APPLICATIONS OF CONTINUOUS WAVELET TRANSFORM TO POLARIMETRY AND INTERFEROMETRY

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Abstract

Usual imaging process makes the assumption that the reflectors are isotropic (i.e. they behave the same way regardless of the angle from which they are viewed) and white (they have the same properties within the emitted frequency bandwidth). New imaging capacities make these assumptions obsolete and some degradation of the image quality occurs when performing the standard imaging process. Multidimensional continuous wavelet transform for radar imaging was initially developed to highlight these effects. The wavelet tool allows to extend the notion of radar image to the hyper-image concept: a function of position \((x,y)\), of the emitted frequency and of the squint angle of the target. The goal of this paper is to present a method based on the wavelet transform, and to widen it for polarimetry and interferometry fields. Wavelet tool is first used for polarimetric parameters in order to improve target identification. Then it is used for coherence optimization in interferometry. The coherence optimization by wavelets is compared with the polarimetric interferometry one. It gives better results on the coherence level, and on the quality of the reconstructed target height thanks the interferometric angle. It is finally possible to combine both methods.

1 - INTRODUCTION

In usual radar systems, a transmitter pulse is radiated by an antenna, backscattered from the terrain and sensed by a receiver (see figure 1). The system is mounted on a moving platform. The wave vector of the emitted signal is written \(\vec{k}\). Thanks the measured backscattering coefficient \(H(\vec{k})\), the image process consists in providing a two dimensional representation of the reflectivity of the scene, written \(I(\vec{r})\). \(I(\vec{r})\) can also be considered as the spatial repartition of the bright scatterers which reflect a part of the emitted radar signal [5, 6].

The square modulus of \(H(\vec{k})\) is called the Radar Cross Section (RCS) of the object for the wave vector \(\vec{k}\) and is expressed in square meter. The wave vector \(\vec{k}\) is related to the emitted frequency \(f\) and to the direction \(\theta\) of radar illumination by relations:

\[
\left| \vec{k} \right| = \frac{2f}{c} \\
\theta = \text{arg}(\vec{k})
\]

where \(c\) is the speed of light, and the extra-factor 2 accounts for the round trip delay (in time) of the signal.

SAR imaging processes generally assumed that the sensor has a fixed orientation with respect to the object and emits with a fixed wave frequency. However, when the object is illuminated using a broad-band signal and/or for and a large angular extent, it is realistic to consider that the amplitude of the reflectors depends on frequency and on aspect angle. The spatial repartition of reflectors \(I(\vec{r})\) depends then on the wave vector \(\vec{k}\). It will be noted \(I(\vec{r}, \vec{k})\) and called “extended image” or “hyperimage”. The following notation will be used too: \(I(\vec{r}, \vec{k}) \equiv I(x, y, f, \theta)\), where \(x, y, f, \theta\) are respectively the range, cross-range in spatial domain, the frequency and the illumination aspect angle.

The multidimensional wavelet transform for SAR imaging has been performed to construct such hyperimages \(I(\vec{r}, \vec{k})\) in order to highlight the frequency and the angular behavior of the reflectors. Indeed, the wavelet tool allows to construct an image of the illuminated scene for each emitted frequency \(f_o\) and for each illumination aspect angle \(\theta_o\): \(I(x, y, f_o, \theta_o)\)
The next section is devoted to the construction of these distributions \( I(\vec{r}, \vec{k}) \) using the time-frequency analysis and the physical group theory. The purpose of the following sections is to widen the wavelet tool to polarimetry and interferometry respectively to improve target recognition and to obtain a better estimation of the target height.

2 - EXTENDED RADAR IMAGING

Time-frequency analysis and the physical group theory allow to construct extended radar images \([1, 2]\). The dimension of these images, called hyperimages, is the product of the dimension of the vector \( \vec{r} \) by the dimension of the vector \( \vec{k} \). The principle of the extended radar imaging \([1]\) is based on a physical group of transformations, the similarity group \( \mathcal{S} \). This is acting on physical variables \( \vec{r} \) and \( \vec{k} \) by rotations \( R_\theta \), dilations \( a \) in length (or time) and translations \( \delta \vec{r} \) according to:

\[
\begin{align*}
\vec{r}' &= a R_\theta \vec{r} + \delta \vec{r} \\
\vec{k}' &= a^{-1} R_\theta \vec{k}.
\end{align*}
\]

(1)

The transformation law of the reflected signal \( H(\vec{k}) \) and its extended image \( I(\vec{r}, \vec{k}) \) in a change of such a reference system is therefore given by:

\[
\begin{align*}
H(\vec{k}) &\rightarrow H'(\vec{k}) = a e^{-2i\vec{k} \cdot \delta \vec{r}} H \left( a R_\theta^{-1} \vec{k} \right) \\
I(\vec{r}, \vec{k}) &\rightarrow I'(\vec{r}, \vec{k}) = I \left( a^{-1} R_\theta^{-1} (\vec{r} - \delta \vec{r}), a R_\theta^{-1} \vec{k} \right).
\end{align*}
\]

(2)

2-1 A general formulation of the extended images

To construct the energy distribution \( I(\vec{r}, \vec{k}) \), a first approach consists in representing it as a hermitian and bilinear form of the signal \( H(\vec{k}) \) reflected by the target:

\[
I(\vec{r}, \vec{k})=\int \int K(\vec{k}_1, \vec{k}_2; \vec{r}, \vec{k}) H(\vec{k}_1) H^*(\vec{k}_2) d\vec{k}_1 d\vec{k}_2,
\]

(3)

where the kernel \( K(\vec{k}_1, \vec{k}_2; \vec{r}, \vec{k}) \) is supposed to be hermitian. This kernel is not known but can be determined with some physical constraint made on the distribution \( I(\vec{r}, \vec{k}) \):

- The distribution can satisfy the property of covariance by the similarity group \( \mathcal{S} \) given by (2),
• The distribution \( I(\vec{r}, \vec{k}) \) can be seen, in \( \mathbb{R}^2 \), as a spatial density (for a given \( \vec{k} \)). Then, the distribution, has to be positive. Its integral on some surface \( \mathcal{D} \) can, therefore, be interpreted as the RCS (Radar Cross Section) contribution \( \sigma_{D}(\vec{k}) \) of all the reflectors contained in \( \mathcal{D} \):

\[
\sigma_{D}(\vec{k}) = \int_{\mathcal{D}} I(\vec{r}, \vec{k}) \, d\vec{r}.
\]  (4)

• If \( \mathcal{D} \) represents the whole plan, the distribution can respect the well known marginal property:

\[
\int I(\vec{r}, \vec{k}) \, d\vec{r} = |H(\vec{k})|^2.
\]  (5)

• The energy conservation between the distribution space and the reflected signal space leads to an important relation (Moyal formula) which connects the inner product between two given reflected signals \( H_1 \) and \( H_2 \) and their associated distributions \( I_1 \) and \( I_2 \):

\[
\left| \int H_1(\vec{k}) \, H_2^*(\vec{k}) \, d\vec{k} \right|^2 = \int \int I_1(\vec{r}, \vec{k}) \, I_2^*(\vec{r}, \vec{k}) \, d\vec{r} \, d\vec{k}.
\]  (6)

The time-frequency analysis has shown that no distribution can satisfy all these properties. For example, the property (6) does not always allow to obtain a distribution everywhere positive, which is inconsistent with the RCS nature of the distribution given by (4) or (5). To overcome this drawback, it is possible to construct a regularized form of these distributions obtained by smoothing the general distribution given by (3). These regularized distributions verify the constraint (2), (4) and (6) but not the marginalisation property (5). The construction of these extended images which introduces the wavelet transform is developed below.

2-2 Construction of the extended images by Wavelet transform

Let \( \phi(k, \theta) \) be a mother wavelet supposed to represent the signal reflected by a reference target. The associated distribution \( I_\phi(\vec{r}, \vec{k}) \) is supposed to be well located around the spatial origin \( \vec{r} = \vec{0} \) and \( (k, \theta) = (1, 0) \). Here a two-dimensional separable gaussian function is used:

\[
\phi(k, \theta) = e^{-\frac{(\sqrt{\epsilon k})^2}{\sigma_k^2}} \cdot e^{-\frac{1}{2} \epsilon \theta^2}.
\]  (7)

where the two free parameters \( \sigma_k \) and \( \sigma_\theta \) control the spread in frequency and in angular domain and play on interrelated resolutions in spatial domain \( \vec{r} = (x, y) \), frequency and angle.

By the action of the group \( \mathcal{S} \), a family of wavelet bases \( \Psi_{\alpha_0, \epsilon_0}(\vec{k}) \) can be generated from the mother wavelet \( \phi(k, \theta) \) according to:

\[
\Psi_{\alpha_0, \epsilon_0}(\vec{k}) = \frac{1}{k_0} e^{-2\pi i \vec{k} \cdot \vec{\epsilon}_0} \phi \left( \frac{1}{k_0} \mathcal{R}_{\alpha_0} ^{-1} \vec{k} \right) \frac{1}{k_0} e^{-2\pi i \vec{k} \cdot \alpha_0} \phi \left( \frac{k}{k_0} \theta - \beta_0 \right).
\]  (8)

A regularized distribution \( \tilde{I}(\vec{r}_0, \vec{k}_0) \) can be constructed by smoothing the general distribution \( I(\vec{r}, \vec{k}) \) given by (3). And using Moyal formula (6), covariance property (2) with \( H_1(\vec{k}) = H(\vec{k}) \), \( H_2(\vec{k}) = \Psi_{\alpha_0, \epsilon_0}(\vec{k}) \), \( I_1 = I_H \) and \( I_2 = I_\phi \), we obtain:

\[
\tilde{I}(\vec{r}_0, \vec{k}_0) = \int \int I_H(\vec{r}, \vec{k}) \times I_\phi^*(\vec{k}) \, d\vec{r} \, d\vec{k} = \int \tilde{H}(\vec{k}) \frac{1}{k_0} e^{2\pi i \vec{k} \cdot \alpha_0} \phi^* \left( \frac{1}{k_0} \mathcal{R}_{\alpha_0}^{-1} \vec{k} \right) \, d\vec{k}.
\]  (9)

The right hand side is nothing but the wavelet coefficient \( C(\vec{r}_0, \vec{k}_0) \) which is introduced as the invariant scalar product of the group \( \mathcal{S} \) between the reflected signal \( H \) and each element \( \Psi_{\alpha_0, \epsilon_0} \) of the wavelet basis:
located at represents a spatial repartition of reflectors which respond at this frequency and this angle. Inversely, for each reflector VH and VV.

The polarimetric generalization of the backscattering coefficient is called the scattering matrix $S$. The matrix consists of 4 complex numbers, representing the complex backscattering coefficients for all four polarization combinations, HH, HV, VH and VV.

$$S = \begin{pmatrix} S_{HH} & S_{HV} \\ S_{VH} & S_{VV} \end{pmatrix}$$ (13)

The purpose of the polarimetric analysis is to separate and identify these mechanisms with the objective of discriminate and recognize targets. Several sets of parameters are available to analyze the distributed targets [7]. Under the reciprocity assumption framework, i.e $S_{VH} = S_{HV}$, the three component scattering vector is used instead of the scattering matrix, for each pixel $(x, y)$:

$$\vec{s}(x, y) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} S_{HH} + S_{VV} \\ S_{HH} - S_{VV} \\ 2S_{HV} \end{array} \right)$$ (14)

Then the coherency matrix, from which all polarimetric parameters sets can be deduced, is written:

$$T(x, y) = \langle \vec{s}(x, y) \vec{s}^T(x, y) \rangle$$ (15)

The wavelet tool allows to calculate a diffusion vector $\vec{s}$ for each angle and frequency : $\vec{s}(x, y; f, \theta)$. All polarimetric parameters can be then expressed in function of the coherency matrix $T(x, y; f, \theta)$ and then are obtained for each angle and each frequency too. A key polarimetric parameter used to distinguish natural targets from artificial one as buildings is entropie $H$: this is a formal measure of the randomness of the scattering features in the scene. By definition

$$H = - \sum_{i=1}^{3} p_i \log_2(p_i), \quad p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2 + \lambda_3}$$ (16)

where the $\lambda_i$ are the eigenvalues of the matrix $T$. When the entropy is near zero, the principal mechanism is said to be the one of a smooth surface or a single object while when the entropy is close to 1, it means that the randomness degree is high as it can be for volume scattering from vegetation. This information can be used to provide automatic classification of radar scenes. Using $\vec{s}(x, y; f, \theta)$, entropy images can be calculated for each frequency and angle. In order to improve target enhancement, an image of minimal and maximal entropy can be obtained, defined as:
An image (see figure 2-a) of 500 × 500 pixels have been selected on the airborne RAMSES X-band data on Bretigny. It contains industrial buildings, trees, a parking lot, and four canonical trihedrons used for calibration. Wavelet coefficients have been calculated for ten angles, ten frequencies. The parameters and are chosen to obtain 30% of the angular and frequency bandwidth. With those coefficients, the maximal entropy has been computed. Its image allows to separate deterministic targets as buildings better than initial entropy (figure 2-a and 2-b). On the contrary, , giving an very low entropy on whole image, is not interesting for contrast enhancement.

4 - INTERFEROMETRY

Interferometry is an efficient approach when used to reconstruct the topography of a given zone. It is based on the measurement of the phase difference between two paired pixels of two complex SAR images, obtained from the data collected by two antennas. The terrain elevation is proportional to this phase difference, known as interferometric phase. If is the scattering coefficient for a pixel of the first image, and is the coefficient for the same pixel obtained from the second image, the coherence of the pixel is mathematically defined as:

\[
\gamma = \frac{<s_1s_2>}{\sqrt{<s_1^2>\sqrt{<s_2^2>}}}
\]
The interferometric coherence computation requires averaging over many samples from the same distribution. Computationally, it is estimated using a boxcar filter where samples are in a N × N window. We choose here N = 3.

\[
\gamma = \frac{ \sum_{i=1}^{N \times N} s_1(i) s_2^*(i) }{ \sqrt{ \sum_{i=1}^{N \times N} s_1(i) s_1^*(i) \sum_{i=1}^{N \times N} s_2(i) s_2^*(i) } } \text{ (19)}
\]

The phase of \( \gamma \) is the interferometric phase and is written \( \Phi \), and its modulus is called coherence. It is known that the height estimation is all the more reliable as this coherence is large. One of the benefits of the wavelet tool is the coherence optimization possibility, and consequently an increase of the accuracy of the reconstructed elevation profiles for each scatterer. Interferometric phase can be computed for each angle and each frequency. For each pixel, we find \( \gamma \) giving the maximum coherence. Then the interferometric phase \( \Phi \) corresponding to this couple \( (f, \theta) \) is used to calculate the elevation map, thanks the relation:

\[
h = \frac{\Phi}{2\pi} e_a \text{ (20)}
\]

\( e_a \), the "ambiguity height", depends on the geometric configuration of the radar and on the distance of the target. This method of optimization is called the "time-frequency optimization". Results are compared with those obtained after the optimization coherence thanks polarimetry [9] on figure 3. On this image, coherence modulus is better optimized with the wavelet optimization than with polarimetric optimization, as shown on table 1.

### Table 1: coherence levels before and after optimizations

<table>
<thead>
<tr>
<th>Coherence level</th>
<th>Mean value</th>
<th>Minimum value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>in vV polarization</td>
<td>0.9533</td>
<td>0.0145</td>
<td>0.9999</td>
</tr>
<tr>
<td>after polarimetric optimization</td>
<td>0.9903</td>
<td>0.5206</td>
<td>1.0000</td>
</tr>
<tr>
<td>after time-frequency optimization in vV polarization</td>
<td>0.9993</td>
<td>0.9577</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

5 - POLARIMETRIC INTERFEROMETRY

The coherence time-frequency optimization thanks wavelet is performed for a given polarization. One way to combine this optimization with the polarimetric one is to perform them successively. On the figure 4, the time-frequency optimization has been first applied to the three scattering vector components, then the polarimetric optimization has been performed. This method, which will be called "method a", gives a very smoothed image, each optimization requiring an additional average to calculate the coherence. The drawback of this is that samples used to compute the coherence can be finally chosen in heterogeneous areas. This leads with our data to an underestimated height on buildings.

To avoid this problem, another idea is to perform the polarimetric optimization after an average performed on all wavelet coefficient of the pixel. That means that the coherence is now computed for each pixel by

\[
\gamma = \frac{ \sum_{f, \theta}^{N_f \times N_\theta} s_1(f, \theta) s_2^*(f, \theta) }{ \sqrt{ \sum_{f, \theta}^{N_f \times N_\theta} s_1(f, \theta) s_1^*(f, \theta) \sum_{f, \theta}^{N_f \times N_\theta} s_2(f, \theta) s_2^*(f, \theta) } } \text{ (21)}
\]

and then polarimetric optimization is performed on this coherence. This method, called "method b" leads to the better results in this paper: elevation profiles are represented on for each method. The estimated heights of three buildings are listed in the table 2 (in meters).

6 - CONCLUSION

The use of time-frequency analysis in order to improve radar images has been presented both in interferometric and polarimetric fields. A combination of time-frequency and polarimetric optimizations has been proposed for the first time in Polarimetric interferometry. Further investigations could allow to improve final results choosing more appropriate averaging, or more appropriate wavelet forms.
Figure 3: elevation map obtained from interferometric angle: a- in polarization hH, b- after polarimetric optimization c- after time-frequency optimization

Figure 4: elevation map obtained from interferometric angle first optimized with wavelets, then with polarimetry
Table 2: ground truth and estimated heights

<table>
<thead>
<tr>
<th></th>
<th>building 1</th>
<th>building 2</th>
<th>building 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>method a</td>
<td>8.3</td>
<td>1.0</td>
<td>4.5</td>
</tr>
<tr>
<td>method b</td>
<td>10.6</td>
<td>3.4</td>
<td>7.1</td>
</tr>
<tr>
<td>ground truth</td>
<td>11.5</td>
<td>3.5 to 7</td>
<td>10</td>
</tr>
</tbody>
</table>

References


