Abstract—By virtue of their capability to penetrate into the vegetation cover, longer wavelengths Synthetic Aperture Radars (SAR) have been widely used for the aim of investigating forested areas at vast scales. A turning point in this kind of studies resulted with the idea to jointly exploit polarimetric and baseline diversity (PolInSAR), which made it emerge the possibility to analyze the vegetation layer by isolating volume only contributions within the SAR signal. Recently, the problem of Scattering Mechanism (SM) separation basing on multi-baseline and multi-polarimetric information has been faced from an algebraic point of view in [1], resulting in a fundamental decomposition based on the sum of Kronecker products (SKP) structure. Such a decomposition allows to tie together the matrix containing structural information with the one containing polarimetric information. Therefore, in this work we discuss the possibility to retrieve the vertical structure of ground and volume scattering by forcing their polarimetric behaviour. To do this, the polarimetric signature of ground and volume scattering will be investigated by considering the application of polarimetric decomposition techniques widely exploited in the analysis of forested areas, such as the Freeman and the Cloude-Pottier decompositions. Experimental results will be shown basing on a data-set of multi-polarimetric and multi-baseline SAR images, at both P and L band, acquired by DLR’s E-SAR over the Krycklan catchment, in northern Sweden, in the framework of the ESA campaign BioSAR 2008.

I. INTRODUCTION

Tomographic SAR (T-SAR) has been recently employed [2] to retrieve structural information about the observed scene because of its capability to resolve multiple targets. T-SAR is able to relate the backscattered power with a specific height above the ground, enabling the separation of targets in the vertical direction. With the introduction of acquisitions collected by exploiting different polarimetric channels it became possible to obtain an estimation of the backscattered power associated with each one of them as a function of the height. With the joint exploitation of Multi-Baseline and Multi-Polarimetric data emerged the chance to retrieve polarimetric and structural information associated with different Scattering Mechanisms (SM) such as volumetric scattering or ground scattering. In such context Algebraic Synthesis (AS) technique has been employed as a working tool to reduce the degrees of freedom of the decomposition by determining the subspaces which the matrices containing structural and polarimetric information are forced to lie in. An important consequence is that the residual ambiguity is governed by two parameters relating the polarimetric behaviour with the structural one. The aim of this work is to study the different polarimetries which can be obtained varying such characteristic parameters.

This article has the following structure. In section II is summarized algebraic synthesis basic theory and are shown the working tools which it provides. Section III describes the two decompositions said above: Freeman and Cloude-Pottier decomposition (CPD). Freeman models are used in section IV to have at disposal reliable ground and volume scattering features whereas CPD is used for analysis purposes. In section V are analyzed and discussed the obtained results. In section VI conclusions are drawn.

II. BASIC THEORY OF ALGEBRAIC SYNTHESIS

Basic theory of AS technique is now shown together with few definitions to make this work self consistent.

Multi-Baseline (MB) acquisitions share the same target which is investigated from slightly different look angles enabling to retrieve its volumetric structure. Organizing the received data in a vector \( y_{MB} = [y_1 \ y_2 \ \cdots]^T \), where the subscript refers to image index, the MB covariance matrix is expressed by:

\[
R = E[y_{MB}^* y_{MB}]
\]

Since such a matrix embodies the information about the vertical structure of the target we’ll refer to it as structure matrix.

Multi-Polarimetric (MP) acquisitions rely on the chance to separately transmit signals with both polarizations and to receive the backscattered echoes again with both polarizations. The matrix formed by mean values of the products between MP data is expressed by:

\[
C = E[y_{MP}^* y_{MP}]
\]

where:

\[
y_{MP} = \begin{bmatrix} y_{hh} & \sqrt{2} y_{hv} & y_{vv} \end{bmatrix}^T
\]

the subscripts in eq. 3 right side refer to the received and transmitted polarization. Because of the fact that matrix \( C \) characterize the polarimetric behaviour of the target we’ll refer to such a matrix \( C \) as polarimetric signature.

Multi-Polarimetric Multi-Baseline (MPMB) acquisitions make it possible to obtain for every resolution cell, one measurement for every explored look angle and for every couple of transmitted and received polarimetry. We’ll refer to such a measurement as \( y_{n,\gamma,\delta} \), where \( n \) is the image index, \( \gamma \)
and $\delta$ are the received and transmitted polarization, that is $h$ or $v$. Organizing MPMB received data as vector

$$Y_{MPMB} = [y_{1, hh} \ y_{2, hh} \ \cdots \sqrt{2} y_{1, hv} \ \cdots]^T$$  \hspace{1cm} (4)

The MPMB covariance matrix $W$ is expressed by:

$$W = E[Y_{MPMB}Y_{MPMB}^*]$$  \hspace{1cm} (5)

In [1] (where more details may be found) three hypotheses was proposed; if they hold MPMB covariance matrix may be expressed by a simple equation. The three hypotheses are:

- statistical uncorrelation of the different SMs,
- invariance of structural parameters with respect to polarization
- data stationarity across different tracks, which may be expected to hold if events like fires, frosts, deforestation do not occur during the acquisition campaign

Under such hypotheses, matrix $W$ may be expressed by sums of Kronecker products:

$$W = \sum_{k=1}^{N_m} C_{mech(k)} \otimes R_{mech(k)}$$  \hspace{1cm} (6)

Where $N_m$ is the number of scattering mechanisms contributing to the data, $R_{mech(k)}$ is the Multi-Baseline covariance matrix and $C_{mech(k)}$ is the Multi-Polarimetric covariance matrix associated with the $k$-th scattering mechanism.

AS technique relies on the fundamental result proved by Van Loan and Pitsianis in [3]. It states that every matrix $W$ may be decomposed as:

$$W = \sum_{k=1}^{N_m} \lambda_k C_k \otimes R_k$$  \hspace{1cm} (7)

Where $R_k$ and $C_k$ are orthonormal in the sense that $tr (R_k R_k^*) = \delta (h - k)$ and $tr (C_k C_k^*) = \delta (h - k)$ and where $\lambda_k \geq \lambda_h \forall h > k$.

Assuming two SMs contributing to the data, truncating summation of eq. 7 after first two terms lead to discard the subspaces in which noise is supposed to prevail. So $R_1$ and $R_v$, structural matrices of ground and volume respectively, are supposed to lie within the subspace spanned by $R_1$ and $R_2$. The same happens for their polarimetric signatures $C_g$ and $C_v$ which are supposed to lie within the subspace spanned by $C_1$ and $C_2$.

Once the signal subspace is determined, it is possible to express the covariance matrix of the ground and of the volume as linear combination of the basis matrices. The parametrization shows an important result: choosing a particular $R_g$ as a linear combination of $R_1$ and $R_2$ univocally determines the only $C_v$ fitting the data and vice-versa. The same happens for $R_v$ and $C_g$. Such a parametrization may be expressed as:

$$R_g = a R_1 + (1 - a) R_2$$  \hspace{1cm} (8)

$$C_v = \frac{1}{a - b} ((a - 1) C_1 + a C_2)$$  \hspace{1cm} (9)

$$R_v = b R_1 + (1 - b) R_2$$  \hspace{1cm} (10)

$$C_g = \frac{1}{a - b} ((1 - b) C_1 - b C_2)$$  \hspace{1cm} (11)

Eq. 8-9 show that, apart from a scaling factor, $R_g$ and $C_v$ share the same dependence from the parameter $a$, so that constraining a particular polarimetry of the volume within the signal subspace, corresponds to setting a certain structure of the ground, and vice-versa. $b$ parameter is the counterpart which connects ground polarimetry with volume structure as it may be seen in eq. 10-11.

It should be remarked that the matrices $R_k$ and $C_k$ obtained thanks to the algebraic decomposition expressed by eq. 7 although inherit Hermitian symmetry from matrix $W$, are not constrained to be semi positive definite (SPD) so their linear combination may be lacking of physical meaning. Due to this the admissible values for parameters $a$ and $b$ are the ones which lead, according to eq. 8, 9, 10 and 11, to SPD matrices $C$ and $R$.

### III. Polarimetric Target Decomposition

In the past years, several efforts have been made in order to interpret the polarimetric covariance matrix $C$ once it was directly estimated from the data as suggested from eq. 2-3. The purpose of such works was to recognize inside of the matrix $C$ the polarimetric signature of the principal scattering mechanisms. To achieve this goal many approaches have been tried however in [4] is recognized that they may be classified in 2 main groups:

- Model-based decompositions
- Eigenvector-based decompositions

The first category of decompositions relies on the chance of expressing the estimated $C$ as a weighted superimposition of known models of polarimetric behaviour. In this work Freeman models will be considered. The second group of decompositions doesn’t use any other information but the data from which the predominant scattering mechanisms are supposed to emerge via an eigenvector analysis. Belonging to this category of approaches, Cloude-Pottier decomposition has been chosen in this work as an analysis tool.

#### A. Freeman Models

Among many polarimetric models developed in order to be applied in a ground/vegetation environment, the ones proposed in [5] are considered in this work. These models are suggested for volume and ground polarimetry:

$$C_g = f_g \begin{bmatrix} 1 & 0 & \alpha \\ 0 & 0 & 0 \\ \alpha^* & 0 & |\alpha|^2 \end{bmatrix}$$  \hspace{1cm} (12)

$$C_v = f_v \begin{bmatrix} 1 & 0 & \rho \\ 0 & 1 - \rho & 0 \\ \rho & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (13)

Where $f_g$ and $f_v$ are scaling factors whereas $\alpha$ and $\rho$ are the parameters which characterize the polarimetric signatures. $\alpha$ is complex; its absolute value ranges from 0 to 1 if scattering
comes from a double bounce ground-trunk (α phase should be about ±π) whereas is greater than 1 if it comes directly from the ground surface scattering. ρ is real, it ranges from 0 to 1 and it is related to the preferred orientation of the elementary scatterers which are supposed to compose the volume. The estimated parameters are the ones which make the measured data consistent with Freeman model expressed by eq. 14.

\[ C = C_g + C_v \]  

Both polarimetric models rely on the hypothesis of targets characterized by reflection symmetry (as defined in [6]) hence zero correlation between co- and cross-polar terms. The ground model is derived considering the soil a Bragg scattering surface; such a model is recognized to be mathematically equivalent with the purpose one for double bounce scattering hence only one out of two SMs may emerge from the data. Its main features are no received power on the hv channel and being rank 1. The volume model is typical of a generic target with azimuthal symmetry ([6]), its main feature is zero co-polar phase that is \( C_v (1, 3) \) is real.

B. Cloude-Pottier Decomposition

It is proposed in [7] an algorithm to perform unsupervised classification of remote sensed targets which relies on features derivable from eigenvector decomposition of the coherency matrix \( T \); such features are entropy \( H \) and angle \( \alpha \). \( T \) may be obtained from \( C \) thanks to a one-to-one transformation and it is defined as:

\[ T = E \left[ y_p y_p^* \right] \]  

\[ y_p = \frac{1}{\sqrt{2}} \begin{bmatrix} y_{hh} + y_{uv} \\ y_{hh} - y_{uv} \\ 2y_{hv} \end{bmatrix} \]

Eigen analysis allows expressing \( T \) as:

\[ T = \sum_{k=1}^{3} \lambda_k v_k v_k^* \]  

\[ v_k = \begin{bmatrix} \cos(\alpha_k) \\ e^{j\beta_k} \sin(\alpha_k) \cos(\beta_k) \\ e^{j\beta_k} \sin(\alpha_k) \sin(\beta_k) \end{bmatrix} \]

Cloude-Pottier Decomposition (CPD) associates to every elementary mechanism a "probability" value defined as:

\[ P_m = \frac{\lambda_m}{\sum_{k=1}^{3} \lambda_k} \]

After which entropy is derived as:

\[ H = \sum_{k=1}^{3} P_k \log_3 P_k \]

Probability values also allow defining a mean \( \alpha \), defined as:

\[ \alpha = \sum_{k=1}^{3} P_k \alpha_k \]

In their work Cloude and Pottier then suggest partitioning \( H - \alpha \) plane with 9 zones each one typical of a certain target class in order to perform classification. Since such a partitioning was intended to be used on remote sensed data gathered from forested areas, it characterizes backscatter from bare soil, rough surfaces, tree canopies and so on.

IV. EXPERIMENTAL RESULTS

Experiments have been carried on P and L band data set collected in Krycklan (northern Sweden) in October 2008 during ESA campaign BioSAR 2008. Each data set is composed by 6 acquisitions characterized by a maximum baseline of 40m for P band and of 30m for L band, resulting in an average vertical Fourier resolution of 23m (near range) and 92m (far range) for P band whereas 9m (near range) and 36m (far range) for L band. It should be remarked that for both P and L band data, more than 90% of the data may be correctly represented by keeping the first two terms of the decomposition shown by equation 7.

![hh and hv height spectrum](image)

Fig. 1. \( hh \) (above) and \( hv \) (below) channels, height spectrum (P-band).

In figure 1 the estimated height spectrum associated with \( hh \) and \( hv \) polarimetric channels are shown. Their structural matrix has been analyzed by means of the Capon algorithm as suggested in [8]. In order to focus on the vegetation contribution, topographic height has been compensated so leading to a zero height ground. It’s easy to recognize from figure 1 areas covered with forest and bare soil. The height estimated by LIDAR measurements are also shown together with the ground reference height (horizontal line). The vertical black line refers to a particular slant range value whose behaviour is now analyzed in detail.

The received data has been decomposed as suggested by eq. 7 thus determining basis matrices \( R_1, R_2, C_1 \) and \( C_2 \). Such matrices have been used according to eq. 9 and 11 to obtain the polarimetric signatures of the ground and the volume: \( C_g \) and \( C_v \). The range of admissible values for parameters \( a \) and \( b \) has been shrunken in order to get (semi) positive definite covariance matrices; from now on, \( a \) and \( b \) range of values will be always considered as the one which leads to SPD \( R_g, C_v, R_v \) and \( C_p \), respectively according to eq. 8, 9, 10 and 11.

Concerning Freeman model we identify as a key feature the co-polar coherence namely \( C(1, 3) \) normalized with respect to \( \sqrt{C(1, 1) C(3, 3)} \) as this term allows for a quick evaluation of the underlying SM. The behaviour of the co-polar coherence
of the volume polarimetry may be observed in figure 2 varying $a$. In the same figure it is shown as a black asterisk the value of the co-polar coherence of the original, not decomposed data.

The set of possible volume polarimetries as a function of the parameter $a$ has been analyzed by means of CPD described in section III-B. In figure 3 it may be appreciated the locus described by volume polarimetry $C_v$ on the $H - \alpha$ plane varying $a$. The partition of the plane follows after the one proposed in [7] for qualitative classification purposes.

As stated by eq. 8, choosing volume polarimetry is equivalent to choosing the spatial structure of the ground. Results are shown in figure 4, where the corresponding ground structure matrix $R_g$ is analyzed by means of the Capon algorithm [8].

Dually, the polarimetric signature of the ground and the spatial structure of the volume are tied and controlled by $b$ parameter. Ground polarimetry key features have been identified in: backscattered power on the $hv$ channel and rank. It is possible to appreciate the way these two key features of $C_g$ change as $b$ varies in figure 5; as a measure related to the rank is proposed the ratio between the second and the first singular value.

In figure 6 it may also be appreciated the co-polar coherence for the ground polarimetry. Again the black asterisk shows the co-polar coherence of the original (i.e. non decomposed) data.

Locus in the $H - \alpha$ plane for the ground polarimetry is shown in figure 7, whereas the resulting spatial structure of volume scattering is reported in figure 8.

The results of the same processing as above on the L band data set are now shown. It should be noticed that because of the shorter wavelength with respect to the P band, a smaller amount of the energy of the signal is able to penetrate the canopy layer and reach the ground.

Figure 9 shows the single polarimetric channels $hh$ and $hv$ height spectrum. The LIDAR height estimations are plotted as a black line; the ground reference height is shown as a black horizontal line whereas the vertical black line specifies a particular range value analyzed here.

The behaviour of the co-polar coherence of the volume
polarimetry may be observed in figure 10 varying a. In the same figure it is shown as a black asterisk the value of the co-polar coherence of the original, not decomposed data.

Figure 11 reports the locus of the volume polarimetries in the $H - \alpha$ plane varying the $a$ parameter.

Ground structures corresponding to different volume polarimetries are shown in figure 12.

Now focusing on parameter $b$, different ground polarimetric signatures may be appreciated together with the corresponding volume spatial structures. In figure 13 are shown the two first eigenvalues ratio and the received power associated to the $hv$ channel of the ground polarimetry.

Figure 14 shows the co-polar coherence.

In figure 15 the locus described by the ground polarimetry on $H - \alpha$ plane is shown varying $b$.

In the end the volume spatial structure estimations may be observed in figure 16 as a function of $b$.

V. DISCUSSION

Concerning volume polarimetry, we observe that the result yielded by the AS technique is consistent with a typical high entropy forest mechanism. In fact, as it may be observed in figure 3 and 11, both volume polarimetries lie in high entropy
Ground polarimetry key features varying $b$ (L-band).

Ground polarimetry key features

$\frac{\lambda_2}{\lambda_1}$

$|hv|/|hh|$

Fig. 13. Ground polarimetry key features varying $b$ (L-band).

Ground polarimetry co-polar coherence

$\text{real}$

$\text{imag}$

$\text{MAX}$

$\text{MIN}$

Fig. 14. Co-polar coherence of the ground polarimetry varying $b$ (L-band).

Ground structure is again controlled by parameter $a$, generally sharper height spectra are determined by more entropic volume polarimetrries. This effect may be appreciated in figure 4 for P band and in figure 12 for L band. Forcing volume polarimetry to be less entropic as possible may lead to ground structure to show contributions from top heights for L band.

With regard to the ground polarimetry it may be seen from figures 6 and 14 that the decomposition exploited by AS results

in an increase of the co-polar coherence and phase with respect to the original (i.e. non decomposed) data. $b$ value which leads to the most coherent ground is generally the same $b$ value which leads to the least focused volume structure. It may be seen from figures 5 and 13 that generally constraining the least entropic ground polarimetry leads to minimizing the $hv$ contribution in the ground polarimetric signature as well as the $\lambda_2/\lambda_1$ ratio.

At both wavelengths, allowed structures for the volume contributions generally vary (with the $b$ parameter) from a very sharp height spectrum whose phase center is located at top height (small $b$), to a more disperse one (increasing $b$) to a bimodal spectrum due to the presence of ground-locked contributions (larger $b$). This may be clearly observed in figure 16 for L band whereas P band lower resolution partially masks this effect as it may be seen in figure 8.

VI. Conclusions

In this paper we have applied the Algebraic Synthesis (AS) technique for the investigation of the polarimetric behaviour of ground and volume Radar scattering from the boreal forest within the Krycklan catchement. The following conclusions can be drawn.

The variation of the polarimetric properties within the region of physical validity (i.e. positive definiteness) is not such as to allow a non ambiguous identification of ground and volume scattering. Yet, polarimetric information allow to further shrink the range of admissible solutions by imposing physical consistency, for example by requiring ground scattering to be less entropic than volume scattering.

Assuming two Scattering Mechanisms, a one to one correspondence is established between volume structure and ground polarimetry. As a result, we observed that at both P-Band and L-Band minimizing ground entropy leads to the arising of volume contributions from the ground level, not present allowing an entropic ground. This result can be interpreted as follows. If the assumption of least entropy ground scattering is correct, then volume specular contributions (or ground trunk double bounce whose polarimetric signature is affected by the propagation through a canopy layer) have to be accounted for, whereas if only volume backscattering is assumed, then entropic ground scattering has to be accounted for.

Further physical insights could be provided by extending the AS technique to three SMs, at the cost of enlarging the number of free parameters from two to six.
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REFERENCES


