Recovery of the gravity field signal due to a low viscosity crustal layer in glacial isostatic adjustment models from simulated GOCE data
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ABSTRACT
We use a glacial isostatic adjustment model, based on a semi-analytical normal mode technique and a pseudo-spectral sea level code, to simulate the effect of a low viscosity zone (LVZ) in the lower crust on GOCE satellite gravity gradient (SGG) measurements. We recover the gravity field from simulated SGG measurements and try to assess how well the signal due to an LVZ can be extracted in the presence of measurement noise and model errors in the ice load history and the amount of isostatic compensation of topography. Using the correlation coefficient between the measurements and different Earth models, we conclude that GOCE data should be able to constrain the properties of the LVZ, if errors due to uncertainties in the ice load history can be minimized.

1 INTRODUCTION
In glacial isostatic adjustment (GIA) studies it is generally assumed that the lithosphere has very high viscosity and is therefore effectively elastic. Based on laboratory experiments and measured geothermal heat flux it can be expected however that parts of the lithosphere have low viscosity. This is supported by results of postseismic deformation and by noted lack of seismicity in some parts of the lower crust. Values for the properties of such a low viscosity zone (LVZ) range from 20 to 60 km in depth, 10 to 30 km in thickness, and \(10^{17}\) to \(10^{19}\) Pas in viscosity. For recent overviews of literature on low viscosity zones we refer to [1], [2] and [3].

Several studies on the effect of an LVZ on GIA observables have been performed in the past years, see e.g. [1], [2], [4] and [5]. In [6] it is shown that an LVZ introduces variations in geoid heights up to 1 meter with spatial scales down to hundred kilometers underneath and just outside former glaciated areas. The response to changes in the properties of the LVZ is shown to be wavelength-dependent.

With the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite mission, the accuracy of a global geoid model will reach the centimeter level at a resolution of 100 kilometer or better [7]. Such precision enables the detection of low viscosity zones and provides constraints on their properties. The success of recovering the gravity signal due to an LVZ from a global geoid model depends largely on the quality of the models for other geophysical processes and on how effectively the LVZ induced gravity signal can be filtered out, see e.g. [8] and [9].

In this paper we investigate whether the gravity signal due to an LVZ computed from a reference model is better correlated with simulated geoid height measurements than the gravity signal computed from models with different properties of the LVZ. To test the sensitivity of our results, we add measurement noise to our gravity gradients and introduce model errors in the ice load history and amount of isostatic compensation of topography. In the next section, we describe our GIA model. We will first describe the forward model, i.e. the computations of geoid heights and gravity gradients and introduce model errors in the ice load history and amount of isostatic compensation of topography. In the next section, we describe our GIA model. We will first describe the forward model, i.e. the computations of geoid heights and gravity gradients, and then our inverse model, in which we recover the gravity field from the gradients, add noise and model errors, and compute the correlation coefficient. In section 3 we present our results in both the spectral and spatial domain and show that due to the polar gap (the GOCE orbit has an inclination of 96.6°) the use of the correlation in the spatial domain, in this case Northwestern Europe, is preferred. We conclude with a short overview of our findings and some ideas for future research.

2 MODEL DESCRIPTION

2.1 Forward Model
We use a semi-analytical viscoelastic relaxation model as described in [10], based on the normal mode formalism ([11], [12]) to compute the response of the Earth to a surface load in the form of viscoelastic load Love numbers. The Earth is modeled as a spherically symmetric, incompressible, Maxwell viscoelastic, self-gravitating sphere. A seven-layer model is employed to represent the major layers in the Earth, see Table 1.

The LVZ is modeled by assigning a finite viscosity value to the middle one of three layers representing the crust and lithosphere. Model Md20t20v18 is our reference model, with an LVZ at a depth of 20 km, a thickness of 20 km and a viscosity of \(1.10^{18}\) Pas. We compare this model with Earth models with changes in depth (Md40t20v18), thickness (Md20t10v18) and viscosity (Md20t20v19) of the LVZ.

The viscoelastic Love numbers are used in a pseudo-spectral sea level code as described in [14] to compute geoid heights.
Table 1: Seven-layer Earth model (Md20t20ela) with a fully elastic lithosphere of 115 km thickness and volume-averaged parameters obtained from PREM [13].

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth [km]</th>
<th>Density [kg/m³]</th>
<th>Rigidity [GPa]</th>
<th>Viscosity [Pas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>lithosphere</td>
<td>0-20</td>
<td>2411</td>
<td>30.7</td>
<td>$1 \cdot 10^{20}$ (elastic)</td>
</tr>
<tr>
<td>lithosphere</td>
<td>20-40</td>
<td>3272</td>
<td>62.7</td>
<td>$1 \cdot 10^{20}$ (elastic)</td>
</tr>
<tr>
<td>lithosphere</td>
<td>40-115</td>
<td>3376</td>
<td>67.6</td>
<td>$1 \cdot 10^{20}$ (elastic)</td>
</tr>
<tr>
<td>upper mantle</td>
<td>115-400</td>
<td>3438</td>
<td>72.9</td>
<td>$5 \cdot 10^{20}$</td>
</tr>
<tr>
<td>upper mantle</td>
<td>400-670</td>
<td>3870</td>
<td>108.0</td>
<td>$5 \cdot 10^{20}$</td>
</tr>
<tr>
<td>lower mantle</td>
<td>670-2891</td>
<td>4891</td>
<td>221.0</td>
<td>$5 \cdot 10^{21}$</td>
</tr>
<tr>
<td>core</td>
<td>2891-6371</td>
<td>10925</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Unless indicated otherwise, we will subtract geoid heights computed from a model without an LVZ (as given in Table 1) from the model with an LVZ. The resulting geoid heights are then due to the difference between the finite viscosity of the LVZ and the elastic limit only, and will be called LVZ induced geoid heights.

The model is forced by two different ice models. Our reference model is a modified version of the ICE3G ice model of [15] (model I3G). A second model is based on ice sheet dynamics (model ISD), similar to the model described in [16]. Both models are preceded by a linear glaciation phase of 90 ka. To facilitate comparison of the two models, we have used the major ice sheets of North America (NAM) and Eurasia (EAS) only. As in [17] we have increased the volume of ICE3G with 20%. Moreover, we have increased the percentage of the major ice sheets in NAM and EAS to 74% of the total volume, in accordance with values of other ice models, see e.g. [18]. In Figures 1 and 2 we have plotted both ice load models for Northwestern Europe at the last glacial maximum (LGM). Note that we have used a filter to remove holes between the discs of ICE3G as described in [6]. In Table 2 we have listed the main characteristics of our ice load models. As we use only the ice sheets of NAM and EAS, the eustatic sea level rise and sea load will be somewhat smaller than in other GIA studies.

The full gravity gradient tensor is simulated along a realistic GOCE orbit with a sampling interval of 5 seconds and a repeat period of 29 days, completing 467 revolutions, which allows a maximum recoverable degree of spherical harmonic expansion of 233. We compute gravity gradients from a geoid model consisting of EGM96 plus LVZ induced geoid.

Table 2: Volumes of ice models I3G and ISD, with between brackets the sea level equivalent ice volume.

<table>
<thead>
<tr>
<th>Ice Model</th>
<th>Total Volume</th>
<th>Volume NAM+EAS</th>
<th>NAM+EAS of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>I3G⁶</td>
<td>$4.95 \cdot 10^{16}$ m³ (126 m)</td>
<td>$3.66 \cdot 10^{16}$ m³ (93 m)</td>
<td>74%</td>
</tr>
<tr>
<td>ISD⁶</td>
<td>$5.10 \cdot 10^{16}$ m³ (130 m)</td>
<td>$3.93 \cdot 10^{16}$ m³ (100 m)</td>
<td>77%</td>
</tr>
</tbody>
</table>

⁶ Modified ICE3G as described in the text
⁷ Model based on ice sheet dynamics
heights (from Md20t20v18 and I3G). We have simulated the noise behavior of the GOCE gradiometer using white noise with a standard deviation of 1.5 mE (1 E = 1 E¨otv¨os = 1 \times 10^{-12} s^{-2}) up to the sampling rate of 0.2 Hz. This corresponds roughly with the expected noise level of 3 mE/\sqrt{Hz} in the measurement bandwidth of GOCE ([7]).

2.2 Inverse Model
The recovery from the simulated gradients is performed using an iterative block-diagonal solution method as described in [19]. From the recovered gravity field we subtract EGM96 to obtain LVZ induced geoid height measurements, which are equal to the model field (from Md20t20v18 and I3G) plus a recovery error. We will estimate how well model values can be correlated with the measurements of the LVZ induced geoid heights using the correlation coefficient:

$$R(x, y) = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$  \hspace{1cm} (1)

with $x, y$ a series of measurements resp. model values, $\sigma_{xy}$ the covariance of $x$ and $y$ and $\sigma_x, \sigma_y$ the standard deviation of $x$ and $y$.

We will use four different models, based on Md20t20v18, Md40t20v18, Md20t10v18 and Md20t20v19, using the I3G model. We then add a model error equal to the difference between LVZ induced geoid heights based on ice model I3G and ice model ISD. Finally we add a model error in the amount of isostatic compensation of topography. This is simulated by comparing the Airy-Heiskanen model of isostasy and the Vening-Meinesz model, which assumes a certain amount of flexural rigidity leading to partial isostasy, see e.g. [20]. We have used ETOPO5 for topographic heights, and a constant crustal density of 2670 kg/m$^3$ and an upper mantle density of 3270 kg/m$^3$. For the Airy-Heiskanen model we have used a crustal thickness of 30 km if no elevation is present ([20]). For the Vening-Meinesz model we have used a crustal thickness of 29 km and a degree-dependent operator $JGK$ that smoothes the compensation surface ([21]). For implementation of both kinds of isostasy in a spherical harmonic representation of the gravity field we refer to [21].

3 RESULTS

3.1 Forward Model
As a reference, we have plotted the total geoid height due to GIA, based on Md20t20ela (with a fully elastic lithosphere, see Table 1) and I3G, for Northwestern Europe in Figure 3. The effect of an LVZ on the geoid height, which is the difference between geoid heights computed using model Md20t20v18 and Md20t20ela, is displayed in Figure 4. The effect of an LVZ is small (tens of centimeters) compared to the total GIA induced geoid height (some meters), but the amplitudes are much larger than the 1 centimeter accuracy level claimed for GOCE and moreover, have a distinct geographical pattern. This pattern is mainly influenced by the ice load history and the resulting sea load. To show the effects of an LVZ on simulated gravity gradients as measured by GOCE, we have displayed in Figure 5 the difference between gravity gradients at satellite altitude computed from EGM96 plus the LVZ induced geoid heights, and EGM96 only. As the total signal from EGM96 is a few thousand E¨otv¨os in the radial direction, the amplitudes for the effect of an LVZ are small. The signal is however well above the expected gradiometer noise level of 1.5 mE. Note that the pattern is very similar to the LVZ induced geoid heights (Figure 4), which indicates that the increase in detail due to
differentiation of the gravity field is largely compensated by upward continuation from the surface of the Earth to satellite altitude.

3.2 Inverse Model

Our iterative block-diagonal solution of the gravity field converges within 6 iterations. The recovery error is dominantly dependent on latitude, which is due to the polar gap and the less closely spaced groundtracks around the equator as compared to high latitudes, see Figure 6. In the same figure we have plotted the recovery error for the case that white noise with a standard deviation of 1.5 mE is added to the gravity gradients. The recovery error is not strongly dependent on the input gravity field, and therefore we use one recovery error for the case without added measurement noise and one recovery error when white noise is added to the gravity gradients. The model errors introduced by using different ice models are given in Figure 7 and by using different isostatic compensation models in Figure 8.

We compute the correlation coefficient both in the spectral and spatial domain. In the spectral domain, the correlation coefficient is very small in the presence of the recovery error due to the polar gap. We therefore only look at the presence of model errors in the ice load history and the amount of isostatic compensation. In Figure 9 we have plotted the degree amplitudes of the measured signal, without errors (blue), with an error due to different ice load histories (red) and with an error in the amount of isostatic compensation (green). In Figure 10 we have plotted the correlation coefficient as a function of harmonic degree for an error in the ice load history (red) and an error in isostatic compensation (green). The correlation for an error in the ice model is poor from degree 50 up to degree 100, i.e. in the area where the LVZ induced gravity signal has its largest power. The correlation for an error in the isostatical compensation method is however good up to degree 100, and very poor for higher degree. This means that this error does not have a large influence in the part of
the spectrum we are most interested in. The correlation is much better in the spatial domain if we only consider a certain region where the recovery error is not too large (as for example in Northwestern Europe, see Figure 6, where the geoid height error is smaller than 1 cm). The recovery error if no noise is added to the gradients is very small, leading to a correlation of 1 for model Md20t20v18. The correlation for the other models (Md40t20v18, Md20t10v18 and Md20t20v19) is around 0.8-0.9, see Table 3. The correlation is worse if white noise is added to the measurements, though the effect is small and Md20t20v18 is still the preferred model.

For an error in the ice load history the correlation is poor, which we already expected from the results in the spectral domain. An error in the isostatic compensation model does not have a large influence, as the error is only dominant for high degree (> 100). If we include all errors, the correlation coefficient is equal to 0.5 or smaller, mainly due to the model error in ice load history.

4 CONCLUSIONS

Due to the large effect of the polar gap on global results, the correlation in the spectral domain is poor in the presence of measurement errors. We have therefore only investigated the influence of model errors in the spectral domain. For a model error in the ice load history the correlation is especially good for high degree (> 100). This is because LVZ induced geoid heights are strongly influenced by the ice load history, and the signal has its largest power up to degree 100. For an error in the amount of isostatic compensation the correlation is good up to degree 100, because this error has its largest power for high degree (> 100).

In the spatial domain, we computed the correlation in a region where the recovery error is relatively low (Northwestern Europe, < 1 cm). We have shown that even in the presence of realistic measurement noise, and errors in the ice load history and amount of isostatic compensation, our preferred model correlates best with the measurements. The correlation coefficient varies however significantly, to a value of 0.51 for our preferred model when all errors are included. An error in the amount of isostatic compensation of topography only influences the high degree harmonics (> 100), and therefore does not strongly influence the correlation coefficients in the spatial domain. As we want to include lateral heterogeneities in future GIA models, which might induce higher harmonics than the low viscosity zone in our current model, incomplete correction for isostasy might be a serious error source.

<table>
<thead>
<tr>
<th>Model</th>
<th>recovery error (no noise)</th>
<th>recovery error with white noise</th>
<th>model error in ice load history</th>
<th>model error in amount of isostasy</th>
<th>total$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Md20t20v18</td>
<td>1.00</td>
<td>0.97</td>
<td>0.58</td>
<td>0.97</td>
<td>0.51</td>
</tr>
<tr>
<td>Md40t20v18</td>
<td>0.92</td>
<td>0.89</td>
<td>0.54</td>
<td>0.87</td>
<td>0.46</td>
</tr>
<tr>
<td>Md20t10v18</td>
<td>0.92</td>
<td>0.89</td>
<td>0.50</td>
<td>0.87</td>
<td>0.41</td>
</tr>
<tr>
<td>Md20t20v19</td>
<td>0.89</td>
<td>0.86</td>
<td>0.49</td>
<td>0.82</td>
<td>0.39</td>
</tr>
</tbody>
</table>

$^a$ sum of the recovery error with white noise, model error in ice load history and in amount of isostasy

Table 3: Correlation coefficients in the spatial domain for Northwestern Europe
The influence of an error in the ice load history is large. This means that models of the deglaciation should be better constrained to detect the presence of an LVZ. On the other hand, it might be possible to constrain both the properties of an LVZ and the ice load history using high resolution geoid models, as for example expected from GOCE.

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References
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