GRAVITY FIELD ANALYSIS FROM PRE-PROCESSED AND CALIBRATED
GOCE OBSERVATIONS: THE DELFT APPROACH

Roland Klees, Pavel Ditmar and Alexis Van Eck van der Sluijs

Delft Institute for Earth Observation and Space Systems (DEOS)
Delft University of Technology
Kluyverweg 1, 2629HS DELFT, The Netherlands

ABSTRACT
The current status of the Delft approach to the gravity field analysis from fully calibrated and validated level 1B GOCE data is presented. The philosophy behind the Delft approach is explained, and the main features and implementation issues are discussed. We present also the results of a numerical study based on simulated 6-month GOCE data sets. In this study, gravity gradients and precise orbit information are processed both separately and jointly. Advantages of a joint data processing are clearly demonstrated.

1 INTRODUCTION
GOCE (Gravity Field and Steady-State Ocean Circulation Explorer) is a dedicated gravity field mission to be launched in 2006 [7]. The aim of the mission is to provide a high-accuracy, high-frequency model of the Earth’s static gravity field. GOCE will acquire observations over two windows of six months with a hibernation period in between. The primary sensor system on board GOCE consists of a three-axis gradiometer, a GPS receiver, a star sensor, and a drag and angular control system. This new sensor system requires the development of a new data processing strategy, comprising pre-processing, external calibration, precise orbit determination, gravity field modeling, and validation.

This paper aims to give an overview about the current status of the data processing strategy developed at the Delft Institute for Earth Observation and Space Systems (DEOS) at Delft University of Technology. The focus will be on the level 1B to level 2 processing system, which aims to compute a gravity field model from satellite gravity gradient (SGG) and GPS tracking data. First, the philosophy behind the Delft approach is outlined. Then, the algorithms are discussed which implement the presented philosophy. Furthermore, some software issues are addressed. The performance of the developed data processing methodology is demonstrated with a 6-month set of simulated GOCE data. The paper ends with an outlook to open issues that will be addressed in the foreseeable future.

2 PHILOSOPHY
The philosophy behind the Delft approach to the processing of GOCE data can be summarized as follows:

- The development of new algorithms and software tailored to the GOCE mission is preferred to the extension of existing software packages.
- Data will not be reduced, transformed, interpolated or re-sampled in order to preserve the stochastic properties of data noise and the information content in the data.
- Existing noise correlations of satellite data in space and time should properly be taken into account to obtain the optimal solution and a proper error propagation.
- The solution strategy and the implemented algorithms should allow to extend the functional model by other types of observations and parameters.
- A proper relative weighting of all input data and models should be done preferably directly from the data.
The numerical efficiency of the algorithms should allow one to select a high maximum degree to be solved for in order to recover all information contained in the GOCE data.

The formal quality of the gravity field solution should be expressed in terms of the full variance-covariance matrix of various types of gravity field functionals such as potential coefficients, geoid heights and gravity anomalies.

3 IMPLEMENTATION

The primary observations are calibrated and validated GOCE data: (1) four components of the gravity gradients tensor ($V_{xx}$, $V_{yy}$, $V_{zz}$, and $V_{zz}$) in the local orbital reference frame (LORF) and (2) 3-component time-averaged satellite accelerations derived from the precise kinematic satellite orbit. The functional model related to the gravity gradient observations and time-averaged satellite accelerations is documented in [1] and [6], respectively. We prefer satellite accelerations to satellite velocities, because the former are expected to yield more accurate gravity field solutions as shown theoretically and supported by numerical studies [6]. Importantly, both the gravity gradients and the satellite accelerations have to be pre-processed in order to remove: (i) signal from a reference static Earth’s gravity field; (ii) signal from temporal gravity field variations; (iii) influence of non-gravitational effects, e.g. the atmospheric drag (if the drag-free control system fails to remove all such effects entirely).

The unknown parameters are estimated using a regularized least-squares technique. Regularization is considered as indispensable in order to fully exploit the high-frequency information content in SGG data. The latter requires that the potential coefficients up to a very high degree (about 300) are estimated. Typically, we apply the "Kaula" or the first-order Tikhonov regularization technique. Furthermore, usage of the covariance matrix of the reference model for the purpose of regularization is being currently studied. The optimal regularization parameter is found by means of the generalized cross-validation technique [4, 11, 12].

The regularized least-squares solution can be computed either with the Cholesky decomposition or with the pre-conditioned conjugate gradient (PCCG) method. One advantage of using the PCCG method is that the explicit computation of the design or the normal matrix is avoided. It is enough to perform several times the spherical harmonic synthesis (the application of a design matrix to a vector) and co-synthesis (the application of a transposed design matrix to a vector). Very fast algorithms have been developed for this purpose [2, 6].

The performance of the pre-conditioned conjugate gradient method relies on a good pre-conditioner. For satellite gravity gradients, a block-diagonal pre-conditioner can easily be derived [cf. 1]. Ditmar and van Eck van der Sluijs [6] extended the concept of a block-diagonal pre-conditioner to time-averaged satellite accelerations. Further numerical experiments have also showed that the performance of a block-diagonal pre-conditioner is not deteriorated significantly if some nuisance parameters are introduced.

In order to obtain the optimal model, data are weighted in compliance with the inverse covariance matrix $C_d^{-1}$. In doing so, an explicit computation of the matrix $C_d^{-1}$ is replaced with a multiple application of this matrix to vectors. If a data set is an uninterrupted, whereas noise is stationary and colored (frequency-dependent), application of the matrix $C_d^{-1}$ to a vector can be closely approximated by filtering. A more advanced procedure is the exact application of the matrix $C_d^{-1}$ to a vector by means of a low-level PCCG scheme. Two basic operations are performed at each iteration of this scheme: (i) the exact application of the matrix $C_d$ to a vector and (ii) an approximate application of the matrix $C_d^{-1}$ to a vector (pre-conditioning). It is important that data with gaps can be accurately handled this way without any edge effects [9]. An extended discussion on data weighting in the context of gravity gradients and satellite accelerations can be found in [10] and [5, 6], respectively.

For the computation of the variance-covariance matrix of gravity field functionals, an explicit assembly of the normal matrix is indispensable. This can be done column-by-column exploiting the fast synthesis and co-synthesis algorithms [cf. 3]. Thereafter, Cholesky decomposition is used to obtain the variance-covariance matrix of the estimated parameters and the mean square error matrix. Depending on the required accuracy of computations, additional simplifications can be introduced in the synthesis and co-synthesis algorithms to accelerate the assembly of the normal matrix further. Alternatively, Monte-Carlo techniques like the Gibbs sampler can be used to compute the variance-covariance matrix of gravity field functionals [cf. 8].
4 SOFTWARE ISSUES

The data processing methodology presented above form the basis of a dedicated software GOCESOFT. This software can handle satellite gravity gradients and time-averaged satellite accelerations either separately or jointly. At present, GOCESOFT has been adapted to and tested on: (i) Linux PC; (ii) multi-processor computer SGI Origin 3800; (iii) multi-processor computer SGI Altix 3700 (with Itanium-2 processors as the elementary base). We expect that GOCESOFT can also be used (as is or with minor modifications) on any other multi-processor platform with the Unix/Linux operating system; the only requirement is the presence of MPI and Lapack/Blas libraries.

We have put significant efforts in the optimization of GOCESOFT both in terms of CPU time and computer memory. In particular, the memory requirements are of the order of 200 bytes per observation point for either type of data. This means that one would need only about 12 Gb of memory to process all the GOCE data (about $3 \times 10^7$ observation points). By the time GOCE flies, this amount can probably be achieved even on a plain PC. As far as the time expenditure is concerned, it is of the order of 1 hour on the SGI Origin 3800 platform with 32 CPUs for a 1-year data set of either type with 1-s sampling, provided that the maximum degree $L_{max}$ is set equal to 300, and the regularization parameter is given. An automatic search for the optimal regularization parameter increases the time of computations by about the factor 30 (which is still well within reasonable limits). Thanks to an efficient parallelization, memory and time expenditures scale down almost proportionally to the number of processors.

An explicit computation of the normal matrix is the most time-consuming procedure. At present, such a computation can only be done for a model with $L_{max} \leq 150$. Further improvements of the software will be needed before a model with $L_{max} = 300$ can be handled.

5 NUMERICAL EXPERIMENT

A numerical experiment has been designed to demonstrate the accuracy and numerical performance of the developed algorithms and software. The goal of computations is to restore a gravity field model from simulated data. The EGM96 model [13] complete up to degree 300 is chosen as the true gravity field. The JGM-3 model [15] is used as the a priori reference gravity field. A realistic 6-month GOCE orbit was designed with 262 km mean altitude. The orbit was sampled with a frequency of 1 Hz. This yields about 16 million 3-D positions. At each position, three components of the gravity gradients tensor $(V_{xx}, V_{yy},$ and $V_{zz})$ were simulated using the difference between the true and the reference gravity field model as input. The simulated gravity gradients were artificially contaminated with realistic colored (frequency-dependent) noise [14]. Furthermore, a reference orbit was computed on the basis of the reference gravity field model. Then, a set of orbit differences was created by subtracting the reference orbit from the true one. To keep the orbit differences within certain bounds, the reference orbit was split into 1-day arcs (for each arc, a suitable initial state vector was determined). The orbit differences were contaminated with 3-cm white noise. After that, time-averaged residual accelerations were derived by means of the a double numerical differentiation.

The simulated data were processed in order to restore the "true" gravity field model. The computations were made with the PCCG method (i.e. without an explicit assembly of the normal matrix). The regularization parameter was fixed. The two data sets were considered both separately from each other and jointly. Results of data processing are shown in Fig. 1 as maps of geoid height errors. One can see that the stand-alone processing of satellite acceleration results in strong high-frequency errors, whereas gravity gradients alone do not allow the lowest frequencies to be recovered accurately. The model obtained from both data sets simultaneously has a significantly higher quality than models obtained from either data set alone. In order to make these findings even more obvious, we have also presented the results as cumulative geoid error plots (Fig. 2). Thus, satellite accelerations and gravity gradients complement each other and have to be processed jointly in order to provide a gravity field model of highest quality.

The wall-clock time of computations is shown in Table 1. One can see that the time expenditures are indeed modest (within 0.5 hour for a stand-alone data processing or about 1.5 hour for the joint data processing).
Fig. 1: Results of the numerical experiment in terms of geoid height errors (only the latitude band ±80° is considered): (a) Processing of satellite accelerations only (rms error = 0.92 m, max error = 14.9 m); (b) Processing of gravity gradients only (rms error = 0.16 m, max error = 1.58 m); (c) Joint data processing (rms error = 0.15 m, max error = 1.56 m).
Table 1: Results of the numerical experiment: number of PCCG iterations and expenditures of the wall-clock time. A SGI Origin 3800 computer with 32 processor elements was used.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Number of PCCG iterations</th>
<th>Wall-clock time (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerations only</td>
<td>40</td>
<td>31</td>
</tr>
<tr>
<td>Gravity gradients only</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>Combined</td>
<td>59</td>
<td>91</td>
</tr>
</tbody>
</table>

6 FUTURE RESEARCH

In the course of next few years, we intend to extend the scope of the Delft approach further. Some of the issues to be addressed:

- Unified approach to regularization, data weighting, and noise modeling using variance component estimation techniques. The aim is to compute the statistically optimal model from multiple data sets without a priori information about noise in each one.
- Fast methods for the computation of the variance-covariance matrix and the mean square error matrix of gravity field functionals.
- Improved procedure to derive satellite accelerations from GPS tracking data. The current procedure, which uses a kinematic orbit as an intermediate product, suffers from ambiguity resolution inaccuracies.
- Incorporation of other types of data (in particular, terrestrial gravity anomalies, airborne gravity data, and low-low satellite-to-satellite tracking data) into the joint data processing.
- Incorporation of time-dependent gravity field parameters into the functional model.

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References


