PHASE UNWRAPPING
Phase unwrapping
1D Phase unwrapping

Problem: given the wrapped phase $\psi = \text{W}(\phi)$ find the unwrapped one $\psi$. The wrapping operator: $\text{W}(\phi) = \text{angle}(\exp(j \phi))$, gives always solution $-\pi \ldots \pi$ and is $\approx$ to the remainder (modulus $2\pi$)

The problem is ill-conditioned due to sampled nature (how many $2\pi$ from sample to sample?)
Solution is not unique. We need some regularization criteria to find one.

We assume that the phase difference between 2 samples (adjacent) is not aliased, hence the discrete gradient $|\Delta \phi| = |\phi_i - \phi_{i-1}|$ should always be $< \pi$.
We estimate it by wrapping the difference of the wrapped phase $\Delta_c \phi = \text{W}(\Delta \psi)$.
For example $\psi_{i-1} = 3\pi/4$; $\psi_i = -3\pi/4 \Rightarrow \Delta \psi = -6/4 \pi$ we correct it into $\text{W}(\Delta \psi) = -6/4\pi + 2\pi = \pi/2$

After “correcting” the “wrong” gradient (assuming that $|\Delta \phi|$ “should” be $< \pi$) we can integrate the gradient to get the estimate of $\phi$ (but for a constant). The technique is highly non linear due to the wrapping operator.
The technique shown is successful provided that the true gradient is bounded to $< \pi$.

It is unlikely so when both the sampled topography is rough and elevations are $> \Delta H_{amb}$.

Slow varying topographies are not a problem even if elevations are large.
Rough topographies are not a problem if elevations are contained (see multi-baseline pu).
2D PU artifacts
1D Phase unwrapping: not successful

For large elevation and large baseline the problem breaks down fast and probability of success (number of singular points) drops.
2D Phase unwrapping

In the 2D we have different possible integration paths and we want to ensure at least the independence of the solution on the path.

The unwrapped phase field should be conservative (rotation free): $\nabla \times \Delta \phi = 0$ where the gradient is now a two component vector $\Delta \phi = [\Delta \phi^i \Delta \phi^k]^T$.

At least, for a close path (a curl), we want to be back at the same height!

This does not hold as soon as the curl embrace a “vortex” (residue), where phase is not defined:

$$\text{real}(s(\vec{P})) = \text{imag}(s(\vec{P})) = 0$$

Amplitude should be zero, and this happens only in the presence of noise.

In absence of vortex, all fringes are well formed: they are closed (eventually to the image edges) and unwrapping is an easy task: just counting them.

In presence of vortex, fringes appears and disappear suddenly and “closing” them is a hard task.
2D Phase unwrapping

\[ \nabla \phi = \begin{cases} 
(\phi_3 - \phi_1)_{rg} \\
(\phi_2 - \phi_1)_{az}
\end{cases} \]
Example: constant slope

We assume a constant range slope (the most common case).

We assume the phase rotating counter clockwise. Real parts zero for \(-\pi/2\) or \(\pi/2\). Image parts zero for 0 or \(\pi\). In absence of noise real and imaginary parts never crosses. However, the steeper the slope, the closest the lines.
Example: constant slope + noise

Noise shifts \( re=0 \) and \( im=0 \) lines. A crossing may happen.

For small noise, crossings are paired and nearby.

When the summation path contours the vortex, the line integral increases by \( +2\pi \) (or \( -2\pi \)) every circuitation.

In fact every time we cross a blue or an orange line, we change to the adjacent quadrant. In the example, the contour integral adds \( -2\pi \) for every circuitation of the vortex.
**Finding vortices (residuals)**

Residuals cannot be found by inspection, since data are known on the sampled grid. However, it is enough to compute the shortest line integral, $\nabla \times \Delta \phi$, on a square of 4 samples around the vortex.

Residuals should never be encircled by integration paths, since solution will be non conservative. Residuals are always in couples of opposite sign (yet one of the two can be outside the image). If we connect each couple by a “ghost” line, never to be crossed, then we avoid adding $2\pi$ to the contour integral.

Residuals can be due to **topography** (elevation aliasing) or **noise** (the sensitivity increases in areas sloping toward the satellite, since the phase contour lines are closer). Low noise creates short dipoles, that can be easily handled. Situation improves with SNR; however, **smoothing the phase field is mandatory**!
A single -1 charge is created.

(I) Due to topography (horizontal shear)

A dipole is created.

(II) Due to noise (enhanced by topography)

Residuals

added noise: -0.4
Residuals due to topography are much more difficult to be connected, since ghost lines can be very long. A foreshortened slope gives a ghost line parallel to azimuth as long as the slope itself (maybe a valley).

In these cases, the phase unwrapping solution is not unique, and different strategies exists for connecting residuals.

**Connecting residuals**
A real example

Original phase

Irrotational phase

Rotational phase

Unwrapped irr. phase

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Connecting residuals

Original phases

Unwrapped phases
In the 1D case the best pu is PLL. This is not for 2D (which direction?)

Several algorithms have been proposed. Definitely, they integrate phase gradients.

The best ones:

- exploits a-priori (existing DEMs, topography statistics)
- derive a-priori e.g. by operating at different resolutions (multi-grid)
- or by exploiting local measures of SNR (region growing)
- estimates gradient by looking over global (large) areas

The most efficient implementations write PU problem as a linear system of equations, where the unknown are the either the unwrapped phases $\phi$ or their gradients $\Delta\phi$ (e.g. the proper number of $2\pi$ to be added the estimated ones, $\Delta\psi$), given the estimated ones, $\Delta\psi$.

A-priori information, like noise estimates or topographies are included as weights.

These systems are overdetermined, since for each pixel/phase we can estimate 2 gradients (a two component vector $\Delta\phi = [\Delta\phi^i \Delta\phi^k]^T$), thus some “regularization” is needed.

**PU - algorithms**
Solutions proposed in literature find the unwrapped phase that are “mostly” consistent with the “observed data” (the gradient estimates), in some norm. Even though they are driven by statistics optimization problems, the actual properties of signal (topography) and noise are usually disregarded.

L0 norm minimizes the global cut length. It is a NP hard problem, that cannot be solved for global optimum on usual interferogram. Local solution of “reasonable” complexity [Golstein cut-branch, Pritt PCG] exist.

L1 norm minimizes the total number of multiple of 2π to be added at each phase (pixel) - and these are the unknown. Its global solution [Costantini, Flynn] is rather fast by exploiting network-flow algorithm in both the weighted and unweighted case.

L2 norm minimizes the energy of the error between computed slopes and the one to be reconstructed. It is the optimal one for gaussian noise (the case of interest), but it does not “honor the data” (the reconstructed - re-wrapped phase usually differs from the given one). Its unweighted solution is fast (2D FFT). L2 requires slopes gradient, (∂²φ/∂r²) a “noisy” |f|² power spectrum operator. Slopes estimates are usually biased (vs 0), hence reconstructed elevation is (even strongly) downsized!

Finding a solution
DEM generation
Focusing and co-registration

**SAR Interferometry: DEM generation (1)**
Fringes generation and baseline estimation

**SAR Interferometry: DEM generation (2)**
SAR Interferometry: DEM generation (3)

Interferogram flattening
SAR Interferometry: DEM generation (4)
Identification of disconnected zones

SAR Interferometry: DEM generation (5)
Unwrapping 2

SAR Interferometry: DEM generation (6)
Interpolation

SAR Interferometry: DEM generation (7)
\[ \Delta y = \frac{\Delta s}{\sin \theta} + \frac{\Delta q}{\tan \theta} \]

**Geocoding**
\[
\Delta \phi = \frac{4\pi}{\lambda R_0 \sin \theta} B_n \Delta q
\]

phase to height conversion

&
geocoding

**SAR Interferometry: DEM generation (8)**

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SAR Interferometry: DEM generation (9)
**DEM generation: MultiBaseline**

The presence of atmospheric noise prevent quality DEM generation from repeat pass InSAR.

Conventional noise sources **can** cause poor SNR in **particular** areas (and acquisitions). SNR can be somewhat be recovered by averaging in space at the cost of a resolution drop.

Atmospheric artifacts are not detected by coherence maps, since they are correlated in space (like the signal). They can contribute with 1-2 fringes (normal condition). 1 fringe = 30-300 m for Bn=300-30 m. Desired accuracy for a 50 x 50 m DEM: $\sigma_q \approx 5$ m.

\[
\phi = \frac{4\pi}{\lambda} R_1(P) + \phi_{scat}(P) + \phi_{nois}(P) + \phi_{atmo}(P)
\]

The problem of DEM generation from InSAR appears ill-posed

We have to retrieve 3 numbers (for each P) given two measures!
**Multibaseline DEM generation**

Let us assume to have many tandem interferograms. For each pixel of the interferogram “m” you can write the following equation:

\[ \Delta \phi(P) = \frac{K}{B_m} q(P) + \phi_{\text{noise}}(P; m) + \phi_a(P; m) \]

\( \Delta \phi \) is the interferometric phase difference with respect of a reference (O), q the elevation difference + noise terms.

“Decorrelation” noise can be identified by coherence maps.

Atmospheric noise is incorrelated with time. The only stable phase is due to topography \( q(P) \), scaled by the baseline of the acquisition that should be known with high accuracy.

The unknown q can be retrieved by averaging the M measures, since incorrelated noises vanishes. The proper weights could be inversely proportional to the baseline, or determined in a ML optimization driven by DEM and atmospheric statistics.

Only very low frequency residuals (tilt plane) remains. Atmosphere itself has strong power at low frequencies (1/f). A few GCP are required, or fusion with other DEM
**Multibaseline phase unwrapping**

A welcome advantage provided by MB interferometry is the practical solution of pu. If one baseline only is available, the height difference between two pixel is ambiguous of an interval that depends on the baseline.

An height $q$ (from two points) is related to the phase gradient (and a noise) by the following expression:

$$ q = \frac{\lambda}{4\pi} \cdot \frac{R_0 \sin \theta}{B_n} \Delta \phi + n = \frac{k}{B_n} \Delta \phi + n $$

Given the pdf of noise $f_n(n)$, of the interferometric phase difference $f_{\Delta \phi}(\Delta \phi)$ and an a-priori for the height difference (slope) $f_\alpha(\alpha)$ we can derive the ML estimate of the unknown $q$.

In single baseline interferometry the most likely $q$ is the one corresponding to usual wrapped gradient. The error probability is however large even in the noiseless case if ambiguity elevation is small compared to the standard deviation of slopes.
Multibaseline phase unwrapping

A priori not applied: ambiguity is close to the prime factor product (Chinese remainder). The probability of error vanishes when ambiguity is large (say >> 100 m)
Maximum Likelihood DEM Reconstruction

- Solves phase unwrapping and multi geometry / wavelength fusion simultaneously
- Incorporates a-priori knowledge

Interferogram 1

Observation 1

\[ R_1(z') \]

Observation 2

Interferogram 2

\[ R_2(z') \]
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DEM generation: MultiBaseline

Vesuvius (MB) elevation map
Vesuvius: Digital Elevation Model from ascending and descending orbits
DEM generation: MultiBaseline

Etna (MB) elevation map + reflectivity
Etna (MB) elevation map + reflectivity (ascending + descending)
Etna (MB) elevation map + contour lines (ascending + descending)
### The best baseline?

The baseline affects strongly the interferometric quality performances.

**Noise** and **signals** acts differently according to the kind of decorrelation.

- **If signal is topography**, it is “amplified” by the baseline. Yet, Bn large leads to a hard phase unwrapping. Multibaseline (MB) techniques helps in overcoming this problem.

- Atmospheric effects acts like motion and are difficult to be separated. In a MB environment atmosphere decorrelates in time, but not motion.

- For **motion detection** topography is “noise” either it can be known a-priori (an existing DEM) and removed, or it can cancelled for Bn=0 or small.

- Temporal decorrelation and system noise affects interferogram independently on Bn.

- Volume scattering and spatial (slope dependent) decorrelation increase with the baseline. Atmospheric artifacts and temporal decorrelation (the strongest effects in repeat pass ISAR) disappear in single pass interferometry.