A Review of Polarization Orientation Estimation from Polarimetric SAR Data

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ABSTRACT

In this paper, we review estimation algorithms and applications of polarization orientation angle induced by terrain slopes. We develop a unified analysis of estimation algorithms based on the circular polarization covariance matrix. The concept of reflection symmetry is used to explain the soundness of the circular polarization method, and to show problems associated with other algorithms. The effect of radar frequency, scattering media, and polarimetric calibration will also be discussed. SIR-C, and JPL AIRSAR L-band and P-band polarimetric SAR images are used for demonstration.

1 INTRODUCTION

Interferometric (SAR) has been successfully applied to measure topography. Radar interferometry requires the use of dual antennas separated by a baseline in a single pass system, or by a repeated pass configuration. Recently, a new technique has been developed using polarimetric SAR (POLSAR) to measure azimuth slopes that are related to shifts in polarization orientation angles [1-4]. Polarization orientation angle is one of the least used parameters among the wealth of polarimetric information when analyzing POLSAR data. The polarization state of an electromagnetic wave is characterized by its polarization orientation angle $\theta$ and its ellipticity angle. The orientation angle, which is of importance to this study, is the angle between the major axis of the polarization ellipse and the horizontal axis. For distributed media, orientation shifts are induced by azimuthal slopes, which cause the polarization rotation about the line of sight.

Fig. 1 Polarimetric SAR derived topography based on SIR-C L-Band POLSAR data. Orientation angles derived from the POLSAR data are used to produce a DEM.

Polarization orientation shifts are frequently considered as a direct measure of azimuthal slopes. This is not correct. Lee [4] and Pottier [5] have found that orientation shifts are also affected by the radar look angle and the range slope. However, in the absence of range slope data, orientation angles can be used as a rough estimate of azimuthal slopes, especially in cases of gentle terrain and a large radar look angle. An example is shown in Fig. 1. A SIR-C L-band
polarimetric SAR data of Camerota, Italy, is used in the computation of orientation angles. The upper right image is the SAR image, color coded by |HV|, |HH|, and |VV| as red, green and blue, respectively. Orientation angles were estimated from the polarimetric data. A DEM was generated by integrating orientation angles as azimuthal slopes. When integrating azimuthal slopes to produce a DEM, we use the coastline to provide a common zero elevation reference. The 3-D representation of the derived DEM, with the polarization SAR image (upper right) draped over it, is shown in the lower part of Fig. 1. It is evident that polarization orientation angles can be extracted from POLSAR data, and that they are related to the azimuthal slopes.

In this paper, we review orientation angle estimation methods, and the radar geometry related to the azimuth and range slopes. Difficulties are frequently encountered in the estimation of orientation angles from POLSAR images. These difficulties will be discussed and the effect of radar wavelength and calibration on the estimation will be investigated. Applications to geophysical parameter estimation, and to ocean surface feature sensing will be specifically mentioned.

2 POLARIMETRIC SAR DATA REPRESENTATION AND ROTATION

To derive the orientation angle estimation algorithm, it is necessary to understand the rotation of polarimetric matrices and the transformation to a circular polarization basis.

2.1 Scattering Matrix

POLSAR data can be represented by the scattering matrix (1) for single-look complex data.

\[
\mathbf{S} = \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \]  

(1)

For backscattering from reciprocal media, \( S_{HV} = S_{VH} \). The rotation of an orientation angle \( \theta \) is achieved by

\[
\tilde{\mathbf{S}} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}
\]  

(2)

The “\( \sim \)” on top of the matrix \( \mathbf{S} \) denotes the matrix after the rotation by \( \theta \). For convenience, this notation will be used throughout this chapter to indicate the matrix after the rotation by \( \theta \).

2.2 Covariance Matrix from Circular Polarizations

The circular polarization components can be derived from the scattering matrix. The three circular components for right-right, left-left and right-left circular polarizations are

\[
\begin{align*}
S_{RR} &= (S_{HH} - S_{VV} + i2S_{HV})/2 \\
S_{LL} &= (S_{VV} - S_{HH} + i2S_{HV})/2 \\
S_{RL} &= i(S_{HH} + S_{VV})/2
\end{align*}
\]  

(3)

The rotation by an orientation angle can be obtained by applying (2). After manipulation, we have.

\[
\begin{align*}
\tilde{S}_{RR} &= S_{RR} e^{-i2\theta} \\
\tilde{S}_{LL} &= S_{LL} e^{i2\theta} \\
\tilde{S}_{RL} &= S_{RL}
\end{align*}
\]  

(4)

Defining a circular basis vector,

\[
c = \begin{bmatrix} S_{RR} \\ \sqrt{2}S_{RL} \\ S_{LL} \end{bmatrix}
\]  

(5)

the circular polarization based covariance matrix \( \mathbf{G} \) is obtained from the vector \( c \) by

\[
\mathbf{G} = \langle c c^T \rangle = \begin{bmatrix} \left\langle S_{RR}^2 \right\rangle & \sqrt{2}\left\langle S_{RR}S_{RL}^* \right\rangle & \left\langle S_{RR}S_{LL}^* \right\rangle \\ \sqrt{2}\left\langle S_{RL}S_{RR}^* \right\rangle & 2\left\langle S_{RL}^2 \right\rangle & \sqrt{2}\left\langle S_{RL}S_{LL}^* \right\rangle \\ \left\langle S_{LL}S_{RR}^* \right\rangle & \sqrt{2}\left\langle S_{LL}S_{RL}^* \right\rangle & \left\langle S_{LL}^2 \right\rangle \end{bmatrix}
\]  

(6)

Applying (4), the upper off-diagonal terms of the rotated circular polarization covariance matrix are modified to (7).
\[
\hat{\mathbf{G}} = \begin{bmatrix}
\langle |S_{RR}|^2 \rangle & \sqrt{2}\langle (S_{RR}S_{RL}^*) e^{\text{i}2\theta} \rangle & \langle (S_{RR}S_{LL}^*) e^{\text{i}4\theta} \rangle \\
\sqrt{2}\langle (S_{RL}S_{RR}^*) e^{\text{i}2\theta} \rangle & 2\langle |S_{RL}|^2 \rangle & \sqrt{2}\langle (S_{RL}S_{LL}^*) e^{\text{i}2\theta} \rangle \\
\langle (S_{LL}S_{RR}^*) e^{\text{i}4\theta} \rangle & \sqrt{2}\langle (S_{LL}S_{RL}^*) e^{\text{i}2\theta} \rangle & \langle |S_{LL}|^2 \rangle 
\end{bmatrix}
\] (7)

The lower off-diagonal terms are the conjugate of their respective upper off-diagonal terms in (7). The diagonal terms are rotation invariant. It is apparent in (7) that the change in the orientation angle affects only the phases of off-diagonal terms. The circular polarization method to be discussed later is directly related to the \((1, 3)\) term.

3 RADAR GEOMETRY OF POLARIZATION ORIENTATION ANGLE

The change in the polarization orientation angle is geometrically related to topographical slopes and the radar look angle [3]. Fig. 2 shows the schematic diagram. The unit vector pair \((\hat{x}, \hat{y})\) defines a horizontal plane, \((\hat{y}, \hat{z})\) defines the radar incidence plane, and the radar line of sight is in the reverse direction of the axis \(\hat{I}_1\). The angle \(\phi\) between \(\hat{I}_1\) and \(\hat{z}\) is the radar look angle. The axis \(\hat{x}\) is in the azimuth direction, and \(\hat{y}\) is in the ground range direction. The surface normal for a ground patch is denoted by \(\hat{N}\). Assume that the polarimetric SAR is calibrated so that the horizontal polarization (H) is parallel to the horizontal plane \((\hat{x}, \hat{y})\), and the vertical polarization (V) is in the incidence plane.

![Fig. 2 A schematic diagram of the radar imaging geometry which relates the orientation angle to the ground slopes.](image)

For a horizontal surface patch, its surface normal \(\hat{N}\) is in the incidence plane, and no orientation angle shift is induced. However, for a surface patch with an azimuthal tilt, its surface normal \(\hat{N}\) is no longer in the incidence plane. The induced polarization orientation angle shift \(\theta\) is the angle that rotates the incidence plane \((\hat{y}, \hat{z})\) about the line of sight to the surface normal by the following equation [3],

\[
\tan \theta = \frac{\tan \omega}{-\tan \gamma \cos \phi + \sin \phi}
\] (8)

where \(\tan \omega\) is the azimuth slope, and \(\tan \gamma\) is the slope in the ground range direction. This equation shows that the orientation angle shift is mainly induced by the azimuth slope, but that it is also a function of the range slope and the radar look angle. For small range slope, the orientation angle tends to overestimate azimuthal slope angle by the factor of \(\left(1/\sin \phi\right)\). In general, orientation angle measurements overestimate the actual azimuth slope angles, when the range
slope is positive (toward the radar), and may underestimate them, if the range slope is negative. The difference between
the orientation angle and the corresponding azimuth slope angle becomes smaller for larger radar look angles. For an
accurate estimate of azimuth slopes, range slope information, therefore, is required. This can be achieved by imaging
the area with polarimetric SAR in orthogonal passes [2].

4 ORIENTATION ANGLE ESTIMATION: THE CIRCULAR POLARIZATION ALGORITHM

The orientation angle shift causes rotation of both the scattering matrix and circular covariance matrix about the line of
sight. Since the orientation angle information is embedded in the polarimetric SAR data, several methods have been
developed to estimate azimuth slope induced orientation angles. The polarization signature method [1] and the circular
polarization method [3] have been proven to be effective. Other methods have also been proposed [5-8]. The
polarization signature method is based on the concept that the angle $\theta$ corresponds to the change in the polarization
orientation angle, and is estimated by the shift of the maximum co-polarization response. The polarization signature, as
proposed by van Zyl [9], gives the polarization responses in the orientation and ellipticity plane, which is used to find
the maximum co-polarization response. To speed up the optimization process, a steepest ascent algorithm was also
developed [1].

In this chapter, we will limit the discussion to the circular polarization method, because it is based on a theoretical
derivation. This method is also simpler and more accurate than any other methods.

The circular polarization method was proposed in [3] to extract the orientation angle, using right-right (RR) and left-left
(LL) circular polarizations from either the single-look complex, or from the multi-look, data. This algorithm has proven
to be successful. The concept of reflection symmetry [10] is used to explain the soundness of the circular polarization
method, and to show problems associated with other algorithms. L-band POLSAR data of Camp Roberts, California
are used to substantiate the value of this method.

Krogager and Czyz [6] proposed estimating orientation angles using the phase difference between right-hand and left-
hand circular polarizations. This method has been further modified and refined by Lee et al. [3]. It is summarized as
follows.

From the circular covariance matrix (7), the right-right and left-left circular polarization term (i.e., the (1,3) term) can be
used to estimate the orientation angle. If the dominant orientation angle shift is induced by azimuth slope, then
\[
<\tilde{S}_{RR}^* \tilde{S}_{LL}^* > = <S_{RR}^* \tilde{S}_{LL}^* > e^{-i4\theta} 
\]

(9)

For a reflection symmetrical medium as associated with a horizontal surface, \(<S_{RR}^* \tilde{S}_{LL}^* > \) is required to be real in
value, so that it will not corrupt the orientation angle related to the phase term $e^{-i4\theta}$. We will prove that
\(<S_{RR}^* \tilde{S}_{LL}^* > \) is real for a reflection symmetrical medium. For a reflection symmetrical medium, the cross-pol and co-
pol correlation terms are zero. Substituting (3) into \(<S_{RR}^* \tilde{S}_{LL}^* > \) and setting terms that contain $S_{HV}$ to zero (except
the \(<S_{HV}^2 > \) term), we have
\[
<S_{RR}^* \tilde{S}_{LL}^* > = \frac{1}{4} (-<S_{HH}^2 - S_{VV}^2 > + 4 <S_{HV}^2 >) 
\]

(10)

This term is real, so the argument of \(<S_{RR}^* \tilde{S}_{LL}^* > \) is zero or $\pi$. Consequently, the phase difference between $S_{HH}$ and
$S_{VV}$ does not cause error in the estimation of orientation angles.

The factor of $4$ in (9) limits the range of $\theta$ to $[-\pi/4, \pi/4]$. To derive a general expression, we express
\(<S_{RR}^* \tilde{S}_{LL}^* > \) in linear basis. We then have
\[
<S_{RR}^* \tilde{S}_{LL}^* > = \frac{1}{4} \left( -4 <S_{HH}^2 - S_{VV}^2 > + 4 <S_{HV}^2 > - i4 \text{Re}(<S_{HH}^2 - S_{VV}^2 \cdot \tilde{S}_{HV}^*>) \right)
\]

(11)

From (9) and (11), we would have
\[
-4\theta = \text{Arg}(<S_{RR}^* \tilde{S}_{LL}^* >) = \tan^{-1}\left( \frac{-4 \text{Re}(<S_{HH}^2 - S_{VV}^2 \cdot \tilde{S}_{HV}^*>)}{-<S_{HH}^2 - S_{VV}^2 > + 4 <S_{HV}^2 >} \right)
\]

(12)
If (12) is applied directly, it would introduce errors, because, for an azimuth symmetrical medium, 
\( <\bar{S}_{HH} - \bar{S}_{VV}> \) is normally greater than \( 4 <|\bar{S}_{HV}|^2> \). The denominator is then negative. Consequently, when the numerator is near zero, the arctangent is near \( \pm \pi \). The orientation angle would be \( \pm \pi / 4 \) rather than near zero as it should be. To match the orientation angle corresponding to the azimuth slope angle, the bias must be removed by adding \( \pi \). The circular polarization estimator is

\[
\theta = \begin{cases} 
\eta, & \text{if } \eta \leq \pi / 4 \\
\eta - \pi / 2, & \text{if } \eta > \pi / 4 
\end{cases}
\]  

(13)

where

\[
\eta = \frac{1}{4} \tan^{-1} \left( \frac{-4 \text{Re}\left( <\bar{S}_{HH} - \bar{S}_{VV}> \bar{S}_{HV}^* \right)}{-<|\bar{S}_{HH} - \bar{S}_{VV}|^2|^2 + 4 <|\bar{S}_{HV}|^2> + \pi} \right)
\]  

(14)

The arctangent in (14) is computed in the range of \( (-\pi, \pi) \).

Fig. 3  Polarization orientation angles extracted from a 600x600 pixel area of Camp Roberts, California.  (A) The L-band span image of an area containing a variety of complex scatterers.  (B) The orientation angle image derived by the circular polarization method.  (C) For comparison, the orientation angles from a DEM generated using C-band interferometry.  (D) Histogram of orientation angles using the circular polarization algorithm.

This algorithm has proven successful for orientation angle estimation [3]. An example is given here of applying it to the JPL AIRSAR L-band data of Camp Roberts, California. The area of Camp Roberts consists of rugged terrain with sparsely distributed oak trees. In the valley, the vegetation is much more dense. JPL AIRSAR simultaneously imaged this area with C-Band TOPSAR to obtain interferometric data. This permits verification of polarimetric SAR derived orientation angles by those obtained from the interferometric generated DEM using Eq. (8).

A small area within this large image is selected which contains a variety of complex scatterers. Fig. 3 shows the span image of the selected area. The image size is 600x600 pixels. Rugged mountain terrain and a valley are present within the image. This POLSAR image contains artifacts, which appear as bright horizontal streaks. The orientation angle image derived by the circular polarization method is shown in Fig. 3B. For comparison, we computed the orientation angles from the interferometry generated DEM (shown in Fig. 3C). The circular polarization derived orientation angles show good agreement with those derived from DEM. However, noisy results are scattered throughout the areas that correspond to bright areas in Fig. 3A. We observed that these areas also represent steep positive range slope areas that produce higher radar returns. For steep positive range slope areas, the scattering approaches the specular case, where
\( \tilde{S}_{HH} = \tilde{S}_{VV} \). In this situation, the measurement sensitivity is low for azimuth slope induced orientation angles. Consequently, the near specular scattering makes estimation very sensitive to vegetation variations.

A secondary effect is that high radar returns produce high speckle noise. Speckle noise is multiplicative in nature in the sense that the noise level is higher in higher return areas [11]. The bright specular returns from distributed media induce higher speckle noise in the orientation angle estimation. Fig. 3D shows the histogram of orientation angles produced by the circular polarization method. The bell shaped curve indicates that it is a good estimator compared with unsymmetrical histograms from most other methods.

Comparisons of this algorithm with the original polarization signature algorithm and other methods based on polarimetric target decomposition [12] have been made by Lee et al.[3]. This algorithm is slightly better than the polarization signature method, but is much better than those algorithms based on target decomposition.

5 DISCUSSION

Besides the selection of algorithms, many factors affect the accuracy of orientation angle estimations. They will be summarized in the following:

Radar Frequency

Orientation angles can be derived from L-band and P-band POLSAR data, but less successfully from C-band or higher frequency data. Higher frequency POLSAR responses are less sensitive to azimuth slope variations, because electromagnetic waves with shorter wavelengths are less penetrative and are more sensitive to small scatterers within a resolution cell. The orientation angles induced from smaller scatterers overwhelm the orientation angle induced from the ground slope. We have found that C-band data produces a very noisy orientation angle image. On the other hand, longer wavelength radars (operating for example at P-band) are more penetrative and are less sensitive to smaller scatterers, and produce better results than L-band. Radio frequency interference, however, is often a problem.

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![P-Band SAR Image](image1) ![P-Band Orientation](image2) ![L-Band Orientation](image3)

Fig. 4 In heavily forested areas, orientation angles can be extracted from P-band data, but not from L-band or higher frequency data. JPL AIRSAR P-band and L-band Data of Freiburg Forest, Germany, is applied to extract orientation angles. (A) \(|HH-VV|, |HV|\) and \(|HH+VV|\) color coded P-Band SAR image, (B) Orientation angle image derived from the P-band data, (C) Orientation angle image derived from the L-band data.

JPL AIRSAR data from Freiburg Forest, Germany is used for illustration and is shown in Fig. 7. The area is heavily forested as shown in Fig 7A. The orientation angles derived from the P-band data (Fig. 7B) are well defined and show the strength of penetration from P-band. The orientation angles derived from the L-band data (Fig. 7C) are noisy, and are less sensitive to the under-canopy topography. C-band data produce even worse results than L-band.

The Importance of Polarimetric Calibration

POLSAR data calibration is a crucial step in the process of deriving accurate orientation angles. Both amplitude and phase calibration accuracies affect the derivation of orientation angles. The \( |S_{HV}|^2 \) term and phase differences
between cross-polarization and co-polarization terms are especially affected. Many polarimetric SAR calibration algorithms assume zero correlation between co-polarization and cross-polarization terms [12]. This assumption could introduce errors in orientation angle estimation. Recently, a revised method has been introduced by Ainsworth et al. [13] to account for this deficiency. In addition, non-zero pitch angles of the radar platform introduce a bias in the orientation angles. These pitch angles should be properly compensated before applying the orientation angle extraction method.

**Dynamic Range of Radar Response**

The dynamic range and polarization channel isolation of the radar receiver are critical to the success of the orientation angle estimation. The success of the circular polarization methods depends on the accuracy of measuring the \(<(\tilde{S}_{hh} - \tilde{S}_{vv})\tilde{S}_{hv}^* > \) term. This term is much smaller than \(<|\tilde{S}_{hh}|^2 >\) or \(<|\tilde{S}_{vv}|^2 >\). A lack of dynamic range makes this correlation term very noisy. In addition, POLSAR data compression, if necessary, has to be carefully devised to preserve the dynamic range. The extraction of orientation angles becomes an impossible task for SAR systems with small dynamic range and poor channel isolation.

### 6 APPLICATIONS INVOLVING ORIENTATION ANGLES

**Polarimetric Data Compensation**

The derived orientation angle can be used directly to compensate POLSAR data in rugged terrain areas. It is important to compensate the polarimetric SAR data to ensure accurate extraction of geophysical parameters, such as, soil moisture, surface roughness, snow cover, and biomass. A study on POLSAR data compensation has been investigated by Lee et al. [3]. The azimuth slope compensated data shows that all components of the coherency matrix have been modified except the \(<(\tilde{S}_{hh} + \tilde{S}_{vv})\tilde{S}_{hv}^* >\) term, which is rotation invariant. The greatest changes occur in the real part of the \(<(\tilde{S}_{hh} - \tilde{S}_{vv})\tilde{S}_{hv}^* >\) term. The reduction in \(<|\tilde{S}_{hv}|^2 >\) is also significant.

**DEM Generation**

The derived orientation angles can be used to generate topography (Schuler et al. [1, 2]). Two orthogonal POLSAR flight passes are required to derive orientation angles in perpendicular directions. By applying equation (8), the ground slopes in two directions can be computed. The slope data is then used to solve a Poisson equation to estimate the elevation surface. This algorithm is similar to the global least-square phase unwrapping algorithm used by SAR interferometry. Digital elevation maps have been generated. Due to the radar layover effect, difficulties were encountered when co-registering these two images. Currently, the accuracy of the DEM derived from this method is inferior to that generated by SAR interferometry.

**Ocean Applications**

Another interesting application is for the direct estimation of ocean surface slopes. Backscattering from the ocean surface can be assumed in most cases to be homogeneous, and is characterized by two-scale Bragg scattering. This type of scattering provides excellent conditions for orientation angle estimation. Adjustment for range slopes according to Eq. (8) may not be necessary, because of small range slopes of the ocean surface, but corrections for radar look angles have to be made.

In a study of convergent current fronts within the Gulf Stream (Lee et al. [14]), it was found that there existed a sudden change in the orientation angle from positive to negative across a convergent front with the maximum slope change being smaller than 2°. This study indicates the potential to use orientation angles to estimate small ocean surface slopes within an accuracy of a fraction of a degree. This study has been expanded by Schuler et al. [15, 16] and Kasilingam et al. [8] to estimate ocean wave slope spectra, and by Schuler et al [17] who applied this technique to study internal wave radar signatures. In addition, Ainsworth et al. [18] used this technique to study ocean surface features.

### 6 CONCLUSION

In this chapter, we have reviewed the recent advances in polarization orientation angle related research. Polarization orientation angle estimations based on the circular polarization covariance matrix are reviewed in detail. The concept of reflection symmetry was used to explain the soundness of the circular polarization method, and to show problems associated with other algorithms. Difficulties encountered in the estimation of orientation angles are discussed. We
believe that this technique should be applied for better accuracy in geophysical parameter estimation by compensating polarimetric SAR data in areas of high topographic relief. The potential for DEM generation and ocean remote sensing was also discussed. SIR-C and JPL AIRSAR L-band and P-band POLSAR data were used for illustrations.

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