Algebraic Synthesis of Forest Scenarios from SAR data:  
Basic Theory and Experimental Results at P-Band and L-Band

Carried out as a part of the ESA campaign BioSAR 2008

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Overview

Algebraic Synthesis (AS) $\Leftrightarrow$ technique for the decomposition of Ground and Volume scattering basing on multi-polarimetric and multi-baseline SAR surveys

$$y_n(w_i) \Leftrightarrow \begin{cases} \text{Track } n \\ \text{Polarization } w_i \end{cases}$$

Algebraic Synthesis:
- Separated Tomographic Imaging of Ground and Volume contributions \textit{(this talk)}
- Separated Polarimetric Analysis of Ground and Volume contributions \textit{(tomorrow in poster session)}
- Interferometric Analysis of Ground-only contributions \textit{(tomorrow in poster session)}
Model of the acquisitions

Three fundamental hypotheses will be retained:

H1): Statistical independence among different SMs (ground scattering, volume scattering, ground-trunk scattering)

H2): Invariance of the interferometric coherences of each SM w.r.t. polarization

=> negligible variation of the EM properties of each SM (subsurface penetration, volume extinction,...) w.r.t. polarization

H3): Invariance of the polarimetric signature of each SM on the choice of the track

=> events like floods, fires, frosts, are expected not to occur during the acquisition campaign

H1), H2), H3) result in the covariance matrix of the multi-polarimetric and multi-baseline data to be expressed as a Sum of Kronecker Products (SKP)

\[
W = E[yy^H] = \sum_{k=1}^{K} C_k \otimes R_k
\]

\[
y = \begin{bmatrix}
y_1(w_1) & \cdots & y_N(w_1) \\
y_1(w_2) & \cdots & y_N(w_2) \\
y_1(w_3) & \cdots & y_N(w_3)
\end{bmatrix}^T
\]

\[
y_n(w_i) \iff Track \ n \ Polarization \ w_i
\]

\[
C_k : \text{polarimetric covariance matrix of the } k\text{-th SM alone } [3 \times 3]
\]

\[
R_k : \text{matrix of the interferometric coherences of the } k\text{-th SM alone } [N \times N]
\]

\[
R_k, C_k \text{ are (semi)positive definite matrices}
\]
Sum of Kronecker Products Decomposition

The key to the exploitation of the SKP structure is the existence of a fast algorithm for the decomposition of any matrix into a SKP:

\[ W = \sum_{k=1}^{K} \lambda_k \tilde{C}_k \otimes \tilde{R}_k \quad \lambda_1 \geq \lambda_2 \geq \ldots \geq 0 \]

where \( \{\tilde{C}_k\} \) and \( \{\tilde{R}_k\} \) are two sets of matrices obtained from \( W \) through an SVD-like analysis.

Two results follow:

1. **Given** \( W \), \( K \) SMs are uniquely identified by \( K(K-1) \) real numbers

   *Corollary:* The Ground/Volume Decomposition problem is defined by exactly 2 real numbers \((a,b)\)

2. The best Least Square (LS) approximation of \( W \) with \( K \) KPs is simply obtained by retaining the first \( K \) terms of the SKP Decomposition of \( W \)
Algebraic Synthesis

General procedure for Ground/Volume Decomposition

- Approximate $W$ by retaining the first two terms of the SKP Decomposition
- Choose the proper values of $a, b$:
  1. Select values of $a, b$ that give rise to (semi) positive definite $R_g, R_v, C_g, C_v$  
     $\Leftrightarrow$ physical validity of the solution
  2. Optimize some criterion in order to pick a unique solution

- The region of physical validity determines the ambiguity of the problem: all solutions in this region are *physically valid* and result in *exactly the same* data covariance matrix
- Valid solutions for ground and volume coherences belong to a line in the complex plane $\Leftrightarrow$ continuity with PolInSAR
- The region of physical validity tends to shrink as new baselines are gathered
The boreal forest in the Krycklan catchment, northern Sweden, has been investigated in the framework of the ESA campaign BioSAR 2008.

Data has been acquired by the airborne system E-SAR, flown by DLR.

Data focusing, calibration, and co-registration have been performed by DLR.

Five PolInSAR data-sets available:

- **P-Band**: 100 MHz, 1.6 azimuth res
  
  6 Tracks – 40 m horizontal baseline aperture. Look Direction: SW and NE

- **BioMass**: 6 MHz, 12.5 azimuth res
  
  6 Tracks emulating BioMass acquisitions, derived by DLR from the airborne data-set. Look Direction: SW

- **L-Band**: 100 MHz, 1.2 azimuth res
  
  6 Tracks – 30 m horizontal baseline aperture. Look Directions: SW and NE
**Tomographic Profiles**

**Backscattered Power Distribution**

**HV channel**
- Method: Capon Spectrum
- Results have been geocoded onto the same ground range, elevation grid to facilitate comparisons
- All panels have been re-interpolated such that the ground level always corresponds to 0 m
- Loss of resolution from near to far range is clearly visible, especially at P-Band ($\Delta z > 70$ m at far ranges)
- Relevant contributions from the ground level below the forest are found at P-Band
- This phenomenon occurs at L-Band as well, even though to a less extent
SKP Decomposition

\[ \hat{\mathbf{W}} = \sum_{k=1}^{\lambda} \lambda_k \tilde{C}_k \otimes \tilde{R}_k \quad \lambda_1 > \lambda_2 > \lambda_3 > \ldots \quad \hat{\mathbf{W}}_2 = \sum_{k=1}^{2} \lambda_k \tilde{C}_k \otimes \tilde{R}_k \]

**P-Band**

- Percentage of information

\[ \| \hat{\mathbf{W}} - \hat{\mathbf{W}}_2 \|_F \leq 0.05 \| \hat{\mathbf{W}} \|_F \]

> 95% of the information can be represented by the sum of just two KPs

**L-Band**

- Percentage of information

\[ \| \hat{\mathbf{W}} - \hat{\mathbf{W}}_2 \|_F < 0.1 \| \hat{\mathbf{W}} \|_F \]

> 90% of the information can be represented by the sum of just two KPs
Algebraic Synthesis: Volume Solutions

Inner Boundary Solution

- This solution corresponds to the polarization which is supposed not to be affected by ground contributions.
- Despite a slight increase in elevation, results are very close to those in HV.
- Significant contributions from ground level both at P and L-Bands are present.

Two possibilities:
- either this solution is wrong, resulting in non-rejected ground components…
- …or contributions from ground level are to be explained in terms of understorey and ground-volume interactions.
Largest Volume Solution (LVS)

- Volume Scattering is associated with the largest SM along the vertical direction.
- Important improvement at P-Band with respect to HV, resulting in a better match with LIDAR.
- Some improvement at L-Band.
- Physically sound solution.

- Contributions from below the ground are observed at the same locations at P-Band and L-Band, mostly in open or very low vegetated areas.

Further researches: artifact? Multiple scattering? Subsurface penetration?
• Forest height has been retrieved through a direct investigation of the shape of the retrieved tomographic profiles

• Relative error is obtained as:
  \[ e = \frac{|\text{HSAR}-\text{H100}|}{\text{H100}} \]

• Results have been obtained by exploiting a 60 x 60 m (ground range, azimuth) estimation window
Height Retrieval

- Good match with LIDAR: Standard Deviation < 3 m; No significant bias
- Estimation loses reliability for forests lower than 7 - 8 m at P-Band at 3-4 m at L-Band
- Good match between SW and NE
Ratio between the backscattered powers associated with ground-only and volume-contributions
Ground to Volume Backscattered Power Ratio

- At both wavelengths it is observed that the HV GVR decreases with forest height, consistently with the enlargement of volumetric structures.

- HV GVR exhibits a dependence on terrain slope at P-Band but not at L-Band. This result indicates that HV ground contributions are due to double bounce contributions at P-Band, but not at L-Band.
Resolution Loss Factor w.r.t. E-SAR = 100/6 •12.5/1.6 > 100 !

• At 30° a 60 x 60 estimation window contains just 5 independent looks ! ⇐ less robust statistics

• Slant range resolution loss further causes a spread of the backscattered power distribution, resulting in a *vertical* resolution loss

• Nevertheless, Tomographic profiles provide information about the forest structure that is consistent with the airborne case
BioMass: Height Retrieval

- Good match with LIDAR
- Standard Deviation < 4 m w.r.t. LIDAR by exploiting a 1 hectare estimation window
- No significant bias beyond 10 m
- Estimation loses reliability for forest lower than 10 m, consistently with the theoretical resolution limit
- More sensitive to the variation of baseline aperture
Conclusions

• Algebraic Synthesis has provided:
  • Agreement with LIDAR as for forest height
  • Physically consistent Ground-only and Volume-only Backscattered Powers

Not enough to claim that the true volume structure is captured. Yet:
  First step towards the solution of the SM separation problem
  Highlights the importance of structure in the understanding of scattering from forested areas.

• At P-Band the most relevant scattering contributions are observed at the ground level, both in co polar and cross polar channels
  After the BioSar 2007 experience, we conclude that HV scattering from the ground level is a distinctive trait of boreal forests at P-Band, implying the presence of specular reflections

• Topography does affect HV Ground to Volume Ratio at P-Band but not at L-Band, consistently with the hypothesis of specular reflections in HV at P-Band

• Forest imaging from BioMass represents a challenging problem. Though:
  No significant bias observed for forests higher than 10 m
  Dispersion has been assessed in about 20% with a 1 ha spatial resolution
  ⇔ Resolution loss is not a show stopper