Refinement of the Stochastic Model of GOCE Scientific Data and its Effect on the in-situ Gravity Field Solution

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- Jeff Ries
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- Jan Martin Brockmann
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Eduard Höck (AAS)
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Pail et al. (2010): GOCE gravity field model derived from Orbit and Gradiometry data applying the time-wise method. (Session 2.3.3)
Motivation - SGG-only solution

**Principles:**

- Extract as much signal from the gravity gradiometer as possible
- Solve for gradiometer only solution (SGG-only) to get unbiased estimators for the characteristics of the gradiometer measurements
- Find an appropriate stochastic model to describe the behavior of the gradiometer

**Target:** a consistent model !!!

agreement between internal and external error estimates

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Motivation - GOCE-only solution

**Principles:**
- tuning a final combined GOCE gravity field solution without using prior gravity fields
  \[\Rightarrow\text{purity requirements:}\]
  - gradiometry (GOCE)
  - kinematic orbits (GOCE GPS only)
  - smoothness conditions
    (polar gap, high degrees)

  \[\Rightarrow\text{GOCE only model}\]

**Target:** a consistent GOCE-only model with optimal data exploitation
(coefficients, variance/covariance matrix, normal equations)
Contents

Motivation
Signals
Filters
Models
Résumé
Motivation

Signals
- signal
- psd $V_{zz}$
- psd residuals
- GOCE residuals

Filters

Models

Résumé

Signals
Motivation
Signals
signal psd $V_{zz}$ psd residuals GOCE residuals
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GOCE - EGG-NOM_2 $V_{zz}$ signal

![Graph showing $V_{zz}$ signal with model and measurements compared to residuals.](image)

- $V_{zz}$ model
- $V_{zz}$ measurements
- residuals

GOCE - $V_{zz}$ power spectrum density

**Motivation**

- GOCE signal
- GOCE residuals

**Signals**

- $V_{zz}$ psd
- psd residuals
- GOCE residuals

**Filters**

**Models**

**Résumé**

- model
- measurements
- residuals

![Graph](image-url)
Motivation

Signals
signal
psd $V_{zz}$
psd residuals
GOCE residuals

Filters

Models

Résumé

GOCE - power spectrum density

Motivation

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Résumé
Correlated measurements:
- white noise in the measurement bandwidth (0.005 Hz - 0.1 Hz)
- $1/f$ characteristic with individual peaks outside the bandwidth

Problems with GOCE data:
- number of measurements (100.000.000)
- long correlation length (1/rev)
Correlated measurements:
- white noise in the measurement bandwidth (0.005 Hz - 0.1 Hz)
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Problems with GOCE data:
- number of measurements (100,000,000)
- long correlation length (1/rev)

Parametrization strategies:
- deterministic approaches
  - nuisance variables (empirical parameters, stochastic impulses)
  - short arc approach
- stochastic approaches
  - covariance functions
  - decorrelation by filtering
**GOCE signal - colored noise**

**Correlated measurements:**
- white noise in the measurement bandwidth (0.005 Hz - 0.1 Hz)
- $1/f$ characteristic with individual peaks outside the bandwidth

**Problems with GOCE data:**
- number of measurements (100,000,000)
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**Parametrization strategies:**
- deterministic approaches
  - nuisance variables (empirical parameters, stochastic impulses)
  - short arc approach
- stochastic approaches
  - covariance functions
  - decorrelation by filtering

**Strategy: tailored approach**
- time-wise approach
- in-situ measurements
- massive parallel computation
- discrete digital filters
- adaptive filter characteristic
Filters

decorrelation
assortment
filtered residuals
differences
variances
filter 9024
filter 9025
Résumé

Models

Résumé
Decorrelation

Motivation
Signals
Filters
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Models
Résumé

input: \( \ell, \Sigma \)

model: \[ Ax = \ell + v \]

principle: \[ v^T \Sigma^{-1} v \]

\[ \Sigma = R^T R \]

\[ \bar{\ell} = F \ell, \quad \bar{\Sigma} = I \]
\[ \bar{A} x = \bar{\ell} + \bar{v} \]
\[ \bar{v}^T \bar{v} \]

\[ (R^{-1})^T = F \]

\[ \bar{\ell} = (R^{-1})^T \ell \]
\[ \bar{A} = (R^{-1})^T A \]
\[ \bar{\Sigma} = (R^{-1})^T \Sigma R^{-1} = I \]
Filter assortment

square root of power spectral density $Z_Z$

residuals $[\text{mE/}\text{sqrt}(\text{Hz})]$

- **1000**: difference filter
- **9295**: bandpass filter
- **9024**: cascaded ARMA filter
- **9025**: cascaded notch-ARMA filter
Filtered residuals

9295:

1000:

9024:

9025:
Differences of coefficients (ref: ITG grace 2010s)

9295:

1000:

9024:

9025:
Standard deviation of the coefficients

9295:

9000:

9024:

9025:
Filter 9024: cascaded ARMA filter

- 2 cascades: warmup=1500 [sec]
- MA(2) ... difference filter
- ARMA(50,50) ... LS process estimation
- Low computational effort
- Low degree strips are not modeled in the covariances
- Efficient near the bandwidth

Filtered residuals

Square root of power spectral density Z

Residuals

Filtered residuals

Low computational effort

Low degree strips are not modeled in the covariances

Efficient near the bandwidth
Filter 9025: cascaded NOTCH-ARMA filter

26 cascades: warmup=200,000 [sec]
- MA(2) ... difference filter
- 24 * ARMA(2,2) ... notch filters
- ARMA(50,50) ...
  LS process estimation

- hugh computational effort
- low degree strips are modeled
Résumé - Filter

- filters are capable of decorrelating colored noise
- filters overcome noise/signal ratios up to 6 orders of magnitude
- filters can span a frequency selective metric
- filters work data adaptive
- filters are extreme flexible
- filters are easy to implement
- filters allow for a sequential approach
- filters are able to handle huge data sets
- filters are predisposed for parallel implementation
Models
Combination: SST + SGG

Degree variances (without near zonals): deviations from EIGEN-5C

SST: 2-20   transition zone: 20-55   SGG: 55-205
### Time-wise gravity field model - final configuration

<table>
<thead>
<tr>
<th>parameter</th>
<th>test configurations / data period: 31.10.2009 - 7.1.2010</th>
</tr>
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<tbody>
<tr>
<td>Filter</td>
<td>1000 9295 9036 9024 9025</td>
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<tr>
<td></td>
<td>some more during tuning of 9024 and 9025</td>
</tr>
<tr>
<td></td>
<td>Simple approximation, avoid long warmup</td>
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<tr>
<td>SST</td>
<td>PKI PKI + PCV</td>
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<tr>
<td></td>
<td>some more SST configurations</td>
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<tr>
<td></td>
<td>GOCE only model to avoid bias against a priori field (e.g. GRACE)</td>
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<tr>
<td>weighting</td>
<td>VCE</td>
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<td>iterative weights determined by VCE</td>
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<tr>
<td>regularization polar gap</td>
<td>Kaula spherical caps</td>
</tr>
<tr>
<td></td>
<td>better cut of behavior, no global smoothing</td>
</tr>
<tr>
<td>Kaula regularization high degrees</td>
<td>150 170 200 none</td>
</tr>
<tr>
<td></td>
<td>stabilization of high degrees, no degradation of well determined parameters</td>
</tr>
</tbody>
</table>
GOCE TIme-wise Model vs. ITGgrace2010s

**ITGgrace2010s**
- signal ITGgrace2010s
- variances ITGgrace2010s

**GOCE TIme-wise Model**
- deviations from ITGgrace2010s
- variances GOCE TIM

Degree variance vs. sh degree graph.
GOCE TI\text{me-wise} Model vs. ITGgrace2010s

20 - 120 : good agreement between deviations of the coefficients and the variances of GOCE TIM  \implies \text{GOCE TIM is consistent}
GOCE Ti\textit{me-wise} Model vs. ITGgrace2010s

20 - 120: good agreement between deviations of the coefficients and the variances of GOCE TIM  \[ \longrightarrow \text{GOCE TIM is consistent} \]

150 - 180: good agreement between deviations of the coefficients and the variances of ITGgrace2010s  \[ \longrightarrow \text{GOCE TIM is superior} \]
GOCE TI \textit{me-wise Model} vs. ITGgrace2010s

\begin{itemize}
  \item 20 - 120 : good agreement between deviations of the coefficients and the variances of GOCE TIM \quad \Rightarrow \text{GOCE TIM is consistent}
  \item 150 - 180 : good agreement between deviations of the coefficients and the variances of ITGgrace2010s \quad \Rightarrow \text{GOCE TIM is superior}
  \item 120 - 150 : transition zone \quad \Rightarrow \text{benefits from GOCE TIM}
\end{itemize}
GOCE TIme-wise Model vs. ITGgrace2010s

20 - 120: good agreement between deviations of the coefficients and the variances of GOCE TIM $\implies$ ITGgrace2010s is superior

150 - 180: good agreement between deviations of the coefficients and the variances of ITGgrace2010s $\implies$ ITGgrace2010s is consistent

120 - 150: transition zone
GOCE TIMe-wise Model vs. EIGEN05c

EIGEN05c

- - - signal ITGgrace2010s
- - - variances ITGgrace2010s

GOCE TIMe-wise Model

- - - deviations from EIGEN05c
- - - variances GOCE TIM

EIGEN05c

10^0 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0

0 50 100 150 200

degree variance vs. sh degree
20 - 80 : good agreement between deviations of the coefficients and the variances of GOCE TIM

⇒ GOCE TIM is consistent
GOCE TI\textit{me-wise Model} vs. EIGEN05c

20 - 80: good agreement between deviations of the coefficients and the variances of GOCE TIM

\[ \Rightarrow \text{GOCE TIM is consistent} \]

80 - 100: agreement between deviations of the coefficients and the variances of EIGEN5c

\[ \Rightarrow \text{GOCE TIM is superior} \]
GOCE Time-wise Model vs. EIGEN05c

- 20 - 80: good agreement between deviations of the coefficients and the variances of GOCE TIM => GOCE TIM is consistent
- 80 - 100: agreement between deviations of the coefficients and the variances of EIGEN5c => GOCE TIM is superior
- 100 - 220: ???
GOCE Time-wise Model vs. EIGEN05c

EIGEN05c

- - - - signal ITGgrace2010s

- - - - variances ITGgrace2010s

GOCE Time-wise Model

- - - - deviations from EIGEN05c

- - - - variances GOCE TIM

20 - 80: good agreement between deviations of the coefficients and the variances of GOCE TIM

⇒ GOCE TIM is consistent

80 - 100: agreement between deviations of the coefficients and the variances of EIGEN05c

⇒ GOCE TIM is superior

100 - 220: ???

⇒ let’s have a look to the spatial domain
deviations of gravity gradients in satellite attitude

ascending $V_{xx}^{GOCE} - V_{xx}^{EIGEN}$ range: [-5 5] mE

<table>
<thead>
<tr>
<th>Statistics:</th>
<th>xx [mE]</th>
<th>yy [mE]</th>
<th>zz [mE]</th>
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<td>max</td>
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<td>37</td>
<td>64</td>
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<td>3.0</td>
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<tr>
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<tr>
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deviations of gravity gradients in satellite attitude

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⇒ accuracy GOCE TIM:
ocean: 0.6 - 1.3 [mE]
land: ???
Résumé
Motivation
Signals
Filters
Models
Résumé

Gradiometry (SGG-part)
- in-situ processing of gravity gradients
- decorrelation with data-adaptive digital filters
- maximal exploitation of the signal
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**high-low satellite tracking (SST-part)**
- kinematic orbits
Résumé - GOCE TI\textsubscript{me-wise} Model

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Gradiometry (SGG-part)
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high-low satellite tracking (SST-part)
- kinematic orbits

smoothness conditions
- polar gaps
- high degrees (>170)
**Résumé - GOCE TI\textit{me-wise} M\textit{odel}**

- **Gradiometry (SGG-part)**
  - in-situ processing of gravity gradients
  - decorrelation with data-adaptive digital filters
  - maximal exploitation of the signal

- **high-low satellite tracking (SST-part)**
  - kinematic orbits

- **smoothness conditions**
  - polar gaps
  - high degrees ($>170$)

---

**GOCE TI\textit{me-wise} M\textit{odel}** is a **consistent GOCE-only model** with maximal data exploitation (coefficients, variance/covariance matrix)