GOCE gravity field analysis in the framework of HPF: Operational software system and simulation results

Graz University of Technology

Roland Pail, B. Metzler, B. Lackner, T. Preimesberger, H. Goiginger, R. Mayrhofer

Austrian Academy of Sciences

E. Höck

University of Bonn

W.-D. Schuh, H. Alkhatib, B. Kargoll, C. Boxhammer, C. Siemes

Technical University Munich

M. Wermuth

3rd GOCE User Workshop, Frascati, Nov. 2006
GOCE gravity field analysis in the framework of HPF: Operational software system and simulation results

Contents

• Software architecture
• Product flow, demonstrated on the basis of a realistic numerical case study
• Discussion of selected topics (SGG filter, regularization, covariance propagation)
• Summary & conclusions
• **Quick-Look Gravity Field Analysis (QL-GFA)**
  - Fast approximate gravity field solutions
  - Analysis of SST and SGG input data
  - in parallel to the mission
  - latency: 4 hours (QL-A: based on Level 1b data)
    - 2 days (QL-B: intermediate Level 2 data)
  - common PC

• **Core Solver (CS)**
  - Rigorous ultimate-precision solution
  - Final Solver: SST, SGG: full var./covar. matrices
  - Tuning Machine: - pcgma
    - Data inspection
  - Parallel processing on Linux-PC cluster
  - SST processing: energy integral approach

---

3rd GOCE User Workshop, Frascati, Nov. 2006
SPF6000: Architectural Design & Product Flow

**INPUT DATA**
- Orbits
- Gravity gradients
- Attitude information
- Auxiliary data

**QL-GFA**
- SST-only solution
- SGG-only solution
- Combined solution
- Quality analysis of SGG residuals

**Tuning Machine**
- pcgma solution
- SGG filter design
- Data inspection

**CORE SOLVER**
- SST NEQ's

**SST proc.**
- SST-only solution

**SGG proc.**
- Assembling of SGG NEQ's

**Solution**
- Solution of combined system incl. regularization/opt. weighting

**EGM**
- EGM_QLA_2
- EGM_QLB_2i
- EGM_QLK_2i

- Gravity field model
- Full variance-covariance matrix

**EGM**
- EGM_TIM_2i
- EGM_TVC_2i
<table>
<thead>
<tr>
<th>SPF6000: HPF Acceptance Review 2*: Test Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Orbit</strong></td>
</tr>
<tr>
<td>• 60 days, 1 Hz sampling rate</td>
</tr>
<tr>
<td>• based on EGM96, degree/order 200</td>
</tr>
<tr>
<td>• noise: orbit positions: $\sigma = 3$ cm</td>
</tr>
<tr>
<td>orbit velocities: $\sigma = 0.1$ mm/s</td>
</tr>
<tr>
<td>• satellite altitude ~240 km</td>
</tr>
<tr>
<td>• realistic accelerometry</td>
</tr>
<tr>
<td><strong>Attitude</strong></td>
</tr>
<tr>
<td>• quaternions describing rotation between</td>
</tr>
<tr>
<td>inertial (IRF) and gradiometer reference</td>
</tr>
<tr>
<td>frame (GRF)</td>
</tr>
<tr>
<td><strong>Gravity gradients</strong></td>
</tr>
<tr>
<td>• gradients in GRF along orbit, 1 Hz sampled</td>
</tr>
<tr>
<td>• based on EGM96, degree/order 360</td>
</tr>
<tr>
<td>• realistic gradiometer noise assumption</td>
</tr>
<tr>
<td><strong>Addit. products</strong></td>
</tr>
<tr>
<td>• Auxiliary data: IERS products, ephemeris,</td>
</tr>
<tr>
<td>temp.var. corr.</td>
</tr>
<tr>
<td>• A priori SGG error model (generated from</td>
</tr>
<tr>
<td>„true“ SGG noise)</td>
</tr>
</tbody>
</table>

* HPF Acceptance Review (AR) 2: test of operational HPF system at the end of Development Phase 2
Gradiometer error PSD
Quick-Look Gravity Field Analysis
QL-GFA: Gravity field results: QL-A vs. QL-B

Coeff.dev. from EGM96

QL-A
(Level 1b data)
SGG-only solution

EGM_QLA_2

QL-B
(Level 2 data)
combined solution

EGM_QLB_2i

MSE estimates
Degree error median

\[ \sigma_i = \text{median}_m \left\{ \left| \overline{R}_{lm}^{(\text{est})} - \overline{R}_{lm}^{(\text{EGM})} \right| \right\} \]

with \[ \overline{R}_{lm} = \{ \bar{C}_{lm} ; \bar{S}_{lm} \} \]

QL-A: • SGG-only
QL-B: • SST-only
• SGG-only
• combined SST+SGG

Cumulative gravity anomaly errors at D/O 200

SGG-only  σ = 2.9 mGal

SST+SGG  σ = 1.8 mGal
SGG error PSD estimate $\rightarrow$ EGM_QLK_2i

$$V_{XX}$$

- **true error PSD**
- **PSD estimate**

Spectral leakage
Core Solver
**SPF6000: Architectural Design & Product Flow**

**INPUT DATA**
- Orbits
- Gravity gradients
- Attitude information
- Auxiliary data

**QL-GFA**
- SST-only solution
- SGG-only solution
- combined solution
- Quality analysis of SGG residuals

- QL gravity field models
- GOCE error PSD est.
- Quality Report sheets

- EGM\_QLA\_2
- EGM\_QLB\_2i
- EGM\_QLK\_2i

**CORE SOLVER**

**Tuning Machine**
- pcgma solution
- SGG filter design
- data inspection

**SST proc.**
- SST-only solution

**SGG proc.**
- Assembling of SGG NEQ’s

**Solution**
- Solution of combined system incl. regularization/opt. weighting

- Gravity field model
- Full variance-covariance matrix

- EGM\_TIM\_2i
- EGM\_TVC\_2i
Core Solver: Tuning Machine

- pcgma solution
- SGG filter design
- data inspection

- stand-alone gravity field solver
- optimum regularization & weighting parameters
- filters describe the metrics of the normal equation systems
- analysis of residuals
- outlier detection
<table>
<thead>
<tr>
<th>Filter</th>
<th># Cascades (xx / yy / zz)</th>
<th>Warmup # positions</th>
<th>effective order (xx / yy / zz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0020</td>
<td>(2 / 2 / 2)</td>
<td>40000</td>
<td>52 / 42 / 32</td>
</tr>
</tbody>
</table>

The graph shows the square root of the power spectral density (sqr.(PSD)) in millielectronvolts per square root of hertz (mE/sqrt(Hz)) on the y-axis, and frequency in Hertz (Hz) on the x-axis. Two lines are plotted: the true error PSD of Vxx in black, and the filter Vxx in red.
Parallel processing on Linux-PC-cluster
(54 Dual-Xeon 2.6GHz PCs, Gigabit-Ethernet
connection, performance ~210 GFlops)

• SST processing: energy integral
• SGG assembling: full normal equations
• Solution (Cholesky reduction)
  → \( \text{EGM\_TIM\_2i} \)
• Inversion → full variance-covar. matrix
  → \( \text{EGM\_TVC\_2i} \)
Assembling of SGG normal equations

- parameterization complete up to D/O 204 (see below)
- SGG filtering: cascaded filters: filter orders 52 (XX), 42 (YY), 32 (ZZ)

Computation of solution (Cholesky reduction and inversion)

- Regularization:
  Spherical Cap regularization, using an independently computed SST-only solution complete to D/O 50 as stabilizing function in the polar cap regions.
  (Expected geoid height errors in the polar cap regions mainly due to spectral leakage of the D/O 50 SST solution: ~2 m)

- Weighting:
  Optimum relative weighting factor of SST solution = 1.0 (since SST normal equations are properly scaled)

- Spectral leakage:
  In order to further reduce (small) leakage effects, the normal equations were originally set-up complete to D/O 204, and finally the coefficients an the variance-covariance matrix was truncated at D/O 200.

→ GOCE-only solution in a strict sense, i.e., NO a priori gravity field information is inherent !!!
**Idea:**
Introducing a stabilizing function at the poles and force the signal toward this function

(*Metzler & Pail, 2005: Studia geophys. geod.*)

**Advantage:**
No significant regularization bias in the regions covered by measurements.

\[
\begin{align*}
(A^T P A + \alpha [A^T A]_{CAP}) x &= A^T P \ell + \alpha [A^T \ell]_{CAP}
\end{align*}
\]
Core Solver: Final Solution

Coeff.dev. from EGM96

Error estimates

Degree error median

→ Coefficients: EGM\_TIM\_2i

→ Var.-covar. matrix: EGM\_TVC\_2i
Cumulative geoid height errors

Cov. prop. geoid height errors

at D/O 200

\begin{tabular}{ |c|c|c|c| }
\hline
$|\varphi| < 83^\circ$ & AR2 (2 months) & 1 MOP* (6 months) & 2 MOPs* (12 months) \\
\hline
$\sigma_N$ [cm] & 2.93 & 1.69 & 1.20 \\
\hline
$\sigma_{\Delta g}$ [mGal] & 0.81 & 0.47 & 0.33 \\
\hline
\end{tabular}

*Numerical case studies (based on longer data sets) revealed that assumption of error decreases with $\sqrt{N}$ is justified.
Be aware: GOCE will provide the FULL variance-covariance matrix

\[ N = A_N \cdot x \quad \text{with} \quad x = \{ C_{lm}; S_{lm} \} \]

\[ \rightarrow \quad \Sigma(N) = A_N \cdot \Sigma(x) \cdot A_N^T \]
Special subject: covariance propagation [1]

\[ N = A_N \cdot x \]  with \[ x = \{ C_{lm} ; S_{lm} \} \]

\[ \Rightarrow \quad \Sigma(N) = A_N \cdot \Sigma(x) \cdot A_N^T \]

Covar. prop. geoid heights [cm]

\[ \Sigma(x) \text{ full} \]

\[ \text{diag}(\Sigma(x)) \]
$N = A_N \cdot x \quad \text{with} \quad x = \left\{ \overline{C}_{lm} ; \overline{S}_{lm} \right\}$

$\rightarrow \quad \Sigma(N) = A_N \cdot \Sigma(x) \cdot A_N^T$

Geoid height error covariances
Summary & conclusions

• Sub-Processing Facility (SPF) 6000: software is now fully implemented, and the hardware and software system is integrated.

• Successful test in the frame of the HPF Acceptance Review 2.

→ SPF 6000 is ready for operation.