A BAYESIAN APPROACH TO INVERT GOCE GRAVITY GRADIENTS

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ABSTRACT

The measurements made by the gradiometer onboard GOCE satellite are gravity gradients in three directions of space. The main goal with gradients measurements is to compute the Earth gravity field at global scale with improved accuracy and spatial resolution. However, it is quite clear that gradiometric data themselves might help to infer the inner structure of the Earth. The vertical gravity gradients computed from the vertical component of the gravity acceleration are classically used in gravity interpretation, and the full gradiometric tensor is now measured in the oil and gas prospecting for imaging subsurface structures. For those reasons, we propose to build an inversion method for gradiometric data, dedicated to geodynamical studies both at global and regional scales, since those scales of study are indeed allowed by GOCE data spatial distribution. We basically choose to use a Bayesian approach, which is seldom applied in the field of gravity inversion, even though often mentioned. This type of approach is less consuming for heavy computations and then should allow us to deal more easily with a great among of data. We discuss here the contribution of the FTG (Full Tensor Gradiometry) data.

1 INTRODUCTION

The aim of the gradiometer onboard GOCE is to provide measurements from which the gravity field is derived. However, gradients themselves may be used directly to infer properties of subsurface structures. Actually, gradiometric measurements were already performed at the beginning of the twentieth century. With his famous Torsion Balance, Baron Eötvös did not only prove the so-called Equivalence Principle, but also drew geological conclusions from a survey on Lake Balaton in Hungary, using maps of the horizontal gradients [18]. Later, the instrument was successfully used for geophysical prospecting. Today, thanks to gradiometers that allow the measurement of all the tensor components, gradiometry data are used in petroleum exploration, for example to determine the geometry of salt domes [16]. Similarly, the global GOCE gravity gradients set might be used, not in local areas for prospecting, but for regional and global geodynamical studies.

In this paper, we investigate properties of the gravity gradiometry tensor components and combinations of components which should lead to a better inversion of gravity anomalies than methods using the vertical component of the gravity acceleration only. We then describe some features of the process that we are building at present time to perform that inversion and the role that GOCE gradients could play.

2 WHAT WILL GRADIENTS BRING TO THE INVERSION OF GRAVITY DATA?

We recall now using examples several properties of the tensor components which indicate that gradients provide information about subsurface structures that cannot be inferred from the only vertical component of the gravity field.
2.1 Gradients characterize the geometry and the number of sources

Fig. 1 describes a known structure at unitary depth $\xi$ whose associated gradiometric signal is investigated on Fig. 2, where we map the signal for each component of the tensor, the vertical component $g_z$ of the gravity field the horizontal gradient:

$$Th = \sqrt{T_{xz}^2 + T_{yz}^2}$$  \hspace{1cm} (1)

Fig. 1. A regular icosahedron and a cube with same density contrasts at unitary depth $\xi$ from the measurement plane.

Fig. 2. Gradiometric signal associated to the structure in Fig. 1.

It is quite clear when looking at the $T_{xz}$ component on Fig. 2 that the edges of the complex geometry are better characterised than they are with $g_z$. In addition, $T_{xy}$ and the horizontal gradient $Th$ give the position of the barycenters of the cube and of the icosahedron.
Besides, when the distance of the sources from the measurement plane varies, \( T_{zz} \) and \( T_h \) still "see" two sources, when \( g_z \) only "sees" one (Fig. 3).

![Fig. 3. Signal associated with the structure in Fig.1 when the depth varies.](image)

So we can say from the previous example that gradients allow to characterize:
- the geometry of the sources,
- the number of different near sources,

better than the vertical gravity component does. This is why computed gravity gradients have been used so far to help with the interpretation of gravity anomalies [3, 8, 9]. Moreover, it has been shown that measurements of the gravity field at different altitudes, (for example at sea surface and sea bottom [1]), give better constraints on the anomalous bodies.

2.2 A not-so-ill-posed problem

It is well-known that the inverse problem in gravimetry is ill-posed. Gradients will naturally not fully solve this under determination. However, the gradiometric signal could help, in several cases, to discriminate signals associated to different structures. We illustrate this by the following example.

Let us consider (Fig.4) the horizontal gradient, \( T_h \), and the vertical component of gravity, \( g_z \), associated to two different structures: one superficial and laterally extended, the other narrow, deeper, and with larger density contrast (2,500 kg.m\(^{-3}\) vs. 500 kg.m\(^{-3}\)). The difference between the two \( g_z \) signals does not exceed 9\( \mu \)Gal (=9 \( \times \) \( 10^{-8} \) m.s\(^{-2}\)), which is under the accuracy of a standard land relative gravimeter. The difference between horizontal gradients, however, reaches 400 mE (=0.4 \( \times \) \( 10^{-9} \) s\(^{-2}\)), which is almost a thousand times the accuracy being reached by the GOCE gradiometer during tests on the ground [5]. Thus two signals, equivalent from the point of view of gravimetry, may be distinguished by gradiometric measurements.

2.3 Measurements vs. Computations

The previous examples show that gradients give information unavailable from the only gravity field. However, one could object that gradients need not be measured, but simply computed from the gravity field. Actually, if we knew the field with infinite accuracy on each point of the space, gradients would not bring any piece of information that is not in the field itself. Since we obviously do not know the gravity field that way, however, the measured gradients do give additional information on the structure and have to be inverted too (see [6] and associated discussion [7,13,17]).
Fig.4. On the left, two structures are represented, whose signals are compared on the right: the differences (absolute values) between the two associated horizontal gradients (top), and between the two associated vertical component of gravity $g_z$ (bottom), are mapped. The respective maximum intensities (high for the horizontal gradient, low for $g_z$) in signal differences show that gradients might help when dealing with the ill-posed inverse gravimetric problem.

3 HOW TO PROCEED?

Since we are actually building the inversion at the present time, we only present a few features of the process under development.

3.1 Which quantities should be inverted?

GOCE non diagonal gradients are sensitive to rotations and then degraded when computed in a geocentric reference frame [4] instead of the measurement frame. So it might be more efficient to invert combinations of components instead of components themselves. We already mentioned the horizontal gradient $Th$ (see Eq. 1). There are other combinations of the tensor components having physical sense [15].

For example, the strike direction $\theta$ given by:

$$
\theta = \frac{1}{2} \arctan \left( \frac{2 \frac{T_x T_{yz} - T_y T_{xz}}{T^2_{yz} - T^2_{xz} - T_{zz} (T_{xx} - T_{yy})}}{T^2_{xz} - T^2_{yz}} \right)
$$

indicates how to rotate the measurement frame around the z-axis to align one of the x- and y-axis with the main directions of the structure (see Fig.5).
However, $\mathbf{Th}$ and $\mathbf{\theta}$ are both dependent on the frame where they are computed. To find quantities that do not depend on the frame, one should address the invariants of the tensor. There are three such invariants (see Eq. 3):

$$
I_0 = \text{Trace} (T)
$$

$$
I_1 = \frac{1}{2} \sum_{\langle \ell, j \rangle = \{x,y,z\}} T_{\ell \ell} T_{jj} - T_{\ell j}^2
$$

$$
I_2 = \text{Det}(T)
$$

The trace will not help since it is everywhere equal to zero, but the two other ones have two advantages:

- since they are (scalar) invariants, they actually do not depend on the frame where they are computed,
- their formulations include all the tensor components, so they contain the whole gradiometric information,

and one drawback:

- they are non linear with respect to the components.

Since they are invariant, we choose to base our inversion on those two quantities.

3.2 How to address the forward modelling?

Subsurface structures will be investigated as an assembly of homogeneous polyhedra with adjustable density contrast. Then we need an analytical expression for the spatial derivatives of the gravity field caused by such an homogeneous polyhedra. We use Okabe [12] formulae which fully fill that criterion.

3.3 What kind of inversion?

There are several ways to perform a gravity data inversion. It mainly depends on what we look for and what we already know about the structure to infer. According to Camacho et al. [2], they range in two main categories:

- linear methods [10,14], which are suitable when we have strong constraints on geometry and search for density contrast;
explorations of model space, which are the most efficient ones when constraints are on density and geometry is unknown.

As previously mentioned, the gradiometric information is mainly about the geometry of the involved sources. Using a method from the first category implies that gradients shall be used as a priori information on geometry. A process based on the exploration of the model space, on the other hand, would use the gradiometric information as a control parameter for the fitting of solutions to data. Presently we are designing a method that mixes the two categories, in order to deal with the widest range of geophysical studies, which GOCE gradients would allow. A probabilistic method [11], providing a suitable importance sampling algorithm where the gradients would be the control parameter, may be efficient to avoid a too long computing time due to the great among of data.

REFERENCES

5. ESA, Gravity Field and Steady State Ocean Circulation Mission, Reports for Mission Selection, the four candidate earth explorer missions, ESA SP 1233(1), 1999.