Modeling of Atmospheric Effects on InSAR Measurements
With the Method of Stochastic Simulation

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ABSTRACT

The method of stochastic simulation is proposed to model the atmospheric effect on InSAR measurements based on sample data. Test results show that 37.44% reduction in the standard devisation of the atmospheric errors can be achieved with the method of stochastic simulation, compared to 25.69% with the method of Kriging interpolator. The relative improvement of the former over the latter amounts to 45.72%. The results suggest that the method of stochastic simulation is more advantageous over the traditional interpolators such as the Kriging method in modeling the atmospheric effects on InSAR measurements.

1 INTRODUCTION

Atmospheric effect is an important error source in InSAR measurements (e.g., [1]). The approaches that have been proposed for mitigating the atmospheric effect include, (1) selecting SAR image pairs acquired under favorable atmospheric conditions, (2) averaging SAR interferograms, and (3) calibration with external data sources. The first two methods are not preferred as some images may be wasted or the temporal resolution of measurements is reduced. As for the calibratory method, since in general the spatial resolution of the external data is lower than that of the SAR images, interpolation of the data to the grid of the SAR image space is required. The quality of the calculated atmospheric effects directly depends on the quality of the interpolator [2].

Among the several interpolators suggested, the Kriging interpolator has been used most commonly (e.g., [2])., The Kriging interpolator however only provides a local best estimate, but not the global best [3, 4]. The method often overestimates small values while underestimates large values (the so-called smoothing effect). More importantly, the method does not reflect the spatial variability of sampled data as modeled by the covariance or the semivariogram. The method of stochastic simulation on the other hand provides global best estimate and reproduce the spatial variability and statistics of the sampled data evenly over a study area.

The method of stochastic simulation will be investigated in this work to model the atmospheric effects on InSAR measurements. We will first provide some brief background on atmospheric effects on repeat-pass InSAR. The limitations of the Kriging interpolator will then be analyzed. The concept and algorithms of stochastic simulation will be introduced next.. Experimental studies will finally be conducted to evaluate the effectiveness of the method of stochastic simulation.

2 ATMOSPHERIC EFFECTS ON REPEAT-PASS INSAR

The general InSAR formulae for geophysical applications is

$$f - f_0 \approx \frac{4p}{l}\frac{B^{\perp}}{r \sin q} h + \frac{4p}{l}\Delta r \tag{1}$$

Proc. of FRINGE 2003 Workshop, Frascati, Italy,
1 – 5 December 2003 (ESA SP-550, June 2004) 16_li

where f_0 is the flat earth phase; $B^{\perp}$ is the perpendicular baseline; h is the topographic height and Δr is the possible ground deformation along the line-of-sight (LOS). When assuming no atmospheric propagation delay, the interferometric phase is

$$f = \frac{4p}{l} r_1 - \frac{4p}{l} r_2 = \frac{4p}{l}(r_1 - r_2) \tag{2}$$

However, when the atmospheric propagation delays Δr_1, Δr_2 are considered, the interferometric phase becomes

$$f = \frac{4p}{l}(r_1 - \Delta r_1) - \frac{4p}{l}(r_2 - \Delta r_2) = \frac{4p}{l}(r_1 - r_2) + \frac{4p}{l}(\Delta r_1 - \Delta r_2) \tag{3}$$

Obviously, an additional item $\frac{4p}{l}(r_1 - r_2)$ is introduced into the interferometric phase due to the atmospheric effect.

3 STOCHASTIC SIMULATION

3.1 Limitations of Kriging interpolator

Kriging is a class of linear estimators, traditionally obtained by minimizing the local error variance. Take Simple Kriging (SK) as an example

$$Z^*_{SK}(\mathbf{u}) = \sum_{b=1}^{n} l_b(\mathbf{u}) Z(\mathbf{u}_b) \tag{4}$$

where $Z^*_{SK}(\mathbf{u})$ is the Simple Kriging estimator for point u; $\{Z(\mathbf{u}_b), b = 1, \cdots, n\}$ are random variables and the sampled values $\{z(\mathbf{u}_b), b = 1, \cdots, n\}$ are one of their realizations.

The SK system is determined by

$$\sum_{1}^{n} l_b(\mathbf{u}) C(\mathbf{u}_a - \mathbf{u}_ß) = C(\mathbf{u}_a - \mathbf{u}), a = 1, \cdots, n \tag{5}$$

where $C(\mathbf{h}) = Cov\{Z(\mathbf{u}) - Z(\mathbf{u} + \mathbf{h})\}$ is the covariance model.

The error variance of SK is

$$s^2_{SK}(\mathbf{u}) = Var\{Z^*_{SK}(\mathbf{u}) - Z(\mathbf{u})\} = C(\mathbf{0}) - \sum_{1}^{2} l_a(\mathbf{u}) C(\mathbf{u}_a - \mathbf{u}) \tag{6}$$

The covariance of any two estimators, i.e., $Z^*_{SK}(\mathbf{u})$ and $Z^*_{SK}(\mathbf{u}')$ however does not reproduce the model value $C(\mathbf{u}-\mathbf{u}')$ [3]. For example, the autocovariance

$$Cov\{Z^*_K(\mathbf{u}), Z^*_K(\mathbf{u})\} = C(\mathbf{0}) - s^2_{SK}(\mathbf{u}) < Cov\{Z(\mathbf{u}), Z(\mathbf{u})\} = C(\mathbf{0}) \tag{7}$$

is smaller than the model variance $C(\mathbf{0})$. Therefore, a map estimated by the Kriging interpolator is always smoothed. This is the well-known smoothing effect. Besides, more importantly, when the value for a point, say point u, is estimated, the Kriging method does not take into consideration of the estimated values of any of the other points. It therefore does not accurately reproduce the spatial structure of the sampled data.

3.2 Stochastic simulation

Stochastic simulation is a class of methods designed to overcome the drawbacks of the Kriging estimator. It is realised by adding to each Kriging estimate or an estimated Kriging field an independently simulated error or an error field. There are three classes of commonly used stochastic simulation methods, sequential simulation, *P*-field simulation and simulation annealing [4]. Sequential simulation is the most straightforward and widely used simulation algorithm. Each variable is simulated sequentially according to its pre-determined conditional cumulative distribution function. The conditional cumulative distribution function is derived according to the Kriging paradigm based on the conditioning dataset, including all the original known data and those previously simulated within the predefined neighborhood of the location to be estimated [5]. The sequential simulation methods can be classified into either sequential Gaussian simulation or sequential indicator simulation, depending on the conditional cumulative distribution function used. The former is used in this work and its implementation follows the following procedure [4, 5]:

(1) Transform the original data into those of standard normal distribution with the normal score transform;
(2) Check the normality of the two-point distribution of the transformed data. If the bivariate normality does not hold, try other simulation algorithm, i.e., indicator algorithm or *p*-field simulation;
(3) If the multivariate Gaussian model holds, then
- Define a random path that include all the nodes of the grid.
- Use the SK to estimate the parameters of the Gaussian conditional cumulative distribution function at each node $\mathbf{u}$. Note that the conditional data set includes a specified number of the transformed data and the original known data;
- Draw a simulated value from the determined conditional cumulative distribution function;
- Add the simulated data to the conditional data set;
- Move to the next node, and repeat the process until all the nodes are simulated.

(4) Back-transform the simulated normal values into the original space.

In practice, multiple realizations are usually generated (just repeat Steps 3 and 4 with a different random path), and in the end a single representation is derived. A number of methods have been proposed to derive the single representation based on the multiple simulations, including the E-type estimate, median estimate, and quantile estimate. [4]. We will derive the final unique representation by averaging the two-side symmetric quantile values at each node,

$$z^*(\mathbf{u}) = (F^{-1}(\mathbf{u}; \frac{a}{2} \mid (n)) + F^{-1}(\mathbf{u}; (1-\frac{a}{2}) \mid (n)) \Big/ 2 \quad (8)$$

where $F(\mathbf{u}; b \mid (n)) = \Pr ob\{Z(\mathbf{u}) < b \mid (n)\}$ is the conditional cumulative distribution function and a takes either 0.05 or 0.1.

4 EXPERIMENT

A differential atmospheric field derived from an ERS tandem pair is used for the experimental study. The ERS tandem pair was acquired on March 18 and March 19, 1996, with track and frame being 404 and 3159, respectively. A reference DEM is used to remove the topographic phase in the interferogram. As the images have only a time interval of one day, it can be safely assumed that there is no surface deformation in the area between the SAR image acquisitions. The phase values in the derived interferogram can be considered caused by the differential atmospheric phases [6]. Fig. 1 shows the amplitude SAR image over Hong Kong and a hilly rectangular region of about *4.5km* × *9.0km* selected for the study. Fig. 2 illustrates the differential atmospheric phases of the selected region.

Fig. 1 SAR amplitude image over Hong Kong. A hilly region (in red rectangle) is selected for the research.

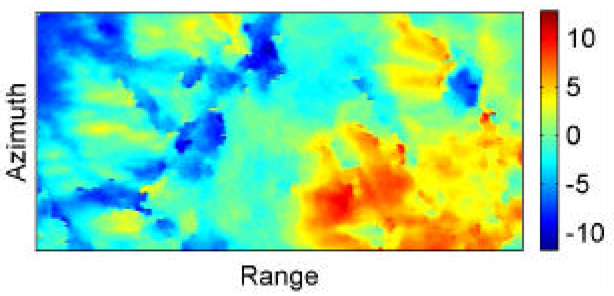


Fig. 2 Unwrapped interferometric phase of the region marked in Fig. 1 (with topographic phase removed) (unit: radians)

In the experiment, 50 points are sampled first from the atmospheric field and considered as known data points. The atmospheric field is then reconstructed from the 50 sampled points with both the stochastic simulation and the Kriging methods for comparison. The sequential Gaussian simulation and the ordinary Kriging (OK) method are used in the process. Fig. 3 shows the reconstructed atmospheric fields with OK and three different stochastic simulations. The results clearly show that the atmospheric field from the OK method is quite smooth, while the ones from stochastic simulations retain more details. Quantitative results also reveal that the spatial structure (semivariogram) is better reproduced by the method of stochastic simulation (see Fig. 4).

To make more reliable comparisons, ten groups of simulations are generated. In each of the groups, 200 realizations (atmospheric fields) are simulated and a final atmospheric field is obtained by taking the simple mean of the 200 simulations as discussed in Section 3.2. Statistics of the differences between the reference and the estimated atmospheric fields are given in Table 1.

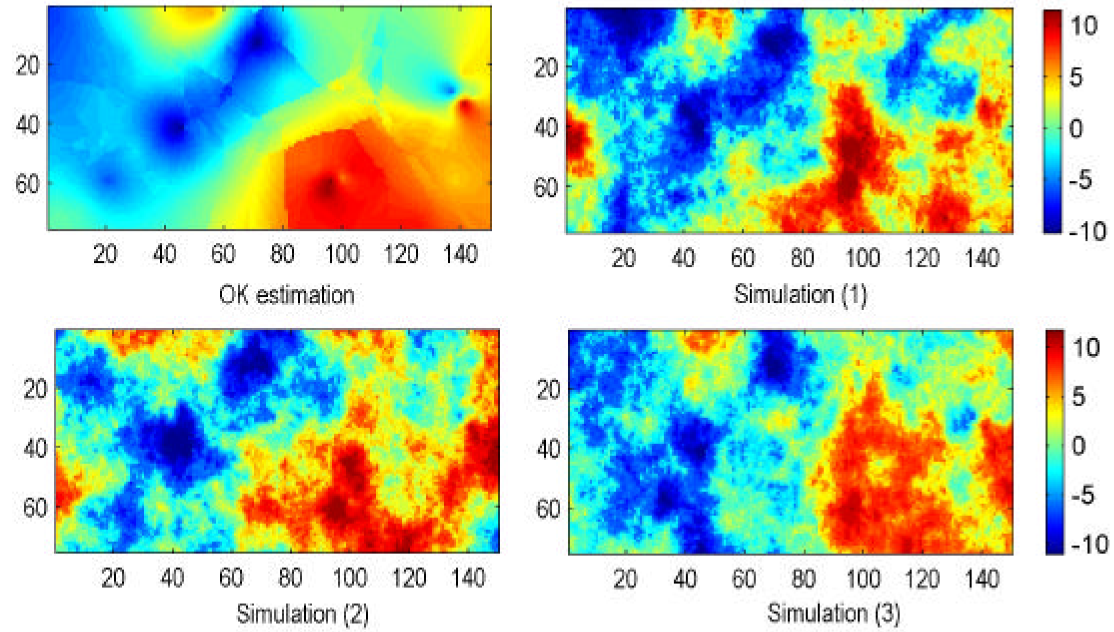
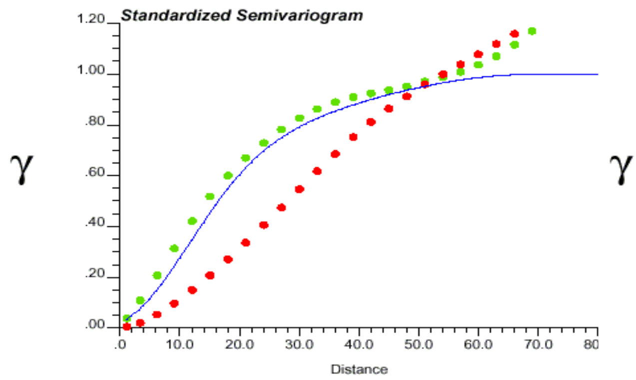
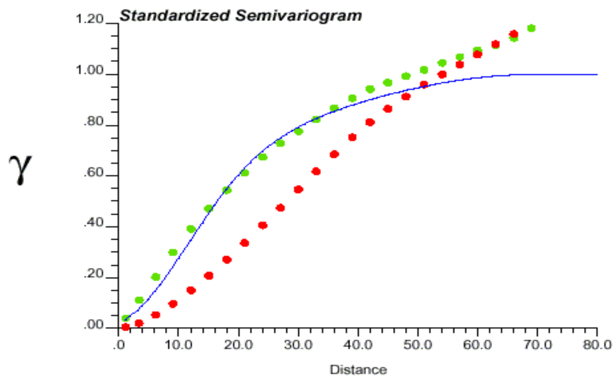


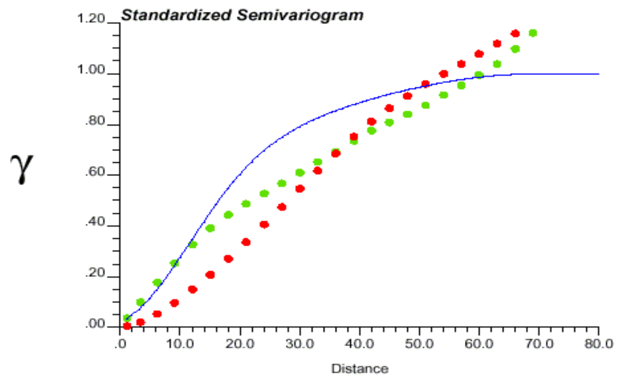
Fig. 3 Reconstructed atmospheric fields with OK and sequential Gaussian simulations



(a) Simulation (1)



(b) Simulation (2)



(c) Simulation (3)

Fig. 4 Comparison of semivariograms of reference model (solid line), OK estimation (red not) and stochastic simulations (green dot).

Table 1 Statistics of the differences between the reference and the derived atmospheric fields

	Mean (radians)	Standard deviation (radians)
1st Group of simulation	-0.890	2.409
2nd Group of simulation	-0.891	2.412
3rd Group of simulation	-0.867	2.388
4th Group of simulation	-0.800	2.374
5th Group of simulation	-0.835	2.413
6th Group of simulation	-0.780	2.368
7th Group of simulation	-0.879	2.382
8th Group of simulation	-0.831	2.378
9th Group of simulation	-0.845	2.364
10th Group of simulation	-0.780	2.417
Mean of the ten groups	-0.840	2.391
Ordinary Kriging	-0.636	2.840

It can be seen from the results given in Table 1 that the results from the stochastic simulations have relatively larger biases but smaller error standard deviations compared to results from the OK. It is considered that the method of stochastic simulation is more advantageous as the common biases in the atmospheric fields do not affect InSAR measurements. The standard deviation of the reference atmospheric field is about 3.822 radians. When the OK-derived atmospheric field is used to correct the interferogram, the standard deviation of the atmospheric errors becomes 2.840 radians, an improvement of 25.69%, while the standard deviation of the atmospheric errors becomes 2.391 radians on average, an improvement of 37.44%, when the atmospheric fileds from the stochastic simulation are used.

5 CONCLUSION

Mitigation of the atmospheric effects on InSAR measurements has long been a difficult problem. The calibrotory method for mitigating the atmospheric effect is strongly dependent on the effectiveness of the spatial interpolator used. The commonly used interpolators, e.g., the Kriging interpolator, however, have significant drawbacks that include the smoothing effects and the less accurate reproduction of the spatial structure of the reference data. The method of stochastic simulation has been proposed and tested to model the atmospheric fields based on isolated sample points. A differential atmospheric field derived from an ESA ERS tandem pair is used to verify the effectiveness of the method. The results have shown that the methods of stochastic simulation and Kriging can improve the results over the original reference data by 37.44% and 25.69%, respectively. The relative improvement of the stochastic simulation method over the Kriging method however amounts to 45.72%.

REFERENCES:

1. Zebker, H. A., et al., Atmospheric effects in interferometric synthetic aperture radar surface deformation and topographic maps, *Journal of Geophysical Research*, Vol. 102, No. B4, 7547-7563, 1997.
2. Williams S., et al., Integrated Satellite Interferometry: Tropospheric Noise, GPS Estimates and Implications for Interferometric Synthetic Aperture Radar Product, *Journal of Geophysical Research*, Vol. 103, No. B11, 27051-27067, 1998.
3. Journel, A. G., et al., Correcting the Smoothing Effect of Estimators: A Spectral Postprocessor, *Mathematical Geology*, Vol. 32, No. 7, 787-813, 2000.
4. Goovaerts, P., *Geostatistics for Natural Resources Evaluation*. Oxford University Press, New York, 1997
5. Deutsch, C.V. and Journel, A.G., *GSLIB: Geostatistical Software Library and User's Guide*. Oxford University Press, New York, 1998.
6. Hanssen, R.F., *Radar Interferometry: Data interpretation and Error Analysis*. Kluwer Academic Publishers, Dordrecht, 2001