GOCE QUICK-LOOK GRAVITY FIELD ANALYSIS: TREATMENT OF GRAVITY GRADIENTS DEFINED IN THE GRADIOMETER REFERENCE FRAME

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ABSTRACT

The purpose of the Quick-Look Gravity Field Analysis (QL-GFA) is to analyse partial and/or incomplete data sets of satellite gravity gradiometry (SGG) and high-low satellite-to-satellite tracking (hl-SST) data, in order to derive a fast diagnosis of the GOCE system performance, by detecting potential distortions of statistical significance (e.g. systematic errors) in the input data, and to give a fast feedback to the GOCE mission control (Payload Data Segment, PDS). The main objective of the present paper is to investigate the impact of the new mission design, i.e. the gravity gradients defined in the Gradiometer Reference Frame (GRF), onto the performance of the QL-GFA software, with special attention given to the proposed functionality such as the handling of data gaps and partial data sets and the estimation of the spectral gradiometer behaviour.

1. INTRODUCTION

The satellite mission GOCE, the first Core Mission of ESA’s Living Planet Programme, is dedicated to the precise modeling of the Earth's gravity field from SGG and hl-SST observations. The mathematical model for its parameterization is based on a series expansion into spherical harmonics, yielding a very large number of unknown coefficients, and efficient solution strategies are required to solve the corresponding large normal equation systems. The QL-GFA tool is based on the semi-analytic method ([6], [10], [12]). Key tasks of the QL-GFA are:

- Check of SGG and hl-SST input data in parallel to the mission and analysis of partial / incomplete SGG and hl-SST data sets to derive a diagnosis of the GOCE system performance.
- Computation of quick-look gravity field models (SGG only, SST only, combined SST+SGG) for the purpose of a fast analysis of the information content of the input data on the level of the gravity field solution. Additionally, quick-look gravity solutions are compared with reference gravity models, and statistically tested using the confidence level of the QL-GFA solution and the covariance information of the a priori gravity model.
- Estimation of the gradiometer error PSD (power spectral density) from the residuals of a SGG-only gravity field analysis, and application of previously defined statistical hypothesis test strategies in time and frequency domain ([6], [11]). Therefore, the question whether the a priori gradiometer error model is realistic can be answered, and optimal filters for an ultimate-precision adjustment (e.g. [3], [4], [7]) can be designed.
- Production of Diagnosis Report Sheets for the PDS with a latency of a few days.

The QL-GFA method was already successfully applied in the framework of realistic GOCE closed-loop simulations ([3], [4]), also in the case of partial data sets (minimum length two weeks), data gaps and non-closing orbits ([5], [6], [8]). In [2] and [6] the strategy for the estimation of the gradiometer error PSD is outlined, and in [6] several hypothesis test strategies for statistically testing both the residuals of the adjustment and the quick-look gravity field solutions in terms of spherical harmonic coefficients are presented and discussed.

QL-GFA solutions complete up to degree/order 250 can be processed within the order of one hour on a standard PC. The efficiency and speed of QL-GFA is founded mainly on the application of FFT techniques, the assumption of block-diagonality of the normal equation matrix, and also on a simplified filter strategy in the spectral domain to cope with the coloured noise characteristics of the gradiometer ([1]). Deviations from this assumption are incorporated by means of an iterative procedure. A detailed discussion of the theory and the mathematical models the QL-GFA software is based on can be found in ([6]).

However, the new developments in the GOCE mission design, particularly ESA's decision to dismiss the FEEP (Field Emission Electric Propulsion) system for the fine-tuning of the satellite’s orientation, leading to a degradation of the drag-free and attitude control, has main consequences onto the gravity field processing and correspondingly to the accuracy of the GOCE gravity field solution. Two key problems may lead to a degradation of the accuracy of the QL-GFA solutions:

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1. Mispointing of the satellite and thus also of the gradiometer with respect to the Local Orbital Reference Frame (LORF);
2. Degraded gradiometer performance, defined in the Gradiometer Reference Frame (GRF).

The first problem is specific for the QL-GFA, because an intrinsic property of the semianalytic approach is that the gravity functionals are evaluated in the Local Orbital Reference Frame. On the other hand, the second issue holds for any gravity field solution strategy.

Therefore, the main objective of the present paper is to investigate the impact of the GRF problem onto the performance of the QL-GFA software, with special attention given to the proposed functionality such as the handling of data gaps and partial data sets and the estimation of the gradiometer error PSD.

2. SIMULATIONS AND RESULTS

Several case studies will be presented to evaluate the performance of QL-GFA on the basis of the new GOCE mission design, i.e. the gravity gradients given in the GRF, and with a lower accuracy compared to the original gradiometer specification ([1]). In order to evaluate the pure effect of the gradiometer rotation onto the gravity field determination, the present paper is restricted to SGG-only solutions, neglecting the contribution of the hl-SST component.

2.1 Test data sets

A realistic non-circular, sun-synchronous 59-days repeat orbit with an inclination of i = 96.6° and a mean altitude of approximately 250 km, based on the global gravity model OSU91A ([9]) complete up to degree/order $l_{max} = 80$, was used. These orbit data were generated applying numerical orbit integration. The orientation of the satellite with respect to the LORF will have two major components: It is expected to be yaw steered, i.e. piloting w.r.t. the LORF, experiencing the maximum drag force, and it will perform a roll motion due to the latitude-dependence of the accuracy of the magnetic torquers. Since so far no realistic simulations on these orientation angles of the GRF w.r.t. the LORF have been available to us, they were modelled to be of sinusoidal shape with a 1/rev. characteristics, and with an amplitude of the yaw angle of ±3.5°, and the roll angle of ±2.2°. In practice, this orientation of the GRF w.r.t. the LORF will not be perfectly known. Therefore, in the following case studies these rotation angles were optionally superposed by a white noise time series with an amplitude of $\sigma = 20$ arcsec. Considering the actual performance of the star tracker, this should be a quite pessimistic assumption.

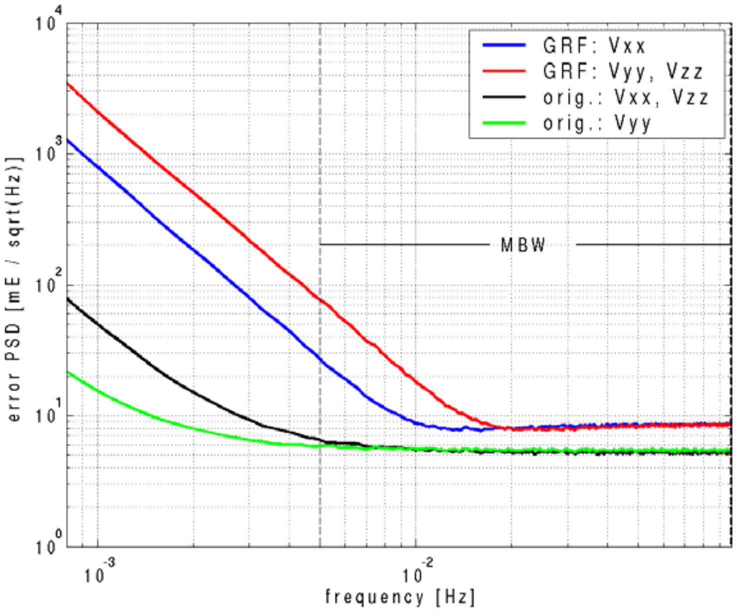

Fig. 1. SGG error PSD of the main diagonal components V_{XX}, V_{YY} , and V_{ZZ}; original performance vs. new GRF specifications.

Gradiometer measurement time series representing the three main diagonal elements of the gradient tensor V_{XX}, V_{YY} and V_{ZZ}, defined in the GRF (according to the rotation defined above), were simulated along the orbit with a sampling interval of Δt = 5 s, leading to a data volume of more than 3 million observations. The SGG data contain gravity field information based on the OSU91A model complete up to degree/order l_{max} = 180. The instrument noise characteristics were simulated based on figures of the new GOCE gradiometer error PSD specified in the ESA fact sheet. Fig. 1 shows the spectral properties of the (smoothed) noise time series in terms of the square root of the PSD. An identical spectral noise behaviour of the V_{YY} and V_{ZZ} components is assumed, while the error PSD of the V_{XX} component is considerably smaller. As a reference, also the error curves related to the original gradiometer specification ([1]) are given. Obviously, the main problem of the new specification is the decreased accuracy in the lower frequency range of the original measurement bandwidth (MBW).

2.2 Case study: Noise-free scenario (proof of QL-GFA concept)

One of the problems caused by the omission of the micro-propulsion system is that the highly precise axes of the gradiometer are no longer oriented in the LORF, but in the GRF. In principle, this new configuration is not critical for any direct or space-wise method for gravity field analysis, because the *base functions* can be rotated and the normal equations can be set up in the GRF. Thus the rotation of the *signals*, which constitute the gradiometer tensor, can be avoided, which would import large errors of the off-diagonal tensor elements V_{XY} and V_{YZ} into the high-sensitive main diagonal components. A straightforward error propagation demonstrates, that such an approach would practically destroy the performance of the mission. However, since the QL-GFA software is based on the semianalytic method, and thus on a few simplifying assumptions, the corresponding normal equations cannot be rotated into the GRF, particularly in the case of a time-varying orientation of the GRF w.r.t. the LORF.

Therefore, an alternative, iterative strategy was implemented in the QL-GFA to adequately treat this mispointing. In the first case study, the QL-GFA software was applied to the three main diagonal components V_{XX}, V_{YY} and V_{ZZ}, defined in the GRF. No noise was superposed to the SGG data, and the rotation matrix LORF-GRF was assumed to be known exactly. Fig. 2 shows the convergence behaviour of the iterative QL-GFA algorithm in terms of the deviations of the estimated coefficients from the initial "true" OSU91A model represented by the degree median,

$$\sigma_l^i = \text{median}_m \left\{ | \overline{R}_{lm,i}^{(est)} - \overline{R}_{lm,i}^{(OSU)} | \right\} \qquad (1)$$

where $\overline{R}_{lm,i} = \{\overline{C}_{lm,i}, \overline{S}_{lm,i}\}$ are the harmonic coefficients, i is the iteration number, *(est)* denotes the adjusted quantities and *(OSU)* refers to the reference model OSU91A.

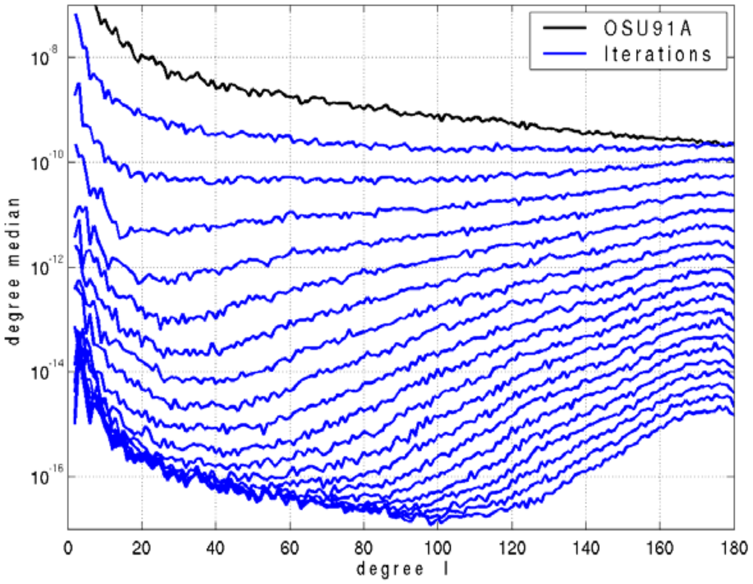

Fig. 2. Convergence behaviour of QL-GFA software, applied to the main diagonal components V_{XX}, V_{YY} and V_{ZZ}, defined in the GRF; noise-free scenario.

Although after 20 iterations convergence is not yet achieved in the high-degree range of the spectrum, the level of numerical accuracy is already reached. This is a prove of the concept, demonstrating that the QL-GFA software can perfectly cope with the misorientation of the satellite w.r.t. the LORF. Concerning the mispointing problem, it performs equivalent to any direct gravity field solver.

2.3 Case study: Realistic scenario: partial data sets, data gaps and PSD estimates

Since it was demonstrated in chapter 2.2 that QL-GFA can adequately treat the problem of the mispointing of the GRF w.r.t. the LORF, in the present case study the second problem, i.e. the effect of the new error budget of the gradiometer (cf. Fig. 1) will be investigated. Additionally, two scenarios, which are quite realistic in the field of applications of the QL-GFA software, were assumed: the processing of fast intermediate gravity field solutions in the case of partial data sets and data gaps, and the spectral analysis of residuals of the SGG component for the cross-validation of the gradiometer error PSD. In [8] the first feature was already extensively discussed, and in the present study this procedure is adapted and improved so that it can be applied in the case of the gravity gradients defined in the GRF.

The first simulation is based on the data sets described in chapter 2.1. The SGG observations (V_{XX}, V_{YY} and V_{ZZ}) were superposed by a coloured noise time series, according to new error specifications. The red curve in Fig. 3 shows the resulting degree median, assuming the rotation between LORF and GRF to be perfectly known. As a reference, the cyan curve shows the same configuration, but applying the original gradiometer error to the SGG observations. The degradation of the gravity field solution due to the degraded gradiometer performance is obvious.

Applying white noise of $\sigma = 20$ arcsec to the rotation matrix results in the dashed blue curve. Evidently, small errors in the orientation information hardly affect the accuracy of the gravity field solution. In order to demonstrate the effect of larger uncertainties of these rotation angles, the last simulation was recomputed, but now applying an unrealistically high rotation error of $\sigma = 5$ arcmin. The green curve shows the corresponding results. Evidently, errors in the orientation information mainly affect the higher harmonic degrees l, while the effect in the low degree range of the spectrum is negligible.

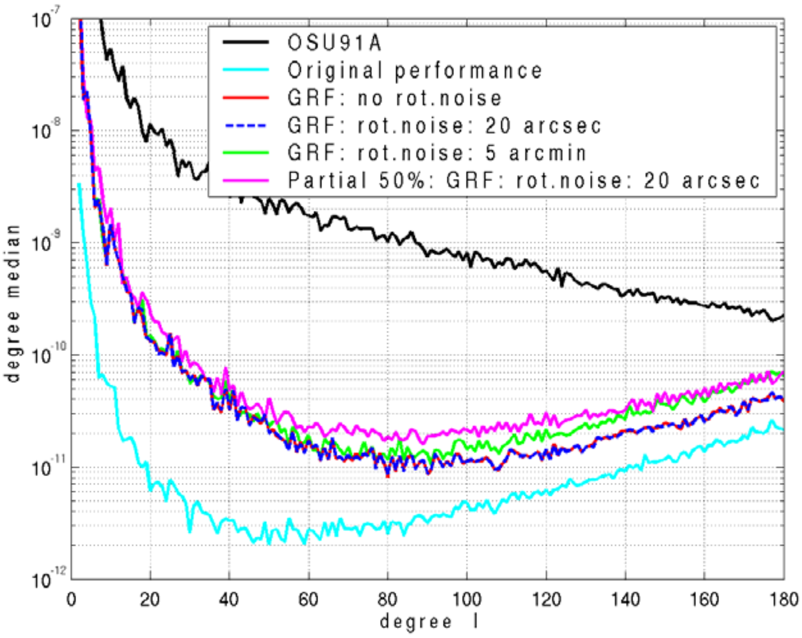

Fig. 3. Degree median of diverse simulation scenarios: SGG-only solutions, resolved complete up to degree/order 180.

One of the main goals of QL-GFA is to analyse partial data sets. Therefore, in the next simulation only the first 50% of the data sets, i.e. the first 29.5 days of the V_{XX}, V_{YY} and V_{ZZ} time series, were used in the processing, and additionally randomly distributed small data gaps of 5 s to 5 min, which occur on an average once per revolution, were introduced. The magenta curve in Fig. 3 shows the degree median for this partial solution with data gaps. Considering that only less than 50% of the data were processed, the quality of the partial solution is very satisfactory. (Following the laws of conventional adjustment theory, a redundancy reduction to one half implies (for uncorrelated observations) an increased standard deviation by a factor of √2.)

The errors of the estimated coefficients can also be expressed in terms of global gravity anomaly differences with respect to OSU91A. These cumulative gravity field errors were analysed in the latitudinal range of 80°N to 80°S, excluding the polar gap regions where no GOCE observations are performed. Table 1 summarizes the main statistical parameters of the simulations described above.

Table 1. Statistical analysis of several simulation results in terms of the minimum, maximum and RMS error of cumulative gravity anomaly deviations [mGal] (1mGal = 10^{-5} m s^{-2}) from the "true" OSU91A reference model at a harmonic degree of l = 180, analysing the corresponding fields in the latitudinal range of 80°N to 80°S.

Case study	no. data proc.	min. [mGal]	max. [mGal]	RMS [mGal]
Original performance	100 %	-2.34	2.15	0.45
GRF: no rot. noise	100 %	-7.71	8.36	1.71
GRF: rot.noise σ = 20 arcsec	100 %	-7.72	8.38	1.71
GRF: rot.noise σ = 5 arcmin	100 %	-9.20	8.18	1.97
GRF: rot.noise σ = 20 arcsec	50 %	-39.06	43.71	2.42

In chapter 1 it was emphasized that another valuable feature of QL-GFA is the estimation of the gradiometer error PSD from the residuals of the adjustment. In the following its feasibility in practical application shall be demonstrated. The theory of the PSD estimation is extensively discussed in [2], [6]. In the present study this concept was adapted for the application to gravity gradients given in the GRF.

Corresponding to Fig. 1, in Fig. 4 the "true" gradiometer error PSDs, derived from the noise time series applied to the SGG observations, are displayed in cyan (V_{XX}) and green (V_{YY}) colour. From the residuals of the adjustment of the gravity field solution, assuming a GRF rotation error of σ = 20 arcsec, the PSD was estimated by an iterative procedure. The black (V_{XX}) and the red (V_{YY}) curves in Fig. 4 a) show the estimated error PSDs, and the blue and magenta lines are the respective deviations from the "true" error PSD. Obviously, based on a spectral analysis of the residuals of the gravity field adjustment the gradiometer transition function can be derived almost perfectly. In order to decide whether there are statistically significant differences between the estimated and the a priori given error PSDs, statistical hypothesis test strategies will be applied. A first investigation into this subject is presented in [6], but will not be subject of this paper.

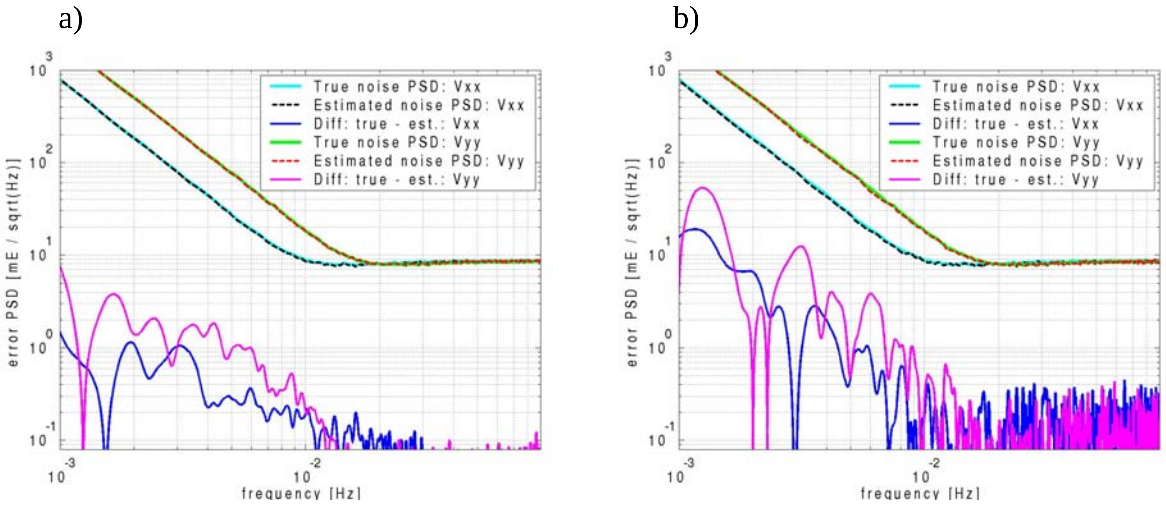

Fig. 4. Estimation of the gradiometer error PSD based on the spectral analysis of the residuals of the adjustment, applied to a) the complete 59-days data sets; b) the 50% partial data sets. The GRF-LORF rotation angles are in both cases affected by white noise of σ = 20 arcsec.

The PSD estimation was also applied to residuals of the simulation related to the 50% partial data sets. Fig. 4 b) shows the corresponding results. Also in this case the actual gradiometer behaviour can be determined with high precision. Compared to the complete data set shown in Fig. 4 a), the deviations from the "true" error PSDs are slightly larger, which is mainly due to the smaller redundancy. It should be emphasized, that the estimation of the gradiometer PSD is not only an important piece of information to evaluate the actual gradiometer performance, but the correctness of the gradiometer transition function is also crucial for obtaining an optimum gravity field solution, and thus it will be used as a priori information for the design of optimum filters for ultimate-precision gravity solvers ([3], [4], [7]).

3. DISCUSSION AND CONCLUSIONS

In this paper the feasibility and applicability of the Quick-Look Gravity Field Analysis software in the case of gravity gradient observations defined in the Gradiometer Reference Frame was demonstrated in the framework of a closed-loop simulation based on a realistic mission scenario. It was shown that the QL-GFA software can perfectly cope with the misorientation of the GRF w.r.t. the LORF. However, generally the reduced accuracy of the gradiometer measurements has a major impact on the quality of the GOCE gravity field solution, where the main problem of the new gradiometer specification is the decrease in accuracy in the lower frequency range. It could be demonstrated that also in this new environment QL-GFA can provide fast and comparatively accurate gravity field solutions, using partial data sets, which are additionally affected by irregularly distributed short-term data gaps. Additionally, QL-GFA successfully applies a spectral analysis of the residuals of the adjustment, for the purpose to validate the a priori information on the gradiometer noise PSD given by Level 0 to 1B processing. The precise knowledge of the spectral gradiometer behaviour is crucial, because it represents the metrics in the framework of a rigorous adjustment solution.

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