GOCE data products for the user community

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1 Introduction

One of the issues raised at the recent GOCE Workshop at ESTEC, Noordwijk, The Netherlands (April 23-24), concerns the format of GOCE data products for (scientific) use. It became obvious that different needs exist, even within one specific discipline such as oceanography. The following contains some thoughts about this issue. Attention will be paid to GOCE data products relevant for (scientific) use and to the question whether these data products accommodate the several (possible) users sufficiently.

This document is not intended to be complete, but merely serves as a means to provide some more information for a discussion that has been going on for already a couple of years in the field of GOCE impact studies. Hopefully, this document sheds some light on this issue.

2 GOCE data products

The objective of GOCE is the provision of a high-accuracy, high-resolution model for the static part of the earth's gravity field. This model will basically be derived from gradiometer (SGG) and GPS tracking (SST) observations. In a good approximation a distinction can be made between calibrated SGG and SST observations in a well-defined reference system, referred to as level 1, and the gravity field model product in the form of a spherical harmonic expansion to a certain maximum degree and order, referred to as level 2. Both the level 1 and 2 data will be complemented by quality measures, such as a covariance matrix for the spherical harmonic expansion or observation error covariance functions.

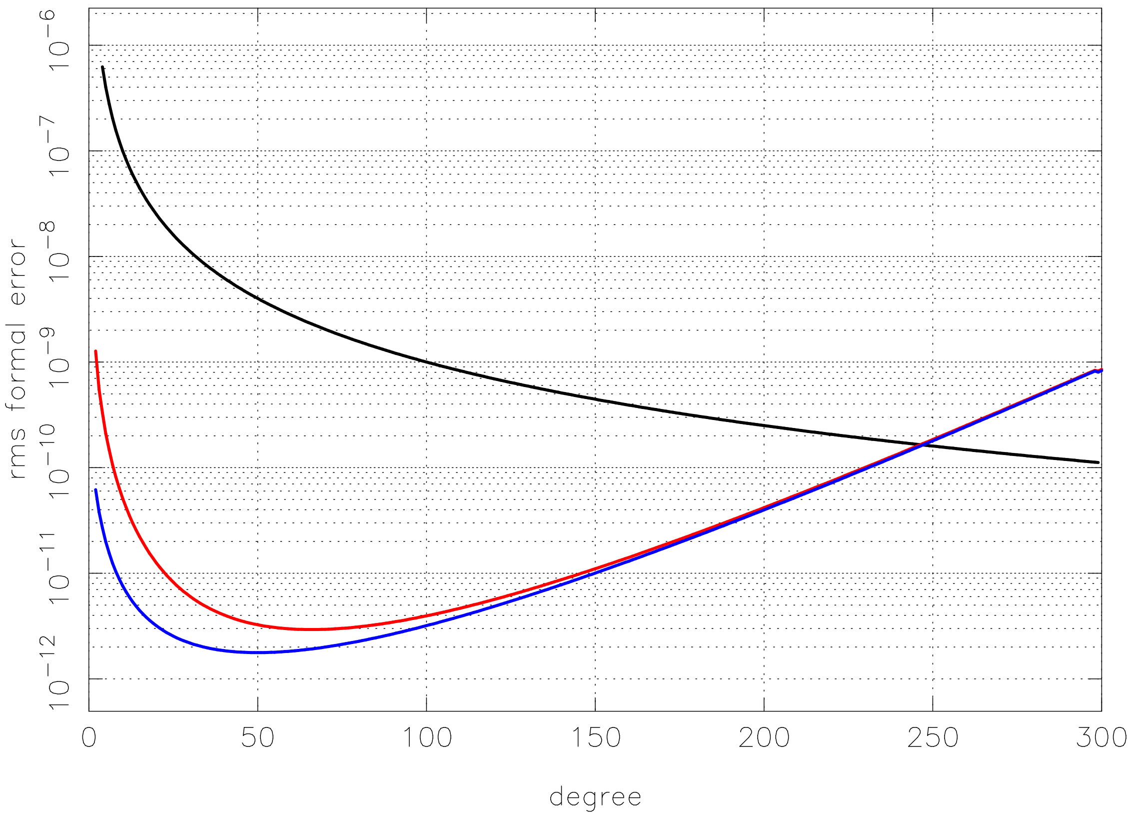

Figure 1. Typical degree error spectra for a GOCE gravity solution based on gradiometer observations (red: colored noise spectrum for SGG, blue: flat)

Concerning level 2, also alternative representation forms can be chosen, such as (local and global) grids of geoid heights, geoid slopes and gravity anomalies, each with appropriate associated covariance functions.

Different representation forms for level 2 come with different kinds of covariance functions, where different kind of commission and omission errors might play a role, as will be made clear by some examples in the following.

When using a spherical harmonic expansion representation, besides commission errors, also aliasing and omission errors will occur due to the truncation at a certain degree. Figure 1 displays a typical curve of degree variances based on error propagation. This curve represents the commission error. Although a spherical harmonic expansion and its associated commission error (covariance matrix) will work fine for many applications which may not be affected very seriously by aliasing and omission errors, for certain other applications it might not work fine.

For certain (oceanographic) impact studies, curves like in figure 1 were converted to covariance functions for geoid heights and geoid slopes. For certain applications these functions have to be valid for point values in space. For other applications, they have to be valid for a certain range of spatial wavelengths.

Sometimes (or many times) the term wavelength w is confused with spherical harmonic degree l, where for convenience wavelength is approximated by for example the following equation:

$$w = 40000/l \tag{1}$$

In certain cases, such an equation might serve well as a rule of thumb, but in other cases it might lead to erroneous interpretations. For example, when a phenomenon with a wavelength cut-off of 100 km (low pass filter) is studied, a covariance function is defined as a series of Legendre polynomials complete to degree 200 with the associated degree variances:

$$C(\psi) = \sum_{l=0}^{200} c_l P_l(\cos\psi) \tag{2}$$

where C is the covariance function, ψ the spherical distance between two data points in for example the ocean, c_l the degree variances (commission error), and P_l the Legendre polynomial of degree l. It has to be clear that this function will not give the proper covariance for wavelengths down to 100 km, since terms with a degree above 200 will still contribute partly to wavelengths of 100 km and larger. In principle the series expansion in equation 2 should be extended until infinity (or to a degree where the signal is not significant anymore) and then smoothed in some way to reflect the 100 km wavelength cut-off.

The following contains an example of covariance function modeling for geoid heights and geoid slopes where the issue of smoothing is addressed as well. The example is relevant to a discussion that took place in the framework of GOCE oceanography impact studies.

Assume a covariance function $C(P1P2)$ that can be used directly to compute the geoid error $\sigma_g(P)$ for point values P and the correlation ρ_{P1P2} between point values:

$$\sigma_g(P) = \sqrt{C(PP)} \tag{3}$$

$$\rho_{P1P2} = \frac{C(P1P2)}{\sqrt{C(P1P1)}\sqrt{C(P2P2)}} \tag{4}$$

The question is how to use this function to do the same for geoid slopes. Some impact studies included geoid slope error estimates at different wavelengths using covariance functions truncated at degree 200, 250 and 300. It was found in these impact studies that e.g. the slope error at a wavelength of 1000 km was an order of magnitude larger using a truncation at degree 300 instead of at degree 200. This does not seem to be correct. This result was due to the different truncations where omission error effect were thus not taken into account, i.e. higher degrees causing errors at long wavelengths (thus confusing wavelength with degree).

In the following, a further explanation of these results is presented. Assume that the geoid slope error σ_{slope} is computed in the following way:

$$\sigma_{slope} = \frac{\sqrt{C(P1P1) + C(P2P2) - 2 \times C(P1P2)}}{dist(P1, P2)} \tag{5}$$

where $dist(P1, P2)$ is the distance between the points $P1$ and $P2$. The slope error at a wavelength of e.g. 1000 km is then obtained by evaluating the covariance function at two points $P1$ and $P2$ that are 1000 km apart.

If the slope error is indeed being computed using this approach, slope errors will be overestimated at long wavelengths. To overcome this problem, the following two possible approaches can be used to obtain more "honest" estimates of geoid slope error at a certain wavelength:

1. assuming a large number ($\rightarrow \infty$) of points P along a track with a length equal to the wavelength and estimating the slope error by assuming that you fit a trend though all these points taking into account all the correlations between the different points. This instead of using two points (equation 5) (method 1);
2. using equation 5, but in computing $C(P1P1)$, $C(P2P2)$ and $C(P1P2)$ smoothing operators can be included taking into account that a certain wavelength is being considered (method 2).

In both approaches the covariance function C must be truncated at a sufficiently high degree to reduce (eliminate) the effect of omission errors.

Concerning the second approach, without using smoothing operators, the covariance function looks like (compare with equation 2):

$$C(P1P2) = \sum_{l=0}^{Lmax} c_l P_l(\cos\psi) \tag{6}$$

where l is the degree of the Stokes coefficients, c_l the degree variance, $P_l(\cos\psi)$ the Legendre polynomial of degree l and ψ the distance between $P1$ and $P2$. The series is truncated at a sufficiently high degree $Lmax$.

By introducing smoothing operators $b_{l,\lambda}$ the equation might look like:

$$C(P1P2) = \sum_{l=0}^{Lmax} b_{l,\lambda} c_l P_l(\cos\psi) \tag{7}$$

The smoothing operators are both a function of the degree and the wavelength λ.

L_{min}	L_{max}	Point value	Mean 100 km (*)
geoid - EGM96			
251	300	8.90	2.86
251	360	11.87	3.13
gravity anomaly - EGM96			
251	300	3.73	1.18
251	360	5.44	1.34
geoid - Rapp'79			
251	300	23.51	7.59
251	360	31.08	7.64
251	500	38.03	8.50
251	1000	42.66	8.72
251	5000	43.28	8.73
251	10000	43.29	8.73
gravity anomaly - Rapp'79			
251	300	9.86	3.13
251	360	14.21	3.51
251	500	20.06	3.78
251	1000	28.22	4.16
251	5000	32.04	4.19
251	10000	32.04	4.19

(*) method 1

Table 1. Geoid height (cm) and gravity anomaly (mgal) power estimates

Coming back to the issue of omission errors, the following serves as an example.

For example, when truncating EGM96 at degree 250, the omission error (degrees 251-360) is equal to 11.87 cm for geoid point values, but only 3.13 cm for 100x100 km^2 geoid mean block values. When using the Rapp'79 model, it can be seen that for geoid point values, there is much power left at degrees above 500, i.e. a spherical harmonic expansion with a resolution (half-wavelength) of 40 km according to equation 1, namely 20 cm ($\sqrt{43.29^2 - 38.03^2}$, where the 43.29 value holds for a truncation at degree 10000 above which no significant power is left). For 100x100 km^2 geoid mean block values, this number is much smaller, namely 2.0 cm ($\sqrt{8.73^2 - 8.50^2}$). Please note that these numbers are global averages. It is fair to assume that these numbers are smaller over e.g. oceanic areas, since today the fit of mean sea surface models complete to degree and order 360 is much better than e.g. the geoid omission error above degree 360 according to the Rapp'79 model (30 cm = $\sqrt{43.29^2 - 31.08^2}$)

The above shows that the estimation of omission errors of course depends on the selection of gravity field model degree variance models. This selection is an issue in itself, but is beyond the scope of this document. Also the question whether degree variances are the appropriate representation form is open for discussion.

Many more examples and gravity field model representation forms can of course be thought of in addition to different methods for treating commission and omission errors, and smoothing to certain wavelengths. However, the above hopefully makes clear that these kind of effects need to be addressed and included in some way in error covariance modeling of GOCE gravity field model products, to satisfy different user requirements. The following section contains some more thoughts concerning this point.

3 User requirements

Concerning the use of GOCE data products, 5 different themes have been distinguished: (1) oceanography, (2) solid-earth, (3) geodesy, (4) glaciology, and (5) sea level change studies. For certain applications within these themes, level 1 data products can be used directly, e.g. calibrated SGG observation in certain solid-earth studies, whereas in many other applications spherical harmonic expansions or a simple linear functional of this can be used directly, e.g. in ocean current derivation from satellite radar altimetry (although not that trivial) and in precise orbit computations.

However, from some GOCE impact studies it became clear that especially for level 2 products other representation forms are needed, some of which may be derived by simple linear transformations or derivations from a spherical harmonic expansion. At least the following forms can be thought of:

1. spherical harmonic expansion;
2. grids of geoid heights, geoid slopes, gravity anomalies, ..., - point values;
3. grids of geoid heights, geoid slopes, gravity anomalies, ..., - mean block values.

In addition, at least the following kinds of quality measures can be distinguished:

1. covariance matrix
2. covariance functions for geoid heights, geoid slopes, gravity anomalies, ..., - point values;
3. covariance functions for geoid heights, geoid slopes, gravity anomalies, ..., - mean block values.

For all these quality measures, special attention has to be paid to omission (and when relevant aliasing) errors, of course in conjunction with commission errors. The nature of these errors will most probably depend on the gravity field recovery method. For example, it is fair to assume that gravity field anomaly errors derived from a spherical harmonic solution obtained by numerical integration techniques ("brute-force") will be different from those obtained by a space-wise least-squares collocation approach where the covariance function is complete to a degree much larger than the truncation degree of the spherical harmonic expansion. In case of wavelength cut-offs (i.e. not spherical harmonic degree cut-offs), special attention has to be paid to smoothing (filtering) operators.

Considering the above, data products provided to the user community have to include at least a description specifying the following:

1. gravity field representation form;
2. gravity field recovery method;
3. nature of quality measure: treatment of commission, omission errors, relevance of aliasing;
4. averaging techniques for e.g. mean block values (optional).

From the above, it is obvious that a wide range of GOCE level 2 data products can be defined. It is fair to assume that the different user communities would actually like to have many of these combinations made available to them. Being able to compute these combinations requires certain tools, some already available, some to be developed and/or extended. The European GOCE Gravity Consortium (EGG-C) can be used as (one of) the platform(s) to address this issue in great detail and take care of product definitions, conversions, etc.