHz]</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slant-rng res. $\Delta$rng [m]</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slant-rng spac. $\delta$rng [m]</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Az res. SLC $\Delta$az [m]</td>
<td>0.59</td>
<td>0.67</td>
<td>2.00</td>
</tr>
<tr>
<td>Az spac. SLC $\delta$az [m]</td>
<td>0.33</td>
<td>0.45</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Focus on L-/P-band, Baselines of 5, 10, 15 m
Experimental data, Summit HH

L-band $|HH|^2$

P-band $|HH|^2$

L-band, $B=5m$ $|\gamma|$

P-band, $B=15m$ $|\gamma|$
Coherence Model
Coherence modelling

Fundamental InSAR observable complex coherence:

\[
\gamma = \frac{\langle S_1 S_2^* \rangle}{\sqrt{\langle S_1^* S_2^* \rangle}}
\]

\[
\gamma = e^{-j \frac{4\pi \Delta R}{\lambda}} \cdot \int W_{ng} (y)^2 e^{-jk_y y} dy \cdot \int \sigma_v(z) e^{-jk_{zvol} z} dz
\]

\[
\gamma = e^{-j\varphi_0} \cdot \gamma_{\text{range}} \cdot \gamma_{\text{vol}}
\]

\[
k_{zvol} = \frac{4\pi \sqrt{\varepsilon}}{\lambda} \frac{\Delta \theta_r}{\sin \theta_r}
\]

\[
k_y = \frac{4\pi}{\lambda} \cos(\theta) \Delta \theta_r
\]

Firm \quad \varepsilon = \text{const}

\[
\gamma = e^{-j\varphi_0} \cdot \gamma_{\text{range}} \cdot \gamma_{\text{vol}} \cdot \gamma_{\text{SNR}} \cdot \gamma_{\text{temporal}} \cdot \gamma_{\text{process}}
\]
Volumetric decorrelation

Assuming scattering medium is:
- homogeneously lossy (i.e. constant extinction)
- consists of uniformly distributed and uncorrelated scattering centres

Where:
- \( \sigma_v^0 \) = averaged normalized RCS per unit vol. [m\(^2\)/m\(^3\)]
- \( \kappa_e \) = extinction coeff. [1/m]
- \( \kappa_e = \cos(\theta_r) / d_{pen} \)
- \( z/\cos(\theta_r) = \) penetration length in the vol. [m]

Evaluate \( \gamma_{vol} \) for \(-\infty < z < 0\)

\[
\gamma_{vol} = \frac{\int \sigma_v(z) e^{-jk_{zvol}z} dz}{\int \sigma_v(z) dz}
\]

\[
\sigma_v(z) = \sigma_v^0 e^{\frac{2z\kappa_e}{\cos\theta_r}}
\]

\[
\gamma_{vol} = \frac{1}{1 + \frac{j \cos \theta_r k_{zvol}}{2\kappa_e}}
\]
**Modelling Coherence**

**Ice properties**
- Consider models characterised by different combinations of ice properties:
  - Volume uniformity (uniform, non-uniform)
  - Volume isotropy (random, oriented)
  - Surface contribution (negligible, significant)

**Models**
- If *volume is uniform* (as assumed in derivation $\gamma_{vol}$), can define 4 models in terms of volume isotropy and surface contribution:
  - Random Volume model (RV)
  - Random Volume under Ground model (RVuG)
  - Oriented Volume model (OV)
  - Oriented Volume under Ground model (OVuG)

- If *volume is non-uniform*, e.g. because firn density/grain size change with depth $z$, then $\varepsilon(z), \theta_r(z), \Delta \theta_r(z)$
Modelling coherence: Uniform volume

- Introduce **volume orientation** with polarisation-dependence on extinction $\kappa_e(\mathbf{w})$
- Introduce **surface scattering** contribution with surface-to-volume scattering ratio $m(\mathbf{w})$
- Define combined coherence from volume + surface scattering + topographic phase as:

\[
\gamma_z = e^{-j\varphi_0} \frac{\gamma_{vol}(\kappa_e(\mathbf{w}), \varepsilon) + m(\mathbf{w})}{1 + m(\mathbf{w})}
\]

**Unknowns:**
- Extinction $\kappa_e(\mathbf{w})$
- Topography $\varphi_0$
- G/V Ratio $m(\mathbf{w})$

**Observables:**
- Complex Coh. $\text{Re}(\gamma_z)$
- $\text{Im}(\gamma_z)$

**Input params:**
- permittivity $\varepsilon$
Coherence Magnitude Uniform Vol. Model Predictions

Volume only, $m=0$

$$|\gamma_{vol}| = \frac{1}{\sqrt{1+(\cos\theta_r k_{zvol})^2}}$$

Volume+Ground, $m\neq 0$, $\kappa_e=0.3$ dB/m

$$|\gamma_z| = \frac{|\gamma_{vol} + m|}{1 + m}$$
Coherence Magnitude: Experimental Data

- Plot $|\gamma_z|$ vs. $k_z$ for single $\theta$
- Use constrained non-linear least squares to find best-fit $m$ and $\kappa_e$ values
- Investigate changes in estimated $m$ and $\kappa_e$ with polarisation ($w$), frequency and test site

$R^2=0.96$

Summit L-band, HV, $\theta=35^\circ$, $\kappa_e=0.40$ dB/m, $m=0.2$
Coherence Magnitude: Experimental Data

Summit L-band, VV, $\theta=35^\circ$, $\kappa_e=0.61$ dB/m, $m=1.0$

$R^2=0.92$
Coherence Magnitude: Experimental Data

Glacier L-band, HH, $\theta=35^\circ$
$\kappa_e=0.51$ dB/m, $m=0$

$R^2=0.83$
Check phase centre of ice near ground reflectors with penetration depth from coherence magnitude model

\[
d_{\text{model}}(50\%, \text{ 2-way power}) = d_{\text{pen}} \cdot \ln(0.5)/2
\]

\[
d_{\text{phs centre}} = \Delta \phi_{\text{corner-ice}}/k_z \text{vol}
\]
Coherence Magnitude: Validation with Ground Reflectors

- P-band model-fit was poor (limited $k_z$ diversity), assume $m=0$ in coherence magnitude model
- $d_{model}$ (50%, 2-way power) = $d_{pen} \cdot \ln(0.5)/2$
- $d_{phs\_centre} = \Delta \phi_{wire-ice}/k_z \cdot vol$

Glacier P-band, HH

<table>
<thead>
<tr>
<th></th>
<th>$d_{phs_centre}$ [m]</th>
<th>$d_{model}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire pt</td>
<td>-14.8 ± 3.3</td>
<td>-17.0</td>
</tr>
<tr>
<td>Wire avg</td>
<td>-9.5 ± 3.3</td>
<td>± 5.0</td>
</tr>
</tbody>
</table>
Some data fit the $|\gamma|$ model very well

As an inversion method is unsuitable for:
- Non-homogeneous areas
- No diversity in $k_z$ (stable flights and/or few baselines)
- Sensitive to decorrelation effects

Use complex coherences…
**Complex Coherence: Simulations**

\[ \theta = 35^\circ, \odot: k_z = 0.1 \]

**Random Volume (RV)**
- \( \kappa_e \neq \kappa_e(w) \)
- \( \gamma_{vol} = \text{const. for all pols} \)

**Oriented Volume (OV)**
- \( \kappa_e(w) \)
- \( \gamma_{vol}(w) \)

\[
\gamma_{vol} = \frac{1}{1 + \frac{j \cos \theta \cdot k_{zvol}}{2 \kappa_e}}
\]

\[
\gamma_{vol}(\vec{w}) = \frac{1}{1 + \frac{j \cos \theta \cdot k_{zvol}}{2 \kappa_e(\vec{w})}}
\]
Complex Coherence: Simulations

Random Volume under Ground (RVuG)
\( \kappa_e \neq \kappa_e(w) = 0.3 \text{ dB/m}, \ m(w) \)
\( \gamma_{vol} = \text{const.}, \ \gamma_z(w) \)

Oriented Volume under Ground (OVuG)
\( \kappa_e(w), \ m(w) \)
\( \gamma_{vol}(w), \ \gamma_z(w) \)

\[ \gamma_z(\bar{w}) = e^{-j\phi_0} \frac{\gamma_{vol} + m(\bar{w})}{1 + m(\bar{w})} \]
**Complex Coherence: Experimental Results**

- **Glacier L-band, (B=5 m, k_z=0.09)**
  - \( RV, \gamma_z = \gamma_{vol} = \text{const. for all pols} \)
  - \( OV, \gamma_z = \gamma_{vol}(w) \text{ on semi-circle} \)

- **Glacier P-band, (B=15 m, k_z = 0.05)**
  - \( RVuG, \gamma_z(w), \text{ line through (1,0)} \)
  - \( OVuG, \gamma_z(w) \)
**Initial results: Polarisation Coherence Tomography (PCT)**

- **PCT (Cloude 2006):** Employs low-order Legendre polynomial decomposition of the vertical structure function ($\sigma_v$) whose coefficients are determined from the interferometric coherence
- **Advantage:** does not assume a particular vertical structure function
- **Has been adapted to a glacier geometry**
- **Initial results confirm exponential distribution of $\sigma_v(z)$**

![Graph showing Relative Intensity vs Depth](image1)

**Glacier L-band, HH, (B=5 m, $k_z=0.06$)**

Estimated Vertical Structure from PCT

![Graph showing Depth vs AZ](image2)
Summary

Coherence model

- Adapted existing Pol-InSAR models created for vegetation scenarios to a glacier geometry

Coherence magnitude

- Modelled coherence magnitudes provide a reasonable match to experimental data at multiple polarizations (L-band)
- Coherence magnitude model predicts depths close to phase centres referenced to ground reflectors (~ 8 m at L-band, ~ 17 m at P-band)

Complex coherence

- Demonstrated position of complex coherences referenced using ground reflectors
- Initial results indicate a possible Oriented Volume under Ground (OVuG) scenario

PCT

- Preliminary results confirm an exponential trend of the vertical structure function
Questions ?