ABSTRACT
It is well known that lack of knowledge of the geoid has been a limitation in the use of satellite altimetry for studies of mesoscale ocean currents. The ocean geoid is at least an order of magnitude larger than the signal from mesoscale ocean currents. This had lead to oceanographers concentrating on the variability of ocean signals from altimetry. Recent improvements in geoid determinations have allowed the large (gyre) scale portion of the mean ocean field to be determined to a fair degree of accuracy. However, at smaller spatial scales, the geoid is still poorly determined.

A method used for some years at Southampton involves determining the surface geostrophic current along a satellite track using a combination of hydrography and ADCP. If the in situ data are collected during a satellite overpass, this current is the current measured by the altimeter. Hence, the total surface current at any time along this track can be determined by taking all other altimetry relative to this overpass.

As a by-product of this process, we are able to determine the shape of the ocean geoid along the altimeter track at the time of the overpass. By removing the contribution of the surface ocean currents from the altimeter derived surface slope, we are left with the slope of the geoid. Integration of this gives the geoid, with an unknown constant offset. By analysing the geoid profiles determined from several such transects, taken at a variety of locations around the world's oceans, we can predict the possible contribution that GOCE may make to determination of absolute surface currents at scales of 10s to 100s of km.

INTRODUCTION
An altimeter measures the sea surface height (SSH) relative to a reference ellipsoid. This measurement can be made to an accuracy of better than 1 cm for TOPEX/POSEIDON (T/P), a little worse for the ERS altimeters. The SSH is dominated by the ocean geoid signal. This is the shape of the zero gravitational potential field, or the shape of the sea surface in the absence of ocean currents and tides. The geoid signal is of the order of 1-200 m amplitude. The SSH signal is modified by water movement. Once height changes due to the tides are removed using a tidal model, the remaining signal is largely due to geostrophic surface currents. The signal due to geostrophic currents is rarely greater than 1-2 m over spatial scales of order 100-200 km. The geoid signal is well known (to an accuracy of a few cm) at wavelengths longer than approx. 1,000 km, but is poorly known at scales less than this. Hence, knowledge of the geoid at smaller scales is a severe limitation in the study of mesoscale (typically 50-500 km) oceanographic features.

As the geoid may be considered constant on oceanographic time scales, changes in the SSH, or more specifically changes in the SSH slope, are dominantly caused by changes in surface ocean currents. Hence, it is possible to monitor changes in ocean currents using altimetry alone.

In the following paper, we will discuss a method used for some years at Southampton Oceanography Centre to retrieve the absolute (i.e. mean and variable) ocean current using a combination of altimetry and in situ data. It will then be shown how this method can also be used to determine the slope of the geoid along the altimeter tracks. An example for a section in the Alboran Sea will be shown and finally some conclusions made for the use of this technique in adding to the results of the GOCE mission.

THE METHOD OF COMBINING HYDROGRAPHY, ADCP AND ALTIMETRY

Theory
An altimeter measures the SSH relative to a reference ellipsoid. After correction for tidal and inverse barometric effects, this height may be considered as the sum of the Geoid height and the height from instantaneous surface currents.

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Hydrographic measurements of temperature, salinity and depth are combined to give us the density along a cruise track. From these data, we can determine the surface geostrophic current. When calculating geostrophic currents from hydrographic data, a 'level of no motion' has traditionally been assumed. This is a level, usually more than 1000 m deep, where currents can be assumed to be negligible. This is rarely true however, and the unknown velocity profile at this level of no motion will cause errors in the absolute value of the geostrophic currents measured.

An Acoustic Doppler Current Profiler (ADCP) measures vertical profiles of the total current, relative to the ship, along the ship track. This current can be placed in an absolute reference frame by careful use of Global Positioning System (GPS) data acquired simultaneously. However, the current measured by the ADCP includes not only the geostrophic, but also the ageostrophic (principally wind driven) parts of the circulation.

Assuming that there is a true level of no motion, let the height of the sea surface above this level be $H$. Let the sea surface height relative to the reference ellipsoid (as measured by the altimeter) be $h$. Since the geoid $G$ is parallel to a level of no motion, at time $t_0$ we have

$$G + k = h_0 - H_0$$

where $k$ is an unknown constant.

By differentiating along the track we have

$$\frac{\partial G}{\partial x} = \frac{\partial h_0}{\partial x} - \frac{\partial H_0}{\partial x}$$

where $x$ is the along track direction.

This gives us:

$$\frac{\partial H}{\partial x} = \frac{\partial H_0}{\partial x} - \left( \frac{\partial h_0}{\partial x} - \frac{\partial h}{\partial x} \right)$$

since $G$ is invariant with time, i.e. the along-track slope of the sea surface relative to a level of no motion at time $t$ is equal to the along track slope at time $t_0$ plus the change in slope measured by the altimeter.

By geostrophy we can calculate the velocity ($v$) from along track slope, acceleration due to gravity ($g$) and the Coriolis parameter ($f$), and at time $t$ we have:

$$v = v_0 - g \left( \frac{\partial h_0}{\partial x} - \frac{\partial h}{\partial x} \right)$$

Given a one-time ship survey, carried out at the same time ($t_0$) as an altimeter overpass, to determine $V$, we can determine the absolute surface geostrophic velocity across the track at any time $t$ for which we have altimetry.

In order to find the absolute surface geostrophic velocity at the time of the satellite overpass we need to combine the ADCP and hydrographic data. Hydrography gives us geostrophic velocity relative to an assumed level of no motion $V_h$. ADCP gives us absolute velocity $V_a$, including ageostrophic components that tend to be greatest near the surface. At some depth, $Z_g$, the vertical current shear measured by the ADCP ($\frac{\partial V_a}{\partial z}$) matches the geostrophic shear ($\frac{\partial V_h}{\partial z}$) measured from hydrography. At this depth, the currents may be assumed to be purely geostrophic and the baroclinic velocity ($V_0$) is given by

$$V_0 = V_a(Z_g) - V_h(Z_g)$$

Hence the surface geostrophic current is given by

$$V_g(0) = V_0 + V_h(0)$$
Synthetic Geoid

As a 'by-product' of the method of combining altimetry and hydrography, we have a method of determining the geoid along-track, free of contamination from the surface currents.

From eq.(1), we have that:

$$G = h_o - H_{rt} + k$$  \(7\)

The altimeter height \(h\) is known, and the dynamic height \(H\) can be calculated from the absolute velocity profile. Hence the geoid profile can be determined \((G+k)\) but as \(k\) is unknown, the absolute height cannot be determined.

This method will allow us to determine the scales of the geoid wherever we have simultaneous altimetry and in situ data. In theory, we are able to determine the geoid accurately to scales less than 10 km. In practise, we are limited by noise levels, and possibly by the limits of geostrophy, to scales slightly longer than this, but still significantly less than those determined by GOCE. This will also give us a validation tool for GOCE that is completely independent of any external geoid information, apart from the gravity field used in the altimeter orbit determination.

By determining the ocean currents and geoid at the same location, we should be able to look more closely at the interactions of ocean currents and topography (as represented in the geoid). Ocean currents have a strong tendency to be controlled by topographic slopes. By determining the relationship of the currents to the geoid slopes, we may be able to get a better estimate of the geoid scales that it is important to resolve in order to capture the major part of the ocean current signal accurately.

DETERMINING THE GEOID IN THE ALBORAN SEA


A Fine Scale Survey (FSS) pattern was designed to cross the Almeria-Oran front in a series of legs parallel to descending ERS passes (Fig. 1). Unfortunately, an error during the survey planning led to some of the legs of the survey not running parallel to the ERS tracks. This fine scale survey pattern was completed three times during the cruise. During the second leg of the cruise a reduced FSS (legs c-j) was completed twice. Additional large-scale surveys were also undertaken, some of which were also along altimeter tracks. The times at which each of the tracks were sampled, by the altimeter and by the ship, are given in Table 1. Even with a survey designed to capture simultaneous altimetry and hydrography the complex ground track led to very few tracks run with the ship on the track when the altimeter flew overhead. For the following examples, we shall use data from ERS track 646, and FSS 2.

![Fig. 1 D224 Survey track with coincident ERS and TOPEX/POSEIDON tracks overlain.](image)
Table 1 Time of sampling altimeter tracks shown in Fig. 1. Tracks sampled simultaneously are shown in bold. Tracks sampled within 1 day of an overpass are underlined.

<table>
<thead>
<tr>
<th>Track</th>
<th>Altimeter Overpass</th>
<th>Ship Transect</th>
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</thead>
<tbody>
<tr>
<td>ERS 029</td>
<td>31/12/1996 22:07</td>
<td>07/12/1996 20:28 — 08/12/1996 09:05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19/12/1996 16:29 — 20/12/1996 06:58</td>
</tr>
<tr>
<td></td>
<td>03/01/1997 10:58</td>
<td>24/12/1996 05:56 — 24/12/1996 19:09</td>
</tr>
<tr>
<td>ERS 487</td>
<td>12/12/1996 22:05</td>
<td>08/12/1996 12:58 — 09/12/1996 00:03</td>
</tr>
<tr>
<td>ERS 646</td>
<td>18/12/1996 11:01</td>
<td>12/12/1996 12:01 — 13/12/1996 06:15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>17/12/1996 17:07 — 18/12/1996 10:44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31/12/1996 02:36 — 31/12/1996 19:35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14/01/1997 15:23 — 15/01/1997 06:14</td>
</tr>
<tr>
<td>T/P 096</td>
<td>01/12/1996 21:06</td>
<td>02/12/1996 16:06 — 02/12/1996 22:44</td>
</tr>
<tr>
<td></td>
<td>31/12/1996 15:02</td>
<td></td>
</tr>
<tr>
<td>T/P 172</td>
<td>04/12/1996 20:18</td>
<td>04/12/1996 19:48 — 05/12/1996 00:15</td>
</tr>
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<td></td>
<td>24/12/1996 16:15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>03/01/1997 14:14</td>
<td></td>
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</tbody>
</table>

**In Situ Data**

Hydrographic data were collected continuously along the survey tracks shown in Fig. 1 using a towed, undulating CTD (Conductivity-Temperature-Depth instrument), which provides vertical profiles to a depth of more than 350 m [3]. These data were initially averaged to one-second values to reduce noise. They were then merged with the ship’s navigation data and averaged to an 8 m vertical by 4 km along-track grid.

Continuous absolute current measurements were taken using the ADCP, with accurate heading data from a Global Positioning System (GPS) system. This is very important when using the heading to determine the direction of current as measured by the ADCP. The ADCP data were referenced to real-time differential GPS derived ship’s position for the first month of the cruise [4].

Leg e of the FSS lies almost along ERS pass 646 (see Fig. 1). Density, calculated from the SeaSoar data for this leg (Fig. 2), shows the structure of the Almeria-Oran front. The front, which is seen at the surface around 36.25°N, can be seen to extend down to 150-200 m. The averaged ADCP vertical current profiles perpendicular to the ship track for leg e are shown in Fig. 2. The currents at the front are in excess of 1.5 ms⁻¹.

![Fig. 2 Density from SeaSoar (left) and averaged ADCP current profiles (right) along FSS leg e, Dec. 17 1996](image-url)
Altimetry

In order to determine the geoid slope, ERS OPR altimeter data [5, 6] were used to determine SSH along ERS pass 646 (Fig. 1) for the overpass on 18 Dec 1996. The data were filtered for bad points and then corrected for atmospheric effects (radiometer wet tropospheric correction; ECMWF dry tropospheric correction; ionospheric correction), tides (ocean tide [7], loading tide CSR3.0 [8], solid earth tide and polar tide [9]) and electromagnetic bias [10, 11]. The orbits provided were replaced with the dgm-e04 orbits provided by DUT [12].

Merging altimetry and in situ data

Because of the offset between ERS tracks 646 and the FSS, the hydrographic data and the ADCP data from FSS legs d and e had first to be interpolated to ERS tracks. Interpolation was carried out using a distance weighted mean, in a direction perpendicular to the front in order to avoid “smearing” the front. The geostrophic velocity relative to 200 m and the ADCP velocity profile were then determined at each altimeter data point.

In order to minimise the effects of noise in the ADCP data, $V_0$ was not determined at a single depth but was calculated using the average difference between ADCP and geostrophic current over a range of depths. In this case (5) becomes

$$V_0 = \frac{1}{N} \sum_{z=z_1}^{z_2} \left( V_a(Z_z) - V_b(Z_z) \right)$$

where $N$ is the number of depth levels. Depths between pressure levels 153 and 253 db were chosen as they consistently produced low standard deviations of differences.

Using geostrophy we integrated the absolute current profile obtained from this method, to give the contribution to the SSH of the surface geostrophic current at the time of the overpass. By removing this from the altimeter derived SSH value, we obtain an estimate of the geoid height along the track, given in Fig. 3. The difference between the SSH signal for 18 Dec 1996 and the synthetic geoid curve is the effect of the surface geostrophic current at this time. The remaining difference between the shape of the mean sea surface height and the synthetic geoid is the result of the difference between the mean current and the current on 18 Dec.

Errors in the Method

There are a number of sources of error within this estimate: measurement errors from the hydrography, ADCP and altimetry, the merging of hydrography and ADCP, instrumental and atmospheric corrections in the altimetry, cross-track geoid slopes and non-coincidence in time and space. A study of the latter has shown that errors of the order of 40 cm s$^{-1}$ could be introduced in this region by a mis-location of the track by less than 10 km, or by a time separation of the data by 3 days. These errors would feed directly into the geoid estimate derived.

All of these errors could be reduced by repeating the sections to obtain multiple estimates of the geoid profile that could be averaged.

![Fig. 3 The Synthetic Geoid, Mean Sea Surface Height and Sea Surface Height for 18 Dec 1996 along ERS Track 646](image-url)
CONCLUSIONS

In this paper we have shown a method that can be used to derive independent Geoid estimates from a combination of altimetry and in situ data. The resultant geoid profiles can be used to study small-scale geoid variability. No verification of the geoid profile obtained in the Alboran Sea has yet been carried out, but the method appears to introduce small-scale features relative to the mean sea surface height, and also introduces a longer scale gradient change.

It is possible that the geoid profiles obtained by this method may help to resolve some of the aliasing issues in the GOCE mission.

Finally, the numerous sections that have been run, in the North and South Atlantic, Mediterranean, Southern Indian and South Pacific, by SOC and other research institutes, may give a useful validation tool for GOCE.

ACKNOWLEDGEMENTS

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REFERENCES


