Data Assimilation Practicals

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Exercise 1

Data Assimilation example and test code

- Pedagogical OI and 2D-var example code (FORTRAN90 with FORTRAN 77 included) for you to take home
- simple test runs with hands-on modifications
- control via file “task.nam”

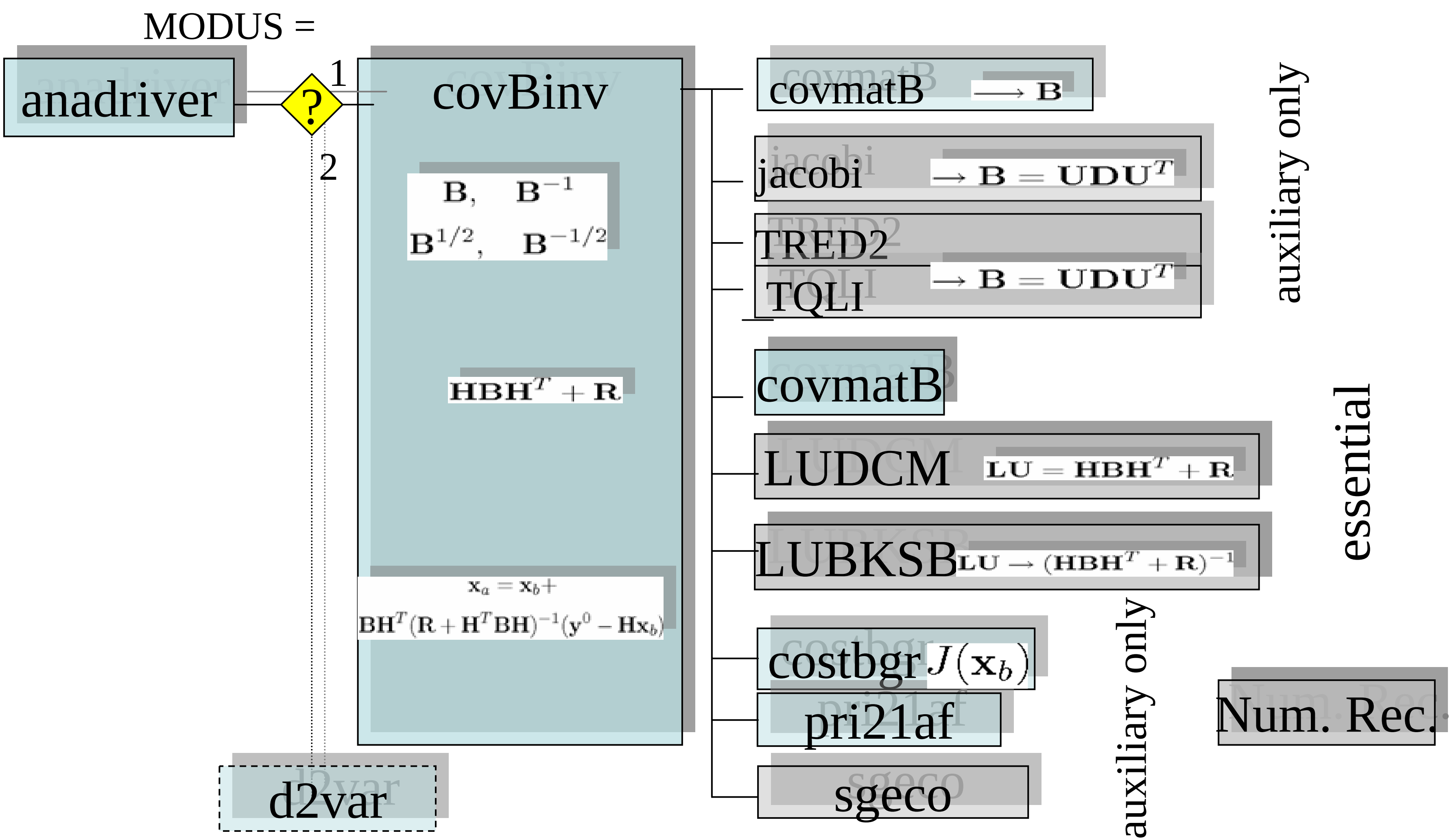
A basic 2D-data assimilation set-up

executable **anaobs.x** allows for performing data assimilation with OI and 2Dvar.

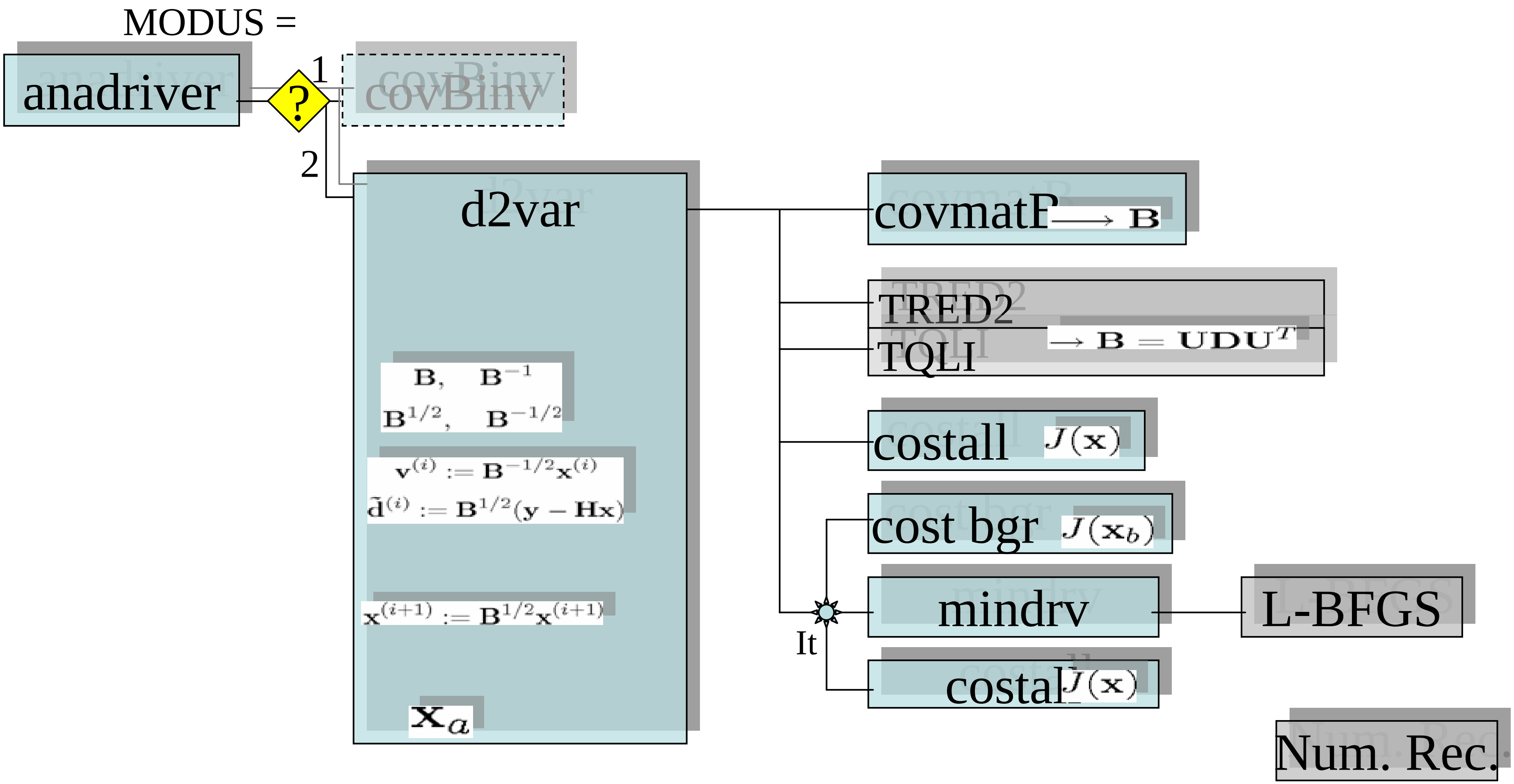
The control of anaobs is via the FORTRAN namelist file **task.nam**

&RADII Rx = 0.1, Ry = 0.1 /	*radii of influence in x and y direction*
&TASK MODUS = 2,	*=1: OI =2: 2D-var*
nobs = 2 /	*number of observations*
&DATA Obsabs = 5. 8. 10. 9. 10. 10. 1. 5. 8. ,	*observation values*
Orte = 5. 5. 10. 12. 14. 2. 6. 8. 10. 11. 11. 11. 9. 12. 10. 12. 11. 12. /	*grid point coordinates: $x_1, y_1, x_2, y_2, \ldots$*

optimal interpolation



2D-var


MODUS =
anadriver
?
1
covBinv
2
d2var
$\mathbf{B}, \quad \mathbf{B}^{-1}$
$\mathbf{B}^{1/2}, \quad \mathbf{B}^{-1/2}$
$\mathbf{v}^{(i)} := \mathbf{B}^{-1/2}\mathbf{x}^{(i)}$
$\tilde{\mathbf{d}}^{(i)} := \mathbf{B}^{1/2}(\mathbf{y} - \mathbf{H}\mathbf{x})$
$\mathbf{x}^{(i+1)} := \mathbf{B}^{1/2}\mathbf{x}^{(i+1)}$
$\mathbf{x}_a$
covmatP $\longrightarrow \mathbf{B}$
TRED2
TQLI
$\rightarrow \mathbf{B} = \mathbf{U}\mathbf{D}\mathbf{U}^T$
costall $J(\mathbf{x})$
cost bgr $J(\mathbf{x}_b)$
It
mindrv
L-BFGS
costall $J(\mathbf{x})$
Num. Rec.

Graphical support tool **batch_isoplot**

(processing of anaobs output)

src = <anaobsoutput>.af	*input file name (.af output file from anaobs)*
output = test.ps	*postscript outputfile*
min = 0.	*minimal value in color table*
max = 0.	*maximum value in color table*
3dflag = 0	*1: 3dimensional plot (surface,contour)* *!= 1: 2dimensional plot*
colort = 34	*IDL colour table*
colorkey = 1	*= 1: draw color code; != 1: do not draw*
n_anot = 11	*number of annotations on colorkey*
f_anot = (F5.1)	*format of annotations on colorkey*

Exercise 2

Generating and testing tangent linear and adjoint code

- Recall related slides of basic data assimilation lecture (replicated below)
- Take the discretised simple diffusion equation

$$u_i^{k+1} = u_i^k + c(u_{i+1}^k - 2u_i^k + u_{i-1}^k)$$

- **Construct its tangent linear and adjoint code**
- **Test the correctness of the tangent linear and adjoint code**

Constructing the adjoint from forward code

Program line

$$x_i^{k+1} = f(x_1^k, \cdots, x_i^k, \cdots, x_n^k)$$

tangent linear form

$$\delta x_i^{k+1} = \frac{\partial f}{\partial x_1^k}\delta x_1^k + \cdots + \frac{\partial f}{\partial x_i^k}\delta x_i^k + \cdots + \frac{\partial f}{\partial x_n^k}\delta x_n^k$$

blown–up equivalent notation as "transformation"

$$\begin{pmatrix} \delta x_1^k \\ \vdots \\ \delta x_i^k \\ \vdots \\ \delta x_n^k \\ \delta x_i^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 & 0 \\ & \ddots & & & & \vdots \\ & & 1 & & & \vdots \\ & & & \ddots & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 & 0 \\ \frac{\partial f}{\partial x_1^k} & \cdots & \frac{\partial f}{\partial x_i^k} & \cdots & \frac{\partial f}{\partial x_n^k} & 0 \end{pmatrix} \begin{pmatrix} \delta x_1^k \\ \vdots \\ \delta x_i^k \\ \vdots \\ \delta x_n^k \\ \delta x_i^{k+1} \end{pmatrix}$$

Constructing the adjoint from forward code

adjoint (transposition)

$$\begin{pmatrix} \lambda x_1^k \\ \vdots \\ \lambda x_i^k \\ \vdots \\ \lambda x_n^k \\ \lambda x_i^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & & & & \frac{\partial f}{\partial x_1^k} \\ 0 & \ddots & & & \vdots \\ \vdots & & 1 & & \frac{\partial f}{\partial x_i^k} \\ 0 & & & \ddots & \vdots \\ \vdots & & & 1 & \frac{\partial f}{\partial x_n^k} \\ 0 & & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda x_1^k \\ \vdots \\ \lambda x_i^k \\ \vdots \\ \lambda x_n^k \\ \lambda x_i^{k+1} \end{pmatrix}$$

Hence

$$\begin{aligned} \lambda x_1^k &= \lambda x_1^k + \frac{\partial f}{\partial x_1^k} \lambda x_i^{k+1} \\ &\vdots \\ \lambda x_i^k &= \lambda x_i^k + \frac{\partial f}{\partial x_i^k} \lambda x_i^{k+1} \\ &\vdots \\ \lambda x_n^k &= \lambda x_n^k + \frac{\partial f}{\partial x_n^k} \lambda x_i^{k+1} \\ \lambda x_n^{k+1} &= 0 \end{aligned}$$

Adjoint code verification

Let F be a coded function, the tangent–linear (TL) and adjoint (AJ) of which be under development. Test the correctness of both the TL and AJ code!

$$\mathbf{y} = F(\mathbf{x}) \quad F : \mathcal{R}^n \to \mathcal{R}^m, \qquad \mathbf{z} = D(\mathbf{y}) \quad D : \mathcal{R}^m \to \mathcal{R}$$

$$\delta \mathbf{z} = \left(\frac{\partial D}{\partial y}\right)\left(\frac{\partial F}{\partial x}\right)\delta \mathbf{x} = \langle \left(\frac{\partial D}{\partial y}\right)^T, \mathbf{H}\delta \mathbf{x}\rangle$$

$$= \langle \mathbf{H}^T \left(\frac{\partial D}{\partial y}\right)^T, \delta \mathbf{x}\rangle = \left(\left(\frac{\partial F}{\partial x}\right)^T \left(\frac{\partial D}{\partial y}\right)^T\right)^T \delta \mathbf{x}$$

Application of $\mathbf{H}$ and $\mathbf{H}^T$ means running the TL and AJ code, respectively. With suitably selected input $\frac{\partial D}{\partial y_i} = (0, \ldots, 0, 1, 0, \ldots, 0)$ and $\delta \mathbf{x}_i := (0, \ldots, 0, 1, 0, \ldots, x_n)^T$ the i^{th} row of $\mathbf{H}$ can be compared with the i^{th} column of $\mathbf{H}^T$. Further, the TL code linearized at input $\mathbf{x}$ be approximated by $\lim_{\delta x_i \to 0} \frac{1}{|\delta x_i|}\mathbf{H}((\mathbf{x} + \delta \mathbf{x}_i) - \mathbf{x})$, where $\delta \mathbf{x}_i = (0, \ldots, 0, \delta x_i, 0, \ldots, x_n)^T$.

direct form

$$u_i^{k+1} = u_i^k + c(u_{i+1}^k - 2u_i^k + u_{i-1}^k)$$

tangent linear form

$$\delta u_i^{k+1} = (1-2c)\delta u_i^k + c(\delta u_{i+1}^k + \delta u_{i-1}^k)$$

adjoint form

$$\lambda u_{i-1}^k := \lambda u_{i-1}^k + c\lambda u_i^{k+1}$$

$$\lambda u_i^k := \lambda u_i^k + (1-2c)\lambda u_i^{k+1}$$

$$\lambda u_{i+1}^k := \lambda u_{i+1}^k + c\lambda u_i^{k+1}$$

$$\lambda u_i^{k+1} = 0$$