IF ESTIMATION BASED ON WAVELET TRANSFORM

Zhang Yanjie   Veronique Prinet

Institute of Automation (IOA), Chinese Academy of Sciences (CAS)

P.O.Box 100080, Beijing, China

E-mail: yjzhang@nlpr.ia.ac.cn   Prinet@nlpr.ia.ac.cn

Abstract

For InSAR processing, the precision of the interferometric phase is the key problem for all kinds of its applications. While it is badly affected by many decorrelation factors such as thermal noise, atmospheric effect, the imaging geometry, the different Doppler Centroid, the processing and so on, all of them add the difficulty to do phase-unwrapping. Nowadays phase-unwrapping based on Instantaneous Frequency (IF) estimation has gained more attention [9][10]. IF can not only be used to do phase-unwrapping, but also can be used to do the interferogram filtering. How to estimate the unbiased IF from a low SNR interferogram is very important and challenging. Here the multi-scale analysis advantage of the WT is used to estimate IF. From the theory of WT, the IF of a signal can be extracted from the ridges of its WT. Beside, the WT can be an efficient tool to analyze multi-component signals. In this work the Morlet wavelet is used.

1. Introduction

Because the space-borne SAR can acquire high resolution images all the time with little affect to the climate and cloud, interferometric SAR (InSAR) has been widely applied to create the digital elevation model (DEM); specially, using the differential interferometric SAR (DInSAR), people can detect subtle change of the surface with a precision of cm-mm. Because the coherence of the SAR images is affected by many factors such as the imaging geometry, the different Doppler Centroid and so on, and the quality of the interferogram is decided by the coherence, phase-unwrapping has to face the problem of speckles due to the decorrelation. Up to now there are many people work on phase-unwrapping and have proposed a lot of methods. Usually phase-unwrapping is composed of two steps: (1) phase gradient estimation of the interferogram, (2) the integration of the estimated phase gradients to obtain an unwrapped phase surface [1]. In fact all kinds of phase-unwrapping methods are related with IF estimation. Discrete phase gradients calculated directly from the noisy data will lead to biased results. The retrieval of an unbiased IF from a low SNR interferogram is still an open issue. The classical IF estimation method is Hilbert Transform (HT). HT based IF estimation is sensitive to noise. Before doing phase unwrapping, the interferograms must be filtered. It has been proved that the local frequency of the interferogram is related to the local terrain slope of the scene, so it can be used to smooth the interferogram and reduce the phase noise by slope-adaptive filtering. In [4], the authors provide a building extraction method by filtering layover areas where the spectral shift can be partially used to separate layover and non-layover areas in high-resolution IFSAR. The estimated local frequency makes phase unwrapping feasible even for very noisy interferograms [3]. There are many methods that can be used to do IF estimation. But how to estimate the IF in a noisy (low SNR) environment is very import. In [2], the authors showed the estimation method using Gabor Transformation. Because the size of the window of the Gabor Transformation is fixed, it can't reflect the fast changing local frequency, while by using the Wavelet Transformation, it can be changed by different scale that is called multi-resolution analysis. Paper [11] has introduced the IF extraction method via WT, in that paper the main idea is according to the Theorem 1 we can calculate the analytic form of a specified mother wavelet. As to the IF estimation, it is the same as that of the HT. In this paper, the IF estimation based Wavelet Transform (WT) is applied. The ridges of Wavelet Transformation can be used to extract the instantaneous characteristic precisely. The experiments show its efficiency.

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2. Methodology

2.1 Introduction of the IF

The Hilbert Transform (HT) of a signal $s(t)$ is defined as:

$$H[s(t)] = \hat{s}(t) = \frac{1}{\pi}\int \frac{s(t^{'})}{t-t^{'}} dt^{'} \tag{1}$$

The analytic signal corresponded with $s(t)$ is defined as:

$$\mathrm{A}[s] = s(t) + \frac{j}{\pi}\int \frac{s(t^{'})}{t-t^{'}} dt^{'} \tag{2}$$

Because the analytic signal is always complex, it can be expressed as:

$$\mathrm{A}[s(t)] = A(t)e^{j\phi(t)} \tag{3}$$

The instantaneous frequency of the signal $s(t)$ is defined as:

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{4}$$

Given a wavelet mother function $\psi(t)$, its central time and time spread around the central time is defined as:

$$< t >= \int t\,|\,\psi(t)\,|^2 dt$$

$$T^2 = \int (t- < t >)^2\,|\,\psi(t)\,|^2 = < t^2 > - < t >^2 \tag{5}$$

Similarly, the central frequency and its frequency bandwidth is defined as:

$$< \omega >= \int \omega\,|\,\Psi(\omega)\,|^2 d\omega$$

$$B^2 = \int (\omega- < \omega >)^2\,|\,\Psi(\omega)\,|^2 d\omega = < \omega^2 > - < \omega >^2 \tag{6}$$

According to the Heisenberg's Uncertainty Principle:

$$T^2 W^2 \geq \frac{1}{4} \tag{7}$$

When $s(t) = e^{-ct^2/2}, c \in R$, the above inequality gets the minimum.

Theorem 1: If $s(t)$ is an arbitrarily given signal and $g(t)$ is an analytic signal, then the convolution of $s(t)$ and $g(t)$ which is $\int \mathrm{A}[g(t^{'})]s(t-t^{'})dt^{'}$ is also an analytic signal.

2.2 Morlet Wavelet

The analytic expression of Morlet wavelet in time domain is $\psi(t)=\exp(i\omega_0 t)\exp(-t^2/2\sigma^2)$, its expression in frequency is $\Psi(\omega)=\sqrt{2\pi}\sigma\exp[-\frac{1}{2}(\omega-\omega_0)^2\sigma^2]$

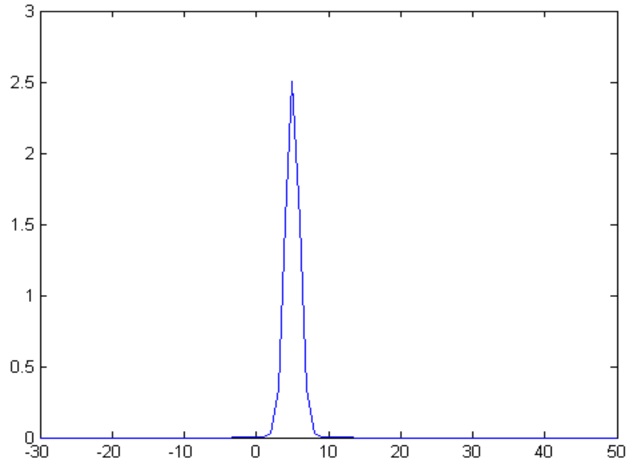

Fig.1 the Morlet wavelet waveform in frequency domain at $\omega_0=5$

The advantage of choosing Morlet wavelet is: it has analytic expression in both time and frequency domain, even though it is infinite support but its envelope decrease rapidly away from the origin, especially for $\omega_0>5$, the admissibility condition $\int_{-\infty}^{\infty}\psi(t)dt=0$ is satisfied.

2.3 Wavelet Transform

For a given wavelet function $\psi_{a,b}(t)=\frac{1}{a}\psi(\frac{t-b}{a})$, in the time domain, the wavelet transform of a given signal $f(t)$ is defined as:

$$W(a,b)=< f(t),\psi_{a,b}(t)>=\frac{1}{a}\int f(t)\psi(\frac{b-t}{a})dt \tag{8}$$

It is easy to know that in the frequency domain, the wavelet transform can be written as:

$$W(a,b)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(w)\Psi(aw)e^{-jbw}dw \tag{9}$$

where $F(w)$ and $\Psi(w)$ are the Fourier transform of $f(t)$ and $\psi(t)$ respectively.

2.4 WT based IF estimation

For an analytic wavelet which can usually be expressed as $\psi(t)=g(t)\exp(i\eta t)$, $g(t)$ is a symmetric window with a support equal to $[-1/2,1/2]$ and a unit norm $\|g\|=1$, the sinusoidal wave is used to shift the center frequency of the window so the spectrum of the wavelet will have no negative frequency. Then for a given signal

$f(t) = a(t)\cos\phi(t)$, its WT is:

$$WT_f(s,u) = \frac{\sqrt{s}}{2} a(u)\exp[i\phi(u)](G(s[\xi - \phi^{'}(u)]) + \varepsilon(u,\xi)) \qquad (10)$$

The corrective term $\varepsilon(u,\xi)$ is negligible if $a(t)$ and $\phi^{'}(t)$ have small variations over the support of $\psi_{s,u}$ and if

$\phi^{'}(u) \geq \Delta w / s$

2.5 Computation of WT via FFT

$$WT_f(a,b) = \int_{-\infty}^{\infty} f(t)\psi^{*}_{(a,b)}(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\Psi^{*}(a\omega)e^{jwb}d\omega = \frac{1}{2\pi} FT^{-1}[F(\omega)\Psi^{*}(a\omega)] \quad (11)$$

2.6 IF estimation of a signal in a noisy environment

For a noise-contaminated signal, its WT will also be contaminated with noise. But from the knowledge of the performance of the noise in WT, the SNR will be the largest in the ridge. So IF estimation from the modulus of the signal's WT will be more reliable compared with the IF estimated directly from the differentiation of the phase. The selection of the scales used to do WT is decided by the signal's sampling frequency and the time domain support length of the wavelet, so WT can be used to process the signal densely and the filters will have the high response at the signal's frequency. For those ridges that have small amplitudes can be considered as the noise and should be removed.

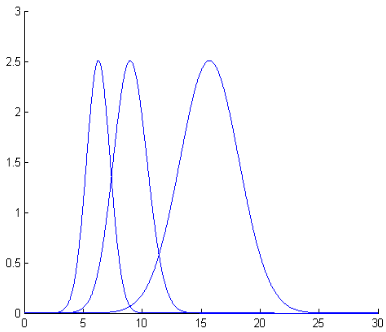

Fig.2 Fourier transform of the Morlet wavelet at three different scales

3 Experiments and Conclusions

In the experiments a chirp signal in four kinds of conditions are considered, they are (1) Signal with no noise (2) Small length signal with weak noise background (3)Small length signal with strong noise (4)Large length signal with strong noise. From the results we can see that the method works well in a weaker noise environment even if the length of the signal is small; in a stronger noise environment, the length of the signal should be larger to get a better estimation. In the future work, the signal reconstruction and interferogram phase-unwrapping will be done using the estimated IF.

4. Acknowledgement

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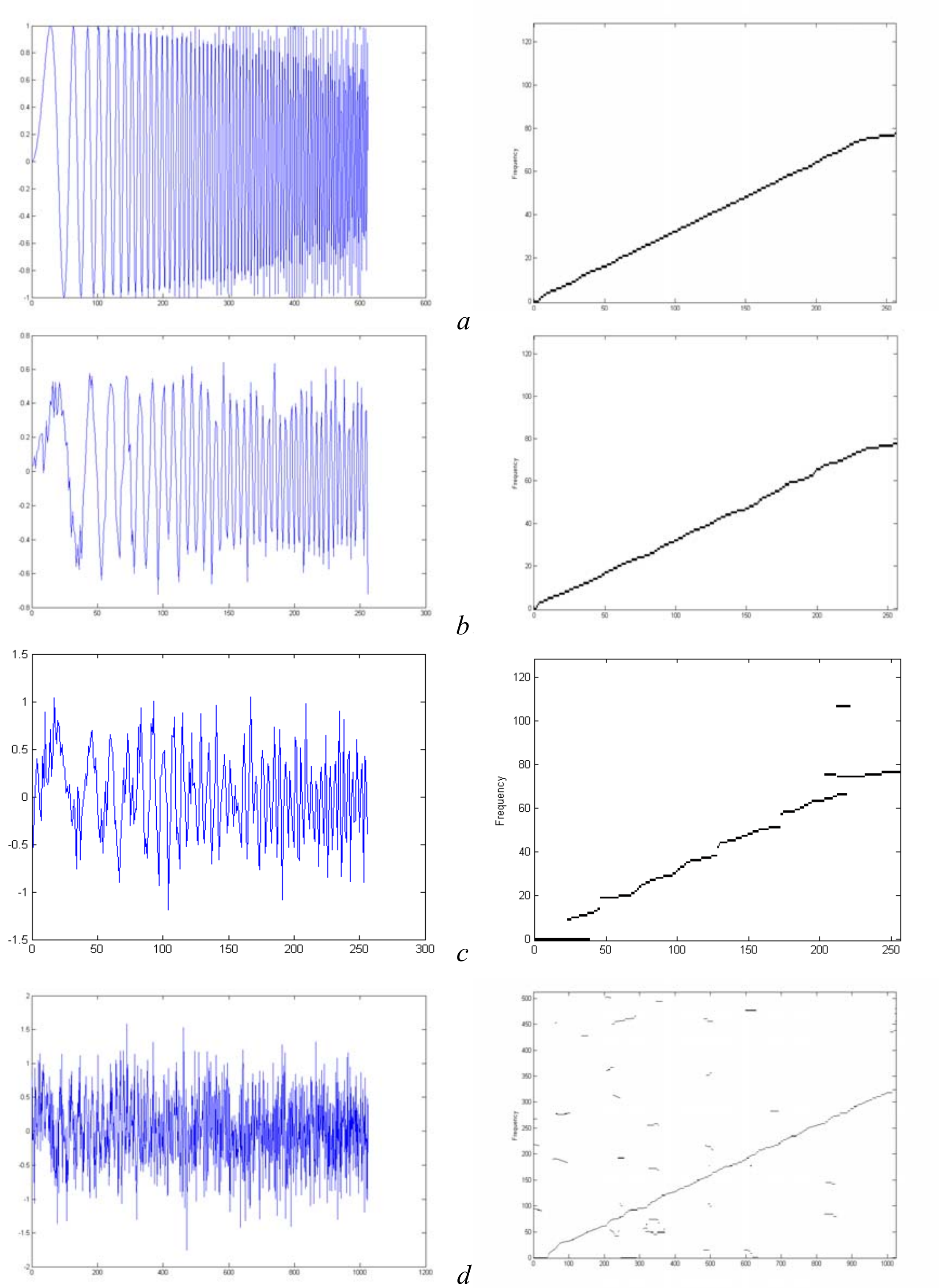

Fig 3. Experiments: Left: original signal, Right: Estimated IF, a) signal without noise, b) short signal with weak noise, c) short signal with strong noise, d) long signal with strong noise

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