Covering the GOCE mission polar data gaps using gradients and ground gravity.

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Abstract

Least-squares collocation error estimates of spherical harmonic coefficients have been simulated using Gravity gradients (T_{zz}) in the area covered by GOCE and either upward continued T_{zz} data or ground gravity anomaly data at one or both poles. An isotropic covariance function using the EGM96 (statistically independent) error degree variances to degree 180 and degree variances derived from Wenzels GPM98A were used at higher degrees. Two kinds of data distributions have been considered. Equal-area distributions with a spacing of 1 degree and 2 deg. and equal-angular distributions with spacing 2 deg., 1 deg. and 0.5 deg.

The error of near-zonal harmonics were typically 2 times larger than the other harmonics when the poles were not covered. A slight improvement was seen when T_{zz} or gravity data on one pole was used, and uniform error estimates were obtained if both poles were covered, however for the gravity data dependent on the supposed associated noise standard deviation and height.

It was found in one example that if ground gravity should be able to "fill the gaps" they should have a resolution twice the one of the T_{zz} data. In the 2-degree experiment, 1 degree-mean gravity anomaly data with a 0.2 mgal error were needed in order to "fill the gap" for the coefficients of degree 28, for example.

1. Introduction.

The selected orbit of the GOCE mission creates gaps in the data coverage at the poles. These gaps may however be covered using other data-types such as ground or airborne gravity data.

How much do we need to fill the gaps, i.e. have a uniform precision of the estimated spherical harmonic coefficients ?

In order to study the influence of adding such data, simulations have been performed using least-square collocation (LSC), (Moritz, 1980) for the computation of the error-estimates of the spherical harmonic coefficients using different data-combinations. We first describe how it is planned to use LSC to process GOCE data using data in the points where they are observed and secondly using gridded data at satellite altitude. Then we describe the results of using the two methods for simulation studies. The first procedure has limitations with respect to how many data can be treated simultaneously. The second have limitations associated with respect to which data-types can be used, but very large data-sets can be used.

2. Use of LSC for the processing of GOCE data.

The use of LSC for the processing of GOCE data is described in detail within the framework of the so-called space-wise approach in the E2M report, (Albertella et al., 2000, Tscherning et al., 2000a, b). Here the main features and some recent developments are summarized.

In general LSC permit the use of all data-types, both satellite gravity gradiometry (SGG) and satellite-to-satellite tracking (SST), data, simultaneously. Also ground data may be added. However a system of equations as large as the number of observations will have to be solved. Methods have been found (Moreaux et al., 1999) which permit the handling of very large datasets using sparse matrix techniques. The present author does not doubt that at the time of the launch of GOCE (2005) it will be possible to treat simultaneously all vertical gravity gradient data simultaneously. Unfortunately data are not strictly vertical gravity gradient data. The data will be given in a satellite reference frame, and it will not be possible to "rotate" the quantities into a reference frame having e.g. the radius-vector as one of its axes. (This requires that all gravity gradient matrix components are measured). At least if one tries to “rotate” new data errors will be introduced.

Instead regional gravity field approximations can be determined from any available data-type. In 2005 it will certainly be feasible to handle at least 200000 observations in one run. (41000 took 3 days on a 500 MHz PC in 2000).

Suppose we have 3 observations every 4 seconds (potential differences from SST, T_{zz} , $T_{xx} – T_{yy}$ from SGG). Then one regional solution will cover a 20 degree x 20 degree area.

The area covered by GOCE will have to be covered by equal-area blocks with e.g. 2.5 degree overlap. This means 24 blocks at Equator, about 190 globally. This is a reasonable number of blocks to handle.

Each regional solution will then be used in the calculation of normal values such as along-track filtered T_{zz} values. (The prediction will take care of the transformation from the satellite frame to the Earth-oriented, radius-vector frame). Such a process should help in assuring that a minimum of information loss will occur when normal point values are constructed. LSC with sparse matrices can then be used to handle global data-sets of such normal values.

Simultaneously a gridding can be performed, so that a regular grid in longitude is constructed. (It does not need to be regular in latitude, or have the same heights for each parallel). These data can be handled using the new method of Fast Collocation (Sanso’ and Tscherning, 2001). This method requires, besides the longitudional gridding, that a uniform noise is used for each data-type at each parallel. This is not unrealistic considering the uniform data noise expected for GOCE data and the fact that the orbits converge towards the poles. The method takes advantage of the repetitive structure of the normal equations. If N parallels are used, systems of equations with dimension N needs to solved. If coefficients up to degree and order M needs to be estimated (and their error estimates computed), M of these systems of equations have to be solved. The value of M can maximally be 180/(grid spacing) for the zero order terms, but the double for the maximal order coefficients. Note that the more general method discussed in 3.1. formally is bound by this Nyquist limit.

For both procedures it is possible to use data of a different kind at the poles. For sparse collocation gravity data must be converted – by upward continuation – to e.g. T_{zz} data at altitude. For the Fast Collocation method no upward continuation is needed, but the gravity data must be gridded in longitude and for each parallel associated with the same height.

3. Simulations using least-squares collocation.

3.1 Using “randomly” distributed data.

Least-squares collocation has been used to determine the error-estimates of (correction to) spherical harmonic coefficients as described in (Tscherning, 2001). Initially a 1 degree approximate equal area data distribution using T_{zz} at 300 km altitude and a data noise of 0.005 EU was used. This corresponds to approximately 41000 “observations” with a normal-equation (upper triangular) matrix of size 6 GB. An isotropic covariance function having degree variances derived from the EGM96 coefficient errors (Lemoine et al., 1997), regarded as uncorrelated, was used. The degree variances above degree 180 were put close to zero. (One may discuss whether it is realistic to use an isotropic covariance function. However, considering that CHAMP and GRACE will be flying before GOCE, and having near-polar orbits, we should expect that the near zonal harmonic coefficients will have errors similar to those of the other coefficients.)

The typical error pattern seen in other studies was also found, i.e. the “near-zonal” harmonic coefficients (order 0, 1, 2, 3) had an error 50% times larger than the other coefficients. (In experiments using a noise variance of only 0.0005 EU the error was 10 times larger).

It was found that for the type of simulation presented here, a 2-degree coverage was sufficient in order to gain insight into the problem. This correspond to about 10000 observations. The resulting error-estimates are shown in Fig. 1.

The results illustrated in this and in the following figures show results for degree 28. This value is chosen quite arbitrarily, but the results are representative for other degrees. Note, that at this degree the standard deviation of the error of a single EGM96 coefficient is 4.3×10^{-9} (unitless).

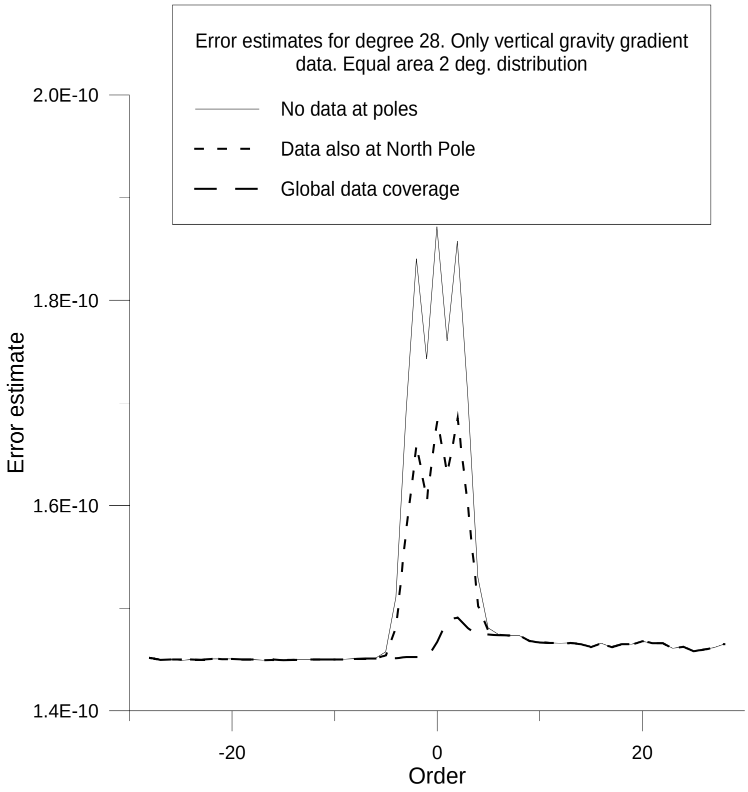

Figure 1.

Ground gravity data may be used to calculate upward continued gravity gradients at satellite altitude. LSC was used to calculate the errors of upward continued air-borne data with a data-distribution like the one obtained for Greenland (see Brozena et al., 1997). As a covariance function the one derived from the Greenland data was used It has a gravity anomaly standard deviation of 42 mgal when subtracting the contribution from a high order reference field to degree 35.

Air-borne gravity data measured with an error of 1 mgal was upward continued to T_{zz} values which resulted in an associated estimated error of 0.006 EU, close to the expected error of the gradiometer. (However, it should be noted that the standard deviation of the signal (T_{zz}) after subtraction of EGM96 to degree 180 is only 0.011 EU).

Two simulations have then been made with these data, where first the North pole and then the South Pole was filled, see Fig. 1. We see that in order to really obtain a substantial reduction of the error of the “near” zonal coefficients, data at both poles are needed.

Least-squares collocation permit the combination of data of different types. Simulations were therefore made using ground gravity upward continued to 20 km and 10 km altitude. (This corresponds to 2 degree-equal area means and 1 degree equal area means). The standard deviation of these values as derived from typical modern air-borne gravity with a 1 mgal error was found to be 0.6 mgal for gravity anomalies at 10 km altitude, and somewhat lower for the 20 km data.

The 20 km altitude gravity data gave a slight improvement. But it was first when the 10 km data was used that error-estimates similar to the ones obtained using gravity gradients were obtained, see Fig. 2.

Data error estimates of 0.5 and 0.2 mgal standard deviation were used, and it was found that 0.2 mgal was needed.

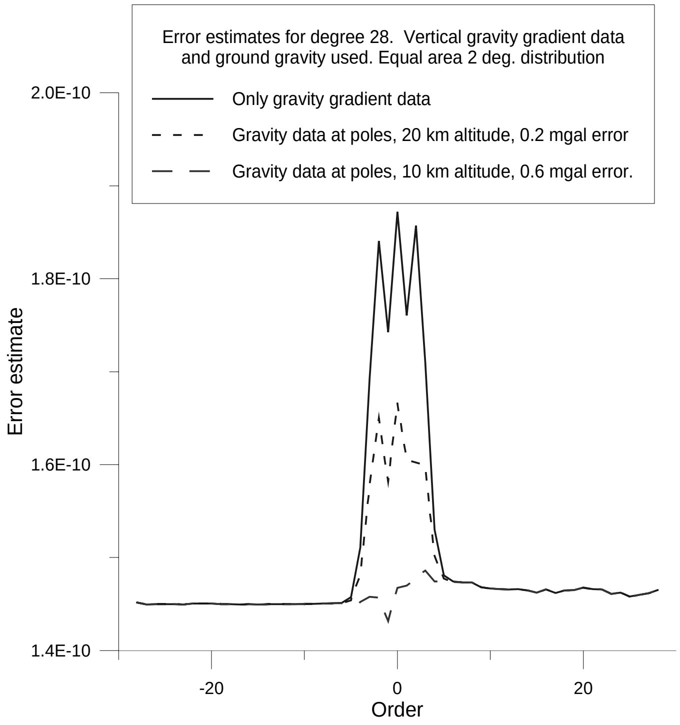

Figure 2.

3.2 Using gridded data.

Grids of data equidistantly located on each parallel with a density of 2 deg., 1 deg. and 0.5 deg. were used. Obviously, the denser the data, the better the result, until a limit where it is the data-noise which determines the error. Results similar to those obtained using equal-area grids of T_{zz} (section 3.1) were obtained. However the coefficient errors were not uniform but had a minimum for orders in between the maximal order and 0 order, see Fig. 3, 4 and 5. Also in these experiments an uncorrelated data noise of 0.005 EU was used. An isotropic

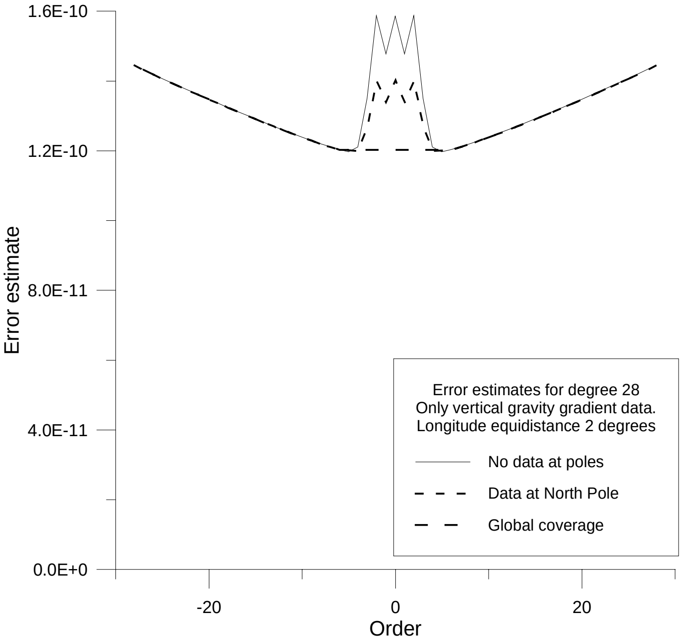

Figure 3.

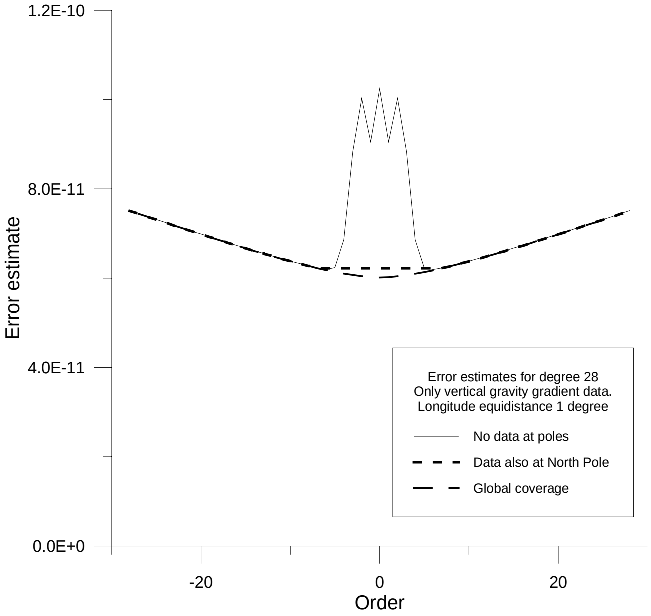

Figure 4.

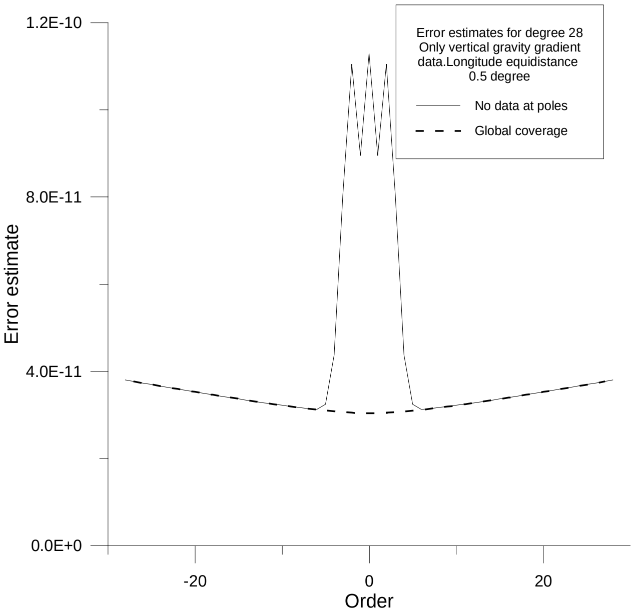

Figure 5.

covariance function with degree-variances equal to the EGM96 error-degree-variances to degree 180 and degree-variances from degree 181 to 1440 equal to the degree-variances of GPM98A (Wenzel, 1998) was used.

Further tests with gravity anomaly data with the same spacing as the gravity gradient data at altitudes 20, 10 and 5 km for the polar areas (above 82 deg. and below –82 deg.) are planned.

4. Conclusion:

Gravity data in the Arctic is available from airborne and sub-marine carried gravimeters, (Tscherning et al., 2000) and has error-estimates of between 5 and 10 mgal. Consequently they will not improve the GOCE solution if data from CHAMP and GRACE are available. The data distribution and quality is not sufficient at present to obtain 0.006 EU for upward continued vertical gravity gradients, but a further improvement of air-borne gravimetry is to be expected. Otherwise we must use airborne gradiometry, which is also now an operational procedure.

A further improvement equivalent to a complete data coverage with gravity gradients is possible only if both the

Arctic and the Antarctic is covered with good quality data.

References:

Albertella A., F.Migliaccio, F.Sanso' and C.C.Tscherning: The space-wise approach - Overall scientific data strategy. H.Suenkel (ED.) Eoetvos to mGal, Final report, pp. 267-297, April 2000.

Brozena, J.: The Greenland Aerogeophysics Project. Airborne Gravity, topographic and magnetic mapping of an entire continent. In: O.Colombo (Ed.): From Mars to to Greenland. Proc. IAG Symp. G3, Vienna, Austria, Aug. 1997, Springer Verlag, 1992.

Lemoine, F.G., D.Smith, R.Smith, L.Kunz, E.Pavlis, N.Pavlis, S.Klosko, D.Chinn, M.Torrence, R.Williamson, C.Cox, K.Rachlin, Y.Wang, S.Kenyon, R.Salman, R.Trimmer, R.Rapp and S.Nerem: The development of the NASA GSFC and DMA joint geopotential model. Proc. Symp. on Gravity, Geoid and Marine Geodesy, Sept. 30 - Oct. 5, 1996. The University of Tokyo, Tokyo, 1996.

Moritz, H.: Advanced Physical Geodesy. H.Wichmann Verlag, Karlsruhe, 1980.

Sanso', F. and C.C.Tscherning: Fast spherical collocation. Paper prepared for IAG2001, Budapest, Sept. 2001.

Tscherning, C.C., R.Forsberg, A.Albertella, F.Migliaccio & F.Sanso': Space-wise approaches to gravity field determination in Polar Areas. H.Suenkel (ED.) Eoetvos to mGal, Final report, pp. 331-336, March 2000.

Tscherning, C.C., G.Moreaux, A.Albertella, F.Migliaccio, F.Sanso' & D.Arabelos: Detailled scientific data processing using the space-wise approach. H.Suenkel (ED.) Eoetvos to mGal, Final report, pp. 299-304. March 2000.

Tscherning, C.C., R.Forsberg, A.Albertella, F.Migliaccio & F.Sanso': Space-wise approaches to gravity field determination in Polar Areas. H.Suenkel (ED.) Eoetvos to mGal, Final report, pp. 331-336, March 2000.

Tscherning, C.C.: Computation of spherical harmonic coefficients and their error estimates using Least Squares Collocation. Accepted Journal of Geodesy, 2001.(Also in E2M report).

Wenzel, H.G.: Ultra hochaufloesende Kugelfunktionsmodelle GMP98A und GMP98B des Erdschwerefeldes. Proceedings Geodaetische Woche, Kauserslautern, 1998.