A Joint Space-Time Approach to Persistent Scatterer Interferometry

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SAR differential interferometry potential and problems

- SAR differential interferometric measurements allows reconstructing from satellite the slow displacements of the terrain occurred between different acquisitions, in principle with sub-centimetric accuracy

- The interferometric phase is measured only modulo $2\pi$

- The phase is affected by:
  - random noise (due to decorrelation between the signals of different acquisitions)
  - systematic effects (due to inaccurate orbital data and reference digital elevation model, and different atmospheric conditions at the various acquisition dates).
Key processing steps in all approaches

- The points characterized by too high random noise must be recognized by means of statistical estimations and discarded, ending up with sparse points.
- The phase must be unwrapped.
- Systematic effects can be recovered using statistical or deterministic models for the disturbances to be eliminated and the signals to be retrieved.
- The models of the signal can help phase unwrapping and phase unwrapping can help modeling the signals.
Why a joint space-time approach

• Significant advances were introduced by the permanent scatterer approach, in particular the ideas of minimizing the amplitude and phase dispersions in long series of SAR acquisitions. This approach exploits mainly the temporal properties of the signal.

• On the contrary, small baselines approaches, more similar to classical differential interferometry, exploit first the spatial and then the temporal properties of the data.

• We have explored an approach that jointly exploits the spatial and temporal properties in order to determine the various components of the signal.

• Considering jointly the spatial and temporal structure of the data can help in recovering the correct solution.
General problem formulation

Model equation

\[ \Delta \phi_{k,j} + 2\pi n_{k,j} = cB_k \Delta h_j + c' \Delta t_k \Delta v_j + \delta_{k,j} \]

Integrality constraints

\[ \sum_{j \in T} n_{k,j} = -\frac{1}{2\pi} \sum_{j \in T} \Delta \phi_{k,j} \]

Irrotationality constraints

\[ \sum_{j \in T} \Delta v_j = 0, \quad \sum_{j \in T} \Delta h_j = 0 \]

Objective function

\[ \min_{\delta, n} \sum_{k,j} \left( w_{k,j} |\delta_{k,j}|^p + \alpha w'_{k,j} |n_{k,j}|^p \right) \]
Particular solution: 2D Phase Unwrap

Limit $\alpha \to \infty$

$$\sum_{j \in T} n_{k,j} = -\frac{1}{2\pi} \sum_{j \in T} \Delta \phi_{k,j}$$

$n_{k,j}$ integer

$$\min \sum_{n} \sum_{k,j} \left( w'_{k,j} |n_{k,j}|^p \right)$$
Particular solution: Temporal Coherence

- Limit $\alpha \to 0$
- **No irrotationality conditions**

\[
\Delta \phi_{k,j} + 2\pi n_{k,j} = cB_k \Delta h_j + c' \Delta t_k \Delta v_j + \delta_{k,j}
\]

\[n_{k,j} \text{ integer}\]

\[
\min_{\delta} \sum_{k,j} \left( w_{k,j} |\delta_{k,j}|^p \right)
\]

or (almost equivalently):

\[
\max_{\delta} \left| \sum_{k,j} w_{k,j} e^{i\delta_{k,j}} \right|
\]
Sub-optimal solution:
velocity and elevation fields robust retrieval

• Irrotationality conditions
• Phase model, $\Delta \phi'_{k,j}$, from 2D unwrap or temporal coherence

\[
\Delta \phi'_{k,j} = cB_k \Delta h_j + c' \Delta t_k \Delta v_j + \delta_{k,j}
\]

\[
\sum_{j \in T} \Delta v_j = 0, \quad \sum_{j \in T} \Delta h_j = 0
\]

\[
\min_{\delta} \sum_{k,j} (w_{k,j} |\delta_{k,j}|)
\]
General problem solution

Using the sub-optimal solution for initialization helps reducing the computational complexity:

\[ \Delta \phi_{k,j} + 2\pi n_{k,j} = cB_k \Delta h_j + c't_k \Delta v_j + \delta_{k,j} \]

\[ n_{k,j} \text{ integer} \]

\[ \sum_{j \in T} n_{k,j} = -\frac{1}{2\pi} \sum_{j \in T} \Delta \phi_{k,j} \]

\[ \sum_{j \in T} \Delta v_j = 0, \quad \sum_{j \in T} \Delta h_j = 0 \]

\[ \min_{\delta, n} \sum_{k,j} \left( w_{k,j} |\delta_{k,j}|^p + \alpha w'_{k,j} |n_{k,j}|^p \right) \]
Test area and coherent scatterers selection

- AOI: Campi Flegrei, Napoli, Italy, 16kmx10km
- 42 ERS SAR images acquired between Apr 1995 – Jan 2000

- Coherent scatterers are usually selected based on the SAR amplitude “dispersion” or more classical coherence estimation

- A method that can improve SAR amplitude “dispersion” approach in terms of probability of detection vs. probability of false alarm has been tested in order to select the coherent scatterers (not described in this presentation)

- The results of the test should be rather insensitive to the method used for the selection of coherent points. For the tests only a subset of the identified coherent scatterers was used: at most one in each area of 200 x 200 m²
Temporal and spatial baselines
Test area and coherent scatterers
Subset of the coherent points used for the tests
Triangulation of the selected points

Coherent points selection
Mean velocities (20 images, 1996-1997)
Comparison tests

- The available 42 ERS SAR acquisitions from Apr 1995 – Jan 2000 were divided in three different subsets made of:
  1. All the 42 acquisitions
  2. 22 acquisitions (approximately one every two images including the first and the last)
  3. 12 acquisitions (approximately one every four images including the first and the last)
- The estimated mean velocities and height corrections should be the same in the three cases
Mean velocities: 41 interferograms, model initialized with temporal coherence
Mean velocities: 21 interferograms, model initialized with temporal coherence
Statistics of the differences between solutions with 41 and 21 interferograms

Sub-optimal sol. starting from temporal coherence:
max error 90% = 0.86 m

Only temporal coherence: max err. 90% = 1.5 m

Sub-optimal sol. starting from temporal coherence:
max error 90% = 0.75 mm/year

Only temporal coherence: max err. 90% = 0.8 mm/year
Mean velocities: 11 interferograms, model initialized with temporal coherence
Mean velocities: 11 interferograms, model initialized with 2D phase unwrap
Conclusions

• An approach is proposed that jointly exploits spatial and temporal properties in order to retrieve information from the interferometric phase

• The general formulation contains 2D phase unwrapping and temporal coherence as particular cases

• The computational needs can be reduced by solving first a 2D phase unwrap or a temporal coherence problem, the choice depending on temporal and spatial density of data

• A sub-optimal solution is possible that is already more stable and reliable than 2D phase unwrap or temporal coherence
Statistics of the differences between solutions with 41 and 21 interferograms

Sub-optimal sol. starting from temporal coherence:
max error 90% = 0.86 m, $\sigma = 5.6$ m

Only temporal coherence:
max 90% = 1.5 m, $\sigma = 6.7$ m
Statistics of the differences between solutions with 41 and 21 interferograms

Sub-optimal sol. starting from temporal coherence:
max error 90% = 0.75 mm/year, $\sigma = 3.1$ mm/year

Only temporal coherence:
max 90% = 0.8 mm/year, $\sigma = 3.7$ mm/year